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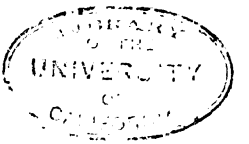

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVIII. NOVEMBER 8, 1907. NO. 1

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Harold A. H. Christie, M.A., Royal Observatory, Greenwich, S.E.;

Bertram Francis Eardley Keeling, The Observatory, Helwân, near Cairo, Egypt; and

Frederick Alexander Lindemann, Sidholm, Sidmouth, Devon,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Henry Boase Austin, J.P., Government Buildings, Bloemfontein, Orange River Colony, South Africa (proposed by R. T. A. Innes);

W. Geoffrey Duffield, Physical Laboratory, Manchester University (proposed by Arthur Schuster);

W. Earnshaw Etzel, B.S., Litt.L., M.S.M.F., Professor of Science, St Bernard's, Rochester, N.Y., U.S.A. (proposed by William R. Brooks);

John M. Field, 1 Hart Street, Edinburgh (proposed by William Peck);

James D. Maddrill, Ph.D., International Latitude Observatory, Ukiah, California, U.S.A. (proposed by H. C. Plummer);

William Henry Rees, B.Sc., Mathematical Master, County School, Pontypridd, 2 Graigeven Place, Pontypridd, South Wales (proposed by Richard Kerr);

Thomas James Forrester Smith, Newstead, Wavertree, near Liverpool (proposed by F. W. Longbottom); and

Captain James Weir, F.R.G.S., Examiner of Masters and Mates in Navigation and Seamanship to the Board of Trade, 5 Clive Crescent, Penarth, Glamorgan, South Wales (proposed by Albert Taylor).

One hundred and sixty-nine presents were announced as having been received since the last meeting, including, amongst others:—

W. W. Bryant, History of Astronomy; S. W. Burnham, General Catalogue of Double Stars (2 vols.); and Sir G. H. Darwin, Scientific Papers, vol. i., presented by the authors; Galileo, Opere, Edizio nazionale, vol. iii. part 2 and vol. xix., presented by the Italian Government; Lady Huggins, Memoir of Agnes and Ellen Clerke, presented by the author; Oxford Astrographic Catalogue, vol. iii.; Observatoire de Paris, Atlas Photographique de la lune (Loewy and Puiseux), fasc. 9; and Perth Observatory, Western Australia, Catalogue of 420 standard stars, presented by the Observatories; Sir N. Lockyer, Report of the Eclipse Expedition to Majorca, presented by the Solar Physics Committee.

Astrographic Chart; 134 charts (Algiers, Bordeaux, Paris, and San Fernando), presented by the French Government; 23 charts, presented by the Tacubaya Observatory, Mexico; and 76 charts, presented by the Royal Observatory, Greenwich.

Three transparencies of the moon from negatives of M. Puiseux, presented by Mr Knobel; 9 transparencies of the northern Milky Way, presented by Professor Max Wolf.

Medal to commemorate the Benjamin Franklin bicentenary (bronze), presented by the American Philosophical Society.

Spectroscopic Observations of Cyanogen in the Solar Atmosphere and in Interplanetary Space. By H. F. Newall, M.A., F.R.S.

The presence of cyanogen in the atmosphere of the Sun seems to be indicated by the distinct appearance of the cyanogen absorption bands at wave-length 3883 in the solar spectrum. But, so far as I am aware, no definite observations have been directed to settle the possible doubt as to whether the cyanogen is confined to the solar atmosphere. The possible alternatives are that it may be in the Earth's atmosphere, or in space between the Sun and the Earth.

[*Note.* Nov. 11.—Professor Dyson has kindly called my attention to evidence which had escaped my memory and is conclusive as to the presence of cyanogen in the chromosphere. Sir N. Lockyer found CN bands in the flash spectrum of the chromosphere photographed in India in 1898 (*Phil. Trans. R.S.*, vol. cxcvii. A., p. 202, and *Mem. R.A.S.*, vol. liv., App. p. [52]), and Professor Dyson himself found them in his observations in Sumatra in 1901 (*Phil. Trans.*, vol. ccvi. A., p. 438, and *Mem. R.A.S.* lvii., App. [36]). I have accordingly modified one or two of the statements in the paper.]

During the past summer I have made some observations in attempting to elucidate this point. The method adopted consisted in photographing side by side on one photographic plate two solar

spectra taken successively in the light coming from the east and from the west ends of the equatorial diameter of the Sun. Lines of truly solar origin should show relative displacement in the two spectra in consequence of the difference of velocity in the line of sight arising from the Sun's rotation. This is a form of the well-known method of Cornu for distinguishing between lines of solar and lines of telluric origin.

In the result I find that my photographs show displacement of the cyanogen bands similar to that exhibited by neighbouring lines of iron and other vapours. There are, however, also evidences in the photographs that there is a trace of superposed cyanogen bands exhibiting displacements other than those attributable to the Sun's rotation. Thus the evidence so far accumulated shows that there is cyanogen in the Sun's atmosphere rotating with the Sun, and there is also cyanogen between the Earth's surface and the Sun showing spectroscopic displacements that may be attributed to independent motion.

Before passing on to give the evidence collected, it may be well to give some considerations which may be held to justify the inquiry as to the seat of cyanogen absorption. I recognise the highly speculative nature of some of them, but I beg leave to set them forth, in company with the statement of several newly observed facts, which go some way towards justifying speculation.

One of the most marked features of the spectrum of comets is the bright bands of cyanogen at 3883 first discovered by Sir W. Huggins. It has generally been held that this incandescent cyanogen and the other carbon compounds evidenced by cometary spectra are emitted from the comet's head, driven out under the influence of the Sun's radiation. On this view the observed fact that periodic comets seldom display brilliant tails receives the explanation that each successive return of a comet to perihelion reduces the quantity of vapour in the possession of the comet's nucleus; such vapours, then, are set free in interplanetary space and seem to be spread out, mainly in the plane of the comet's orbit, under the action of repulsive forces. The old view that the freed vapours are swept up by the gravitative attraction of the planets and the Sun is now being replaced by the modern view that radiation-pressure drives the larger aggregates of molecules or ions outwards from the Sun. Schwarzschild's work has shown that things as small as molecules cannot, as such, be repelled by radiation-pressure. It is generally held that molecules are of linear dimensions of the order of 10^{-8} cms., whilst the wave-length of green light is of the order 5×10^{-5} cms. And Schwarzschild has shown that things smaller than a tenth of the wave-length of the radiation are more attracted by gravitation to the Sun than repelled by the pressure of the Sun's radiation.

But it is held that a great deal of the display of comets' tails is due to illuminated dust as is shown both by the spectrum and also by the polariscopic phenomena observed by Prazoumowski and others. Here again the dimensions of particles giving the polarisa-

tion effects afford some difficulties; but it might be held that the gases, which the spectroscope shows to be present in the tail, are carried out by the repelled dust. Arrhenius, however, as far as I understand his present views, is ready to regard the molecules of gases and vapours as grouped in aggregates large enough to bring them under the sway of radiation-pressure.

The work of Deslandres and Bernard on the spectrum of Daniel's comet 1907 *d* (*C.R.*, cxlv. 445) shows that bands of some gaseous substances were visible in the tail at 45 minutes of arc from the nucleus in the middle of August; and I learn from a letter from Mr. Evershed that, at Kodaikánal, he and Mrs. Evershed have succeeded in photographing the same comet with a prismatic camera, with the following results:—"The hydrocarbon bands extend a long way up into the tail, whilst the cyanogen bands near 3883, although the most brilliant radiations in the comet spectrum, are concentrated round the head."

Now, both in the case of the absorption bands of cyanogen seen in the solar spectrum and in the emission bands of the same vapour seen in the cometary spectrum, a peculiar feature is that the ultra-violet bands at 3883 are the most marked, and that the blue and violet bands are far more feebly seen than under certain conditions attainable in the laboratory (Liveing and Dewar, *Proc. R.S.*, xxx. 494). Some years ago I made a series of observations on the spectra of carbon compounds commonly present in vacuum tubes. My observations (unpublished) related chiefly to phenomena in electrodeless discharges brought about by Professor J. J. Thomson's electro-magnetic method (*Phil. Mag.*, 1891, vol. xxxii. p. 321, and *Proc. Roy. Inst.*, xiv. p. 243), wherein a tube or bulb of considerable diameter, ranging from about 12 mm. to 100 mm., is surrounded by a few coils of thick wire, the ends of which are connected with two Leyden jars or plate condensers. The discharge of such charged condensers is accompanied by exceedingly rapid electrical oscillations, which give rise to induced oscillations (accompanied by luminosity) in the rarefied gas in the enclosed tube. The method made it possible to produce brilliant luminosity in the gas at much lower pressures than those then attainable in ordinary vacuum tubes provided with electrodes. The pressures measured were between 0·60 mm. and 0·005 mm. of mercury.

I will refer here only to two of the results indicated by my observations. One was that as the pressure was diminished the bands of nitrogen and cyanogen, which I specially studied, became faint and disappeared at the red end of the spectrum, whilst those in the violet and ultra-violet were left distinct, or even intensified. I found no case which did not conform with the generalisation, that as pressure is diminished the maximum intensity of the banded spectra is pushed towards the shorter wave-lengths—the electrical conditions being kept constant. The observations of Hasselberg and of Deslandres support this view with respect to nitrogen; and it would not be difficult to devise a set of experiments to decide whether my observations can be reconciled with those of Nutting

(*Astroph. Jour.*, xx. 131), which, so far as they go on these lines, seem to indicate an opposite conclusion for the much higher pressures between 1 mm. and 21 mm. At any rate my own (laboratory) experiments have led me to the belief that in the solar spectrum the evidence points to the conclusion that cyanogen is at low pressure. It is obvious that the pressure must be low in the case of a comet's tail extending several thousands (if not millions) of miles from the nucleus.

The other observation to which I now wish to refer was that the spectra of hydrocarbons and nitrocarbons were far the most easily elicited in such tubes and bulbs, and that the processes involved in the production of luminosity in the vapours could be started under conditions which proved instability, in the sense that the gas was luminous in regions of the vacuum bulb where the electromotive intensity (E.M.F. per cm.) was zero.

This observation suggested an alternative to the usually accepted view of the origin of the cyanogen, etc., in cometary spectra, as follows:—Is it not possible that the hydrocarbons, nitrocarbons, etc., which seem to be evidenced by the spectra of all comets, are *always* present in circumsolar space, and rendered incandescent by some processes connected either with the motion of the solid parts (including dust) of the head of the comet through the vapours, or with the emission of some influence from the comet head? Are we to say that all comets, wherever they may come from in the universe, and whatever their main material may be, always bring with them the cyanogen and hydrocarbons which give them luminosity? Or is it not more rational to say that the spectra of all comets are approximately similar, because they always find the same vapours spread in their path as they approach the Sun, and can only elicit the spectra of these vapours?

If we adopt this latter view, then, apart from the phenomena of repulsion, we have only to explain the production of luminosity, and variations in the intensity of it. And out of the embarrassing wealth of suggestions connected with the development of the corpuscular theory of matter, there is no difficulty in picking one or two modes of influence to which incandescence might be attributed. I will not dwell on this aspect, but will call attention to another mode of producing luminosity—wherein there is no obvious sign of high temperature—I mean the luminosity of air when it rushes through a small crack in the glass walls of a vacuum tube, a phenomenon that must be familiar to most of those who have been engaged in experimental study of such tubes. Two or three times I have been able to recognise the bands of nitrogen in the spectrum of the glow in such a crack. Now, such luminosity cannot be peculiar to nitrogen. If it is legitimate to assume that hydrocarbons and cyanogen would behave similarly, it is not a long step to assume that it is as likely that luminosity will arise when small specks of cometary dust or solid particles are rushing through the gas (possibly in a sensitive state), as that luminosity should be produced when the gas rushes past the walls of a small crack into vacuum.

However the luminosity may be produced, the absence of brilliant display in periodic comets might, on the hypothesis of the constant existence of the vapours in interplanetary space, be attributed to the gradual change in the surface of the solid parts of the head of the comet, or to the exhaustion of the emitted influence.

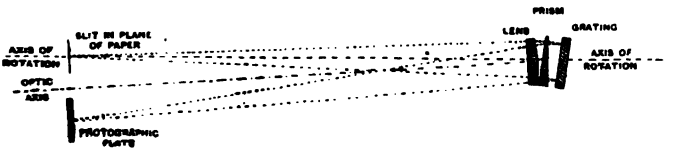
My present purpose is, however, not to elaborate views about comets' tails, but simply to point out that if one accepts radiation-pressure and the phenomena of the repulsion of comets' tails, one is driven to admit the existence of vapours and gases in circumsolar space, however difficult it may be to give account of their quantitative distribution in such space. The great extensions of the corona (as was pointed out by Huggins in his Bakerian Lecture, 1885), and the equally great extension of comets' tails, demand the admission that at any rate temporarily gases and vapours must exist in "free space." How long the clearing process lasts, which depends on the conflict between the pressure of radiation and the gravitative attraction of the Sun and the planets, is another matter.

It is difficult to devise crucial observations which would decide the points raised by these views. For it is to be expected that gases and vapours, if present at all, would be in such a turmoil of rotation round the Sun that the integrated effect of their absorption could hardly give defined lines or bands of absorption, except in the case of accidentally related motions in the vapours at different distances from the Sun. The best way to proceed seemed to be to make special observations of the cyanogen bands, and so learn whether there really are any peculiarities to be found in their behaviour. Professor Hale has pointed out (*Astroph. Jour.*, xxv. 310) that the cyanogen fluting at 3883 is very decidedly weakened at the limb of the Sun relatively to the centre of the disk. Humphreys has shown (*Astroph. Jour.*, xxvi. 28) that the lines of the cyanogen bands are not appreciably displaced in the spectrum of the electric arc, even at the highest pressures used by him (100 atmospheres).

The instrument used for the observations of the cyanogen bands is a new grating spectroscope made by the Cambridge Scientific Instrument Company, and recently mounted for solar observation at the Cambridge Observatory. It forms part of the installation which is in course of construction for solar work. The general form of the installation has been indicated in a previous paper communicated to the Society (*M.N.*, lxvii. p. 161.) The observatory is indebted for this equipment to the munificence of the late Frank McClean, F.R.S.

The spectroscope is of Littrow type, *i.e.* one and the same tube and object-glass serve both as collimator and camera, the slit and photographic plate being slightly separated in the focal plane of the common object-glass. The lens has an aperture of 4 inches [101 mm.], and is of focal length 14 feet [4267 mm.]. A Rowland

plane grating, with ruled surface $5 \times 3\frac{1}{2}$ inches [127×89 mm.] with 14,438 lines to the inch, is used to produce the spectrum. In order to allow of the use of spectra of high order without the troubles introduced by superposed spectra, a prism of small angle 5° and of circular aperture 4 inches [101 mm.] has been inserted, with its refracting edge perpendicular to the rulings of the grating between the grating and the object-glass. Both object-glass and prism are the work of Messrs Cooke & Sons of York, and are of excellent performance. Provision is made for tilting the spindle and frame, which hold the grating, about an axis parallel to the refracting edge of the prism; in this way any part of any spectrum is brought to the centre of the photographic plate, and the dispersion is such that the ordinarily superposed spectrum is thrown above or below. Then, on turning the grating about its axis of rotation, the succession of appearances in the field of view is such that the blue end of the spectrum is seen at the bottom of the field of view; as we turn towards the green, the spectrum mounts to the middle of the field; and on turning further, the red is to be seen at the top of the field. By the tilting of the spindle



on which the grating turns, any part of the spectrum can be brought to the middle of the field.

The instrument, which is of very rigid construction and weighs about 6 hundredweight (300 kilogrammes), has been mounted horizontally on roller bearings, which allow of easy rotation of the whole about a horizontal axis, so that the slit can be turned into any position angle. A position circle is provided, divided to half degrees. The axis on which the instrument turns coincides with the line joining the middle point of the slit to the middle point of the object-glass. In this way a horizontal pencil of rays incident from the proper direction on the centre of the slit always passes through the centre of the collimator-camera lens, whatever may be the position angle of the slit. The true optic axis passes through the focal plane at a point midway between the middle point of the slit and its image; this point thus describes a small circle round the middle of the slit as the whole instrument is rotated on its roller bearings. The middle point of the photographic plate similarly describes a concentric circle of twice the radius.

Black opaque diaphragms of the proper size have been inserted in the proper places between the slit and the object-glass, in order to prevent the light reflected from the surface of the lenses from falling on the spectrum on the photographic plate.

A convenient device has been added to limit the length of the slit used, and to allow of the juxtaposition of spectra in successive exposures on one and the same photographic plate; thus a spectrum of the east limb of the Sun can be photographed between two spectra of the west limb, or two spectra of the east limb can be alternated with two spectra of the west limb.

In the preliminary work with the instrument great care had to be exercised in overcoming the erratic performance of the grating; and in order to secure the defining power required for this work it was found necessary to limit the area of the ruled surface by means of diaphragms carefully placed. When all the ruled surface except a strip 5 inches wide and about $1\frac{3}{4}$ inches high was blocked out, it was found that excellent definition could be attained with full resolving power; and the exposures needed for photographing the cyanogen band at 3883 in the spectrum of the third order seldom exceeded a minute, and in a bright sky an exposure of 30 seconds was ample.

The photographs obtained with the instrument exhibit the spectrum with very open scale, and many of the close double lines even in the cyanogen band are well seen. The linear dispersion of the negatives is such that we have about 1·29 tenthmetres per millimetre. And it should be remembered that the total exposure of a plate in the instrument was completed generally in less than 3 minutes.

Examination of the photographs obtained disclosed peculiar features. It was at once evident that the spectrum of the east limb was nearly always less well defined than the spectrum of the west limb over a considerable region of the spectrum near the cyanogen bands. The photographs dealt with may be divided into two classes: firstly, about 35 "adjustment photographs" taken for finding the best adjustment of the instrument and for testing the effects of some exaggerated maladjustments; and secondly, about 10 "rotation photographs," each exhibiting two spectra of the east limb alternating with two spectra of the west limb, all taken with approved adjustment of the instrument.

A great deal of care and time have been devoted to the critical study of the evidence; and though at first I was led to suspect that, in spite of great care in adjusting the instrument, and in making sure that the optical beam was arranged exactly similarly both for east limb and for west limb, some unexpected error had crept in, I am now convinced that the right interpretation of the peculiar effects lies in accepting them as real, and not instrumental.

The spectrum of the east limb was in nearly every case less clearly defined than that of the west limb. Now, in a malfocussed photograph, one can, from the appearance presented by a single isolated line, predict with considerable certainty what the appearance of various groups of lines will be. But the appearances presented by the spectra of the east limb do not agree with prediction based on study of isolated lines or edges. In particular, the cyanogen band itself at the east limb shows in general a double edge, as if

two similar bands are superposed, with slight relative displacements which vary from day to day. The cyanogen band at the west limb shows in general a single slightly intensified edge, as if there might be two bands superposed, with such slight displacement as not to be measurable. There are many other minute differences between corresponding lines in the spectra of the east and west limbs.

Three of the "rotation photographs" were picked out as showing the best definition: namely

L. 32 1907 Sept. 15, 10.23 a.m. G.M.T (civil).
L. 41 „ „ 11.54 „ „
L. 42 „ „ 12.20 „ „

These have been measured by my assistant, W. H. Manning, under my direction. Six iron lines, two chromium lines, and five cyanogen lines were chosen for measurement between wave-lengths 3871 and 3898. Each plate was measured twice in reversed positions on the Zeiss Comparator. The values of the displacements (δR), East–West, are given for each line in the following table; the values are expressed in millimetres. Four of the five cyanogen lines are really close double pairs, which were bisected as single lines. They are indicated in the table.

Wave-length.	Origin.	L. 32.	L. 41.	L. 42.	Mean.	
3871·963 ?	Fe	0·0330	0·0344	0·0304	0·0326	Fe
73·065	CN	347	297	397	347	CN
73·903	Fe	336	316	312	321	Fe
76·815	Fe	363	342	355	353	Fe
82·{224/309}	CN	329	317	334	327	CN
82·{439/530}	CN	322	368	359	349	CN
82·{650/733}	CN	275	345	280	300	CN
82·{828/893}	CN	287	344	284	305	CN
83·778	Cr	255	267	301	274	Cr
85·657	Fe	299	326	306	310	Fe
86·942	Cr	323	338	342	334	Cr
90·986	Fe	321	343	330	331	Fe
3897·596	Fe	301	321	337	320	Fe
				Mean	0·0323	

It will be seen that

		mm.
6 iron	lines give a mean displacement, E.–W.,	0·0327
2 chromium	„ „ „	·0304
5 cyanogen	„ „ „	·0326

It thus appears that the cyanogen lines give displacements (and consequently also a solar rotation-period) sensibly the same as those given by iron lines of solar origin. These displacements of lines relate to spectra taken from solar light gathered at the east and west ends of the solar equator, within about 5° of solar latitude from the equator, at points situated within the limb by about $\frac{1}{20}$ of the radius of the Sun's apparent disc. I do not put them forward as satisfactory determinations of the equatorial velocity of the Sun.

To deduce the velocities corresponding to the displacements, we must use a factor, $L/\lambda \cdot d\lambda/dR$, which works out to be sensibly 100. For the velocity per tenthmetre (L/λ) is

77·49 km/sec at wave-length 3872

and 76·96 „ „ 3898

whilst the linear dispersion is such that $d\lambda/dR$ is equal to

1·297 tenthmetres per millimetre at 3872,

and to 1·299 „ „ „ 3898.

Thus the factor ranges from 100·6 to 99·8, and for our present purposes we may take its value as 100 over the range of spectrum measured.

Hence the displacements expressed as velocities are

for the 6 iron lines, 3·27 km/sec

2 chromium „ 3·04

5 cyanogen „ 3·26.

The corresponding equatorial velocities are got by halving these numbers. It is seen that the uncorrected numbers are a little lower than Dunér's and Halm's.

The main point, however, is that for these three determinations cyanogen is shown to be in the solar atmosphere in the same sense that iron and chromium are.

The wave-lengths of the cyanogen lines used in this determination have been also deduced with reference to three iron lines as standards. They are found to differ from Rowland's values by less than one hundreth of a tenthmetre, as will be seen from the accompanying table.

Wave-length (Rowland).	Adjustment Plate L. 20.	Rotation Plates.			Mean.	Origin.
		L. 32.	L. 41.	L. 42.		
3871·963 ?	3871·887	·881	·887	·881	·884	?
73·073	73·072	·069	·073	·069	·071	CN
73·903	Standard	...	...	...	(·903)	Fe
76·815	76·813	·805	·812	·813	·811	Fe
82·$\left\{\begin{matrix}224\\309\end{matrix}\right\}$(266)	82·272	·266	·271	·274	·271	CN
82·$\left\{\begin{matrix}439\\530\end{matrix}\right\}$(485)	82·483	·472	·479	·483	·479	CN

Wave-length (Rowland).	Adjustment Plate L. 20.	Rotation Plates. L. 32.	L. 41.	L. 42.	Mean.	Origin.
82·{650 733}(691)	82·682	·673	·677	·679	·678	CN
82·{828 893}(860)	82·856	·850	·856	·853	·854	CN
83778	83·782	·774	·774	·772	·775	Cr
85·657	Standard	...	...	...	(·657)	Fe
86·942	86·945	·949	·949	·951	·948	Cr
90·986	90·989	·984	·983	·988	·986	Fe
97·596	Standard	...	...	...	(·596)	Fe

The apparent duplication of the edge or head of the CN band is, as has already been stated, more marked at the east limb than at the west. It is much more difficult to measure the displacement of edges than the displacement of lines in the band. Nevertheless I have made some measurements of the displacement; and if these are interpreted as indicating a Doppler effect, the results may be summarised as follows, the velocity of the east limb always being towards Earth.

Plate.	Date. 1907. h m	East Limb.	West Limb.
L. 6	Aug. 17, 5.27 p.m.	Spectra similar. No measurable displacement of edges.	
L. 8	Sept. 6, 10.50 a.m.	Velocity 16 km/sec	Displacement very small.
L. 10	Sept. 8, 10 ,,	,, 14 km/sec	,, ,,
L. 14	,, 3.55 p.m.	,, 12 km/sec	,, ,,
L. 32	Sept. 15, 10.23 a.m.	,, ?4 km/sec	,, ,,
L. 41	,, 11.54 ,,	,, ?5 km/sec	,, ,,
L. 42	,, 12.20 p.m.	,, ?5 km/sec	,, ,,

This progressive change of the duplication of the edge is seen to go on for 9 days, and so can hardly be connected with ordinary disturbances of a local nature on the Sun's surface. The cyanogen involved cannot be far from the Sun, for otherwise it would be detected in the spectra of both limbs. It is seen that nothing of the order of cometary velocities is as yet detected.

I have spent some time in considering the possible influence of Daniel's comet which reached perihelion on September 4, with a velocity of 58 km/sec at a distance 0·512; the Earth passed closest to the path of the comet on September 12, as was pointed out by Kritzinger (*Ast. Nach.*, 4198). The comet itself had passed the point on July 27. But in the solar spectrum no sign has been detected that can fairly be attributed to the presence of the comet near the Sun.

Much more evidence is required before the case can be completely elucidated, and this note must be regarded as of a preliminary nature.

The Ultra-Violet Region in Sun-spot Spectra.

By John Evershed.

It is well known that the more conspicuous of the lines which are intensified or otherwise affected in the spectra of sun-spots are found in the less refrangible region of the spectrum, those in the red being by far the most easily seen, whilst in the ultra-violet the lines of the same elements, although far more numerous in this region, seem to be totally unaffected.

A considerable number of photographs of sun-spot spectra, taken in the ordinary way, in the region more refrangible than G, have from time to time been examined in detail by the writer, but, excepting always the hydrogen lines and H and K, these have always shown an entire absence of details which could be certainly distinguished from the adjacent photosphere spectrum.

Recently, however, with improved apparatus and exceptionally favourable atmospheric conditions, a series of spectra have been obtained at this observatory showing a considerable number of affected lines in the region λ 3990–4350.

Owing to the intrinsic darkness of the spot band in the ultra-violet, a very long exposure is necessary, in photographing the umbral spectrum, to give a density equal to that of the spectrum of the surrounding photosphere. Thus I have found for the region near K the umbra requires from ten to twelve times the exposure needed for the photosphere, and it is obvious that in long exposures the diffuse light from the sky will impress the plate more or less, superposing the general solar spectrum on the true spot spectrum. It is questionable, however, whether this admixture of skylight is sufficient to explain the apparent poverty of detail in the ultra-violet, as compared with the red and yellow regions of the spot spectrum.

The photographs were taken in the fourth order of a Rowland plane grating, having 14,438 lines to the inch and a ruled surface of 3 inches. The collimator used is a visually corrected lens of $2\frac{1}{2}$ inches aperture and 36 inches focus. The camera lens is a plano-convex of 4-in. aperture and 7-ft. focus for H. The slit is provided with a sliding shutter, having a $<$-shaped aperture, which enables its length to be varied within wide limits during an exposure. In photographing a spot spectrum, the slit is reduced to a length equal to about two-thirds or less of the diameter of the umbral image. A long exposure is then given, after which the slit is lengthened, and the exposure continued for a short interval to impress the adjacent photosphere spectrum.

The spectrograph was erected in the spectroheliograph room, using as image lens the 12-in. photo-visual lens usually employed for the spectroheliograph. The exposures given on spot and photosphere were as follows:—

Plate exposed 1907 June 20. λ 3900-4140.

No.	Umbra.	Photosphere and Umbra.	Total Exposure.
1	120 seconds	15 seconds	135 seconds
2	180 ,,	25 ,,	205 ,,
3	205 ,,	20 ,,	225 ,,

Plate exposed, 1907 July 16. λ 4090-4350.

1	20 seconds	7 seconds	27 seconds
2	30 ,,	5 ,,	35 ,,
3	40 ,,	5 ,,	45 ,,

The first set of three exposures represents the spectrum of the large southern spot, which passed the central meridian on June 21. The second series represents the same spot after a rotation of the Sun. The latter was not obtained under nearly as favourable conditions as the former, and probably many more affected lines will be added to the list in subsequent photographs.

The two plates were examined and measured quite independently; and in the region where they overlap, the agreement in the estimates of intensity was remarkably good, although, as was to be expected, fewer affected lines were noted on the July plate.

In the following table I give only those lines in the spot spectrum which can be quite certainly distinguished from the photosphere spectrum by an increase or decrease of intensity, or by the absence of a line in the one spectrum which is present in the other. Since a considerable number of the lines visible in the umbral spectrum could not be traced on the photosphere spectrum, the positions of these were measured with a micrometer with respect to neighbouring known lines, from which the wave-lengths were deduced. The measured wave-lengths are given in the first two columns to two places of decimals only, to distinguish them from those lines which could be identified with solar lines by inspection: to these Rowland's values of the wave-lengths are given. The third and fourth columns give the estimated intensities in photosphere and umbra respectively, the former being from Rowland's table. In estimating intensities the spot line is compared with one or more of the neighbouring solar lines having the same apparent intensity, and the intensities of these comparison lines are taken from Rowland's table after comparison with Higgs' map.

It is to be noted that, as in the visible region of the spectrum, titanium and vanadium figure largely among the strengthened lines, whilst of the seven affected iron lines six are weakened in the spot. The hydrogen lines γ and δ are notably weakened, both in width and intensity. Both are displaced relatively to the photosphere lines, but, curiously enough, in opposite directions, γ ·05 towards the red on the July plate, δ the same amount towards the violet on both plates.

List of Lines affected in Sun-spot Spectra in the region between Hζ and Hγ.

Plate Exposed June 20.	Plate Exposed July 16.	Intensity in Photosphere.	Intensity in Umbra.	Element.	Remarks.
λ	λ				
3911·316	...	o	3	Nd	
3914·87	...	o n	1	Fe	The solar line is at 3194·880.
3930·022	...	2	3	Ti	
3933·825 K	...	...	...	Ca	Narrow bright line. Not displaced with respect to the chromosphere absorption line K3.
3960·422	...	4	3	Fe	
3964·663	...	3	<2	Fe	
3996·752	...	oo	2	...	
3968·625 H	...	...	...	Ca	Narrow bright line. Not displaced with respect to the chromosphere absorption line H3.
4015·532	...	o	1	...	
4017·925	..	o	1	Ti	
4019·450	...	o	1	Co	
4023·834	...	2	4	Sc	
4047·93	...	...	2	...	Absent in photosphere.
4048·224	...	1`n	...	...	Absent in umbra.
4050·830	...	2	1	Fe	
4054·69	...	...	3	...	The solar line at 4054·591 is distinctly more refrangible and gives the umbral line a shading on the violet side.
4064·36	...	1	2	Ti	
4095·423	...	o	...	Mn	Absent in spot.
4095·633	4095·633	o	2	V	
4097·61	...	ooo	o	...	
4098·335	...	5	4	Fe	
4099·327	...	oo	2	Ti	Probably the line at 4099·207, intensity o, is included.
4101·921	...	2	1	Fe	
4101·85 Hδ	4101·85	40	10	H	Much reduced in intensity in umbra and displaced to violet ·05 A.U. in both spectra.
4102·31	4102·31	o	2	V	
4102·52	4102·55	...	o	...	Probably this is the yttrium line at 4102·541, intensity o.
4102·71	...	...	oo	...	
4105·318	4105·318	2	5	V	
4105·98	4105·92	ooo	o	...	
...	4111·154	1	oo	Mn?	Almost obliterated.

List of Lines affected in Sun-spot Spectra, etc.—continued.

Plate Exposed June 20.	Plate Exposed July 16.	Intensity in Photosphere.	Intensity in Umbra.	Element.	Remarks.
λ	λ				
4112·869	...	1	3	Ti	
4113·68	4113·69	000 n	1	...	Probably a double line.
4114·28	4114·18	00	2	...	
4117·113	...	0	...	...	Absent in umbra.
4117·63	4117·71	...	0	...	
4120·74	4120·74	...	1	...	Distinctly less refrangible than the solar line at 4120·775.
4123·664	...	1	5	Mn	The intensities of these two lines seem exactly reversed in the umbra.
4123·907	...	5	1	Fe	
4125·52	...	000	3	...	
4128·46	...	00	2	...	
4128·894	4128·894	2	...	...	Absent in umbra.
4129·10	...	...	2	...	Absent in photosphere.
4131·50	4131·507	0	1	Cr	
4133·60	...	...	0	...	Absent in photosphere.
4135·915	...	0	1	Zr	
4139·60	...	...	3	...	Probably a double line. Absent in photosphere.
Spectrum ends at 4140	4140·910	00	0	...	
	4146·78	0	2	...	
	4153·20	00	1	Cr	
	4159·805	0	1	Ti	
	4163·18	...	1	...	
	4252·468	0	>1	Co	
	4262·086	2	1	..	
	4262·142	...	...	...	
	4281·530	0	2	Ti	
	4299·803	2	3	Ti	
	4300·732	3	4	Ti	Rowland gives intensity 2, which seems too small.
	4340·31	0	1	Cr?	
	4340·68	20	10	H	The line is about ·60 wide in sun, and ·35 in spot, where it is displaced towards red, ·05 unit.
	4341·167	0	2	V	
	Spectrum ends at 4350				

Kodaikánal Observatory.

The Spectrum of Comet 1907 *d* (*Daniel*). By John Evershed.

Notwithstanding the very unfavourable conditions prevailing here during the south-west monsoon, attempts were made to photograph the spectrum of this comet on several occasions in August and September. Owing to the short intervals of time practically available for making exposures on the comet (three-quarters of an hour at most), it seemed necessary to use a spectrograph having the greatest possible light-efficiency. I therefore determined to try whether an objective prism spectrograph of short focal length could be successfully employed. On account of the almost constant presence of cloud, more or less thick, in the eastern sky, the results do not come up to expectation, but are perhaps of sufficient interest to place on record.

A prismatic camera was constructed with the two very perfect 60° prisms, of specially selected crown glass, which were successfully used at recent eclipses. The prisms have an effective aperture of 42 mm., and the photographically corrected lens has a focal length of about 600 mm. The instrument was attached to the 6-in. Cooke equatorial in the south dome of this observatory, the plane of dispersion being made parallel to a circle of declination. For guiding, a straight-edged bar was placed in the eyepiece of the finder attached to the equatorial, and this was made parallel to the diurnal motion. In making an exposure, the comet's nucleus was kept on the edge of the bar, and nearly in the centre of the field (defined by a cross-wire), by means of the slow-motion handles of the equatorial.

In the course of several weeks, about the time of the comet's maximum brightness, only three fairly favourable opportunities occurred. These were August 28, September 3, and September 15. The best result was obtained on the first-named date, with an exposure of half an hour. On September 3 the comet was obscured by cloud for about half the available time before daylight, and during this interval the opportunity was taken to photograph the spectrum of Procyon on the same plate, the star being brought to the edge of the bar in the eyepiece, but at a little distance from the centre of the field. In the plate exposed for twenty minutes on September 15, under apparently favourable conditions, no trace of the spectrum can be seen. This is probably due to the low altitude of the comet, for the spectrum of Regulus, then about 3° N. of the comet, is only faintly impressed on the plate. Wratten's "Drop Shutter" plates were used in all the trials.

In the spectrum of August 28 the most striking features shown are a pair of extremely bright lines in the ultra-violet, corresponding with the nucleus of the comet,* and a series of monochromatic images of the tail, having the following approximate wave-lengths :—

$$\lambda\ 358,\ 369,\ 378,\ 401,\ 426,\ 455.$$

* Owing to a slight drift of the comet in R.A., the images of the nucleus are linear.

These radiations are not specially bright in the comet's nucleus, but they extend from ·5° to 1·5° into the tail, whilst the above-mentioned bright lines seem to be confined to the region of the nucleus. Altogether 12 lines or groups of lines have been measured on this plate, and the wave-lengths determined graphically, using the 12 hydrogen lines from $H\beta$ to $H\nu$ inclusive in the spectrum of Procyon and of Sirius to obtain the dispersion curve. In the spectrum of September 3, with Procyon photographed on the same plate, the pair of bright lines in the ultra-violet are seen to be a little more refrangible than $H\zeta$ (λ 3889); and as their distance apart corresponds with about 11 units in wave-length, there can be no hesitation in ascribing them to the cyanogen lines at λ 3871·5 and λ 3883·5. In this plate, however, the images are too faint for measurement.

In the table which follows are summarised the results of measures of the spectrum of August 28. As most of the spectral images on this plate are broad and ill-defined, I give in the second column the approximate limits of wave-length of each line or band; and in the third column the mean wave-length, or the positions of maximum brightness. In the fourth column is entered the length of tail in degrees which can be traced in each radiation. I can assign no origin for the radiations, which seem characteristic of the tail, except the band at λ 455, which, according to Campbell, may probably be due to cyanogen.* The continuous spectrum, which is faint throughout, ends at about λ 490.

No.	Limits of λ.	Mean λ.	Length of Tail in Degrees.	Probable Identification.	Remarks.
1	357	358±	·3±		
	359	...	...	Cyanogen ?	
2	367	369±	·4±	...	Exceedingly faint images of tail.
	370	...	...	...	
3	377	378±	·5±	...	
	380	...	...	...	
4	3863	3873	...	Cyanogen.	Very intense lines: 3884 the strongest in the whole spectrum.
5	3885	3884	·2	Cyanogen.	
6	3995	4015	1·6	...	
	4034	...	...	...	
7	...	4129±	...	...	Extremely faint traces of tail. Campbell gives a faint line at 4126 in comet *b* of 1893.
	...	...	...	...	
8	4190	4200	...	Cyanogen.	No tail. Traces of lines at these positions.
	4221	4215	...	Cyanogen.	

* *Pub. Ast. Soc. Pacific*, vol. v. p. 209.

No.	Limits of λ.	Mean λ.	Length of Tail in Degrees.	Probable Identification.	Remarks.
9	4239 } 4282 }	4260 ...	1·4 ...	... ...	A very faint line is given by Campbell at λ 426 in comet *b* of 1893.
10	...	4360	...	...	Faint bright spot in continuous spectrum. A bright line is given by Campbell at λ 4366 in comet *b* of 1893.
11	4523 } 4584 }	4553 ...	1·1 ...	Cyanogen ? ...	Campbell gives a band near λ 455 in comet *c* of 1893.
12	4650 } 4736 }	4682 ...	·4 ...	Blue band of carbon.	Max. brightness estimated at λ 4682.

Kodaikánal Observatory.

Note on the Ancient Solar Eclipses discussed by Mr. Cowell.
By A. C. D. Crommelin.

The results derived from these eclipses by Mr. Cowell are so important and far-reaching that it is desirable to remove all uncertainty as to possible errors, especially in view of the fact that Mr. Nevill published some results purporting to be deduced from the same formula, but inconsistent with those given by Mr. Cowell himself; I accordingly undertook an independent calculation of the six eclipses −1062, −762, −602, −584, −430 and +197, which are the most important and best authenticated ones. I obtain results practically identical with Mr. Cowell, the discordance in no case exceeding 2″, which is an absolutely negligible quantity.

Several smaller terms in latitude were then introduced, which had some slight effect on the residuals, but not enough to affect the argument in any way; they possibly produce a small diminution in the relative acceleration of the Sun and the node, but only to the extent of ½″ or less.

Two points suggest themselves as worthy of mention regarding the eclipse of −1062; viz.—(1) Mr. King's recently published book gives a translation of the inscription in which it occurs, which is a record of omens occurring in the city of Babylon (wild beasts entering the streets, dogs entering the temples, etc.); hence there

can be no reasonable doubt that the extinction of the Sun, recorded as an omen, was likewise witnessed from Babylon itself.

(2) The date of the phenomenon is given as Sivan 26; now, as the months began with the first observation of the Moon, the 26th of the month must be within a day or two of new moon: this decidedly increases the probability that the recorded solar darkening arose from an eclipse.

Both Professor Newcomb and Mr. Nevill have rejected the suggested explanation of the apparent acceleration of the Sun as arising from a diminution of the Earth's rate of rotation. The point, however, is such a simple one that it seems worth while to restate it in other words, in the hope that they may more definitely indicate their reasons for dissenting from it.

(1) A slowing of the Earth's rotation from tidal friction necessarily involves an increase in the Moon's distance, in order that the moment of momentum of the Earth-Moon system may be conserved.

(2) A rough calculation was given showing that the lengthening of the Moon's period arising from this may be nearly as great as the apparent shortening due to the increase in the length of the day.

(3) In the case of the Sun's period, the shortening due to the latter cause would act unimpeded, so that an apparent acceleration of the Sun comparable with that of the Moon would result.

Note on the Permanency of some Photo-visual Lenses.

By William J. S. Lockyer, M.A., Ph.D. (Plates 1, 2, 3.)

The Solar Physics Observatory at South Kensington is in possession of numerous photo-visual lenses which are employed in several instruments in frequent use. All these objectives have in time developed curious markings on one or more of their internal surfaces. The object of the present note is to draw attention to, and to give a brief description of these markings.

Each of the lenses in question consists of a combination of three elements, the front and back lenses being double convex and the middle double concave. The front and middle lenses are nearly in contact, but a small air-space separates the middle from the back lens.

The material with which the lenses are made is as follows:—

The front lens is made of Schott's baryta light flint glass (O 543), the negative or middle lens is of Schott's boro-silicate flint glass (a variety of their O 164), while the third is made of a light silicate crown (Schott's O 374).

The lenses to which reference will be made vary in aperture from 3 to 12 inches, and have been left, as regards their inner surfaces, untouched in their respective instruments for periods varying from 23 to 83 months.

Aperture in Inches.	Date of Mounting.	Date of Dismounting.	Interval in Months.	Notes on Markings.	Conditions of Situation.
12.	March 12, 1905.	Feb. 18, 1907.	23.	Very fine curly markings closely packed.	In good airy room, temperature variable.
4.	March 16, 1904.	Feb. 21, 1907.	35.	These lenses look quite clear, but on close examination it is found that there is a distinct indication of the beginning of the markings in the form of minute streaks.	In a room well heated throughout the year.
4.	March 16, 1904.	Feb. 21, 1907.	35.		
3·5.	October 1903.	Feb. 21, 1907.	40.	Beautiful feather-like markings, fairly evenly distributed, but denser near centre.	Well aired room, temperature variable.
3.	January 1900.	Dec. 14 1906.	83.	Markings all over the lens, but smaller and more closely packed towards centre.	Subjected, possibly, to vapours from arc in laboratory. Room always well warmed.
3.	January 1900.	Dec. 14, 1906.	83.	Well separated curly markings, more numerous near centre.	

The preceding table contains a summary in convenient form indicating the intervals between the times of mounting and dismounting the objectives, the conditions of their situation, and brief notes on the markings developed.

In order, in the first instance, to indicate the appearance of the markings in question, the following four selected photographs will serve as examples. It may, however, be first mentioned how these photographs were secured.

The whole combination was first removed from the instrument in which it is employed and placed in a V-shaped wooden stand in front of a window. The camera was placed about 2 feet away from the lens and about 3 feet from the window, when the markings were carefully focussed on the ground-glass. The system of illumination of the lens under examination was as follows. The space between the lens and the camera was first covered in with black velvet. About a foot away from the lens, but on the opposite side of it to the camera, a large dull black piece of cardboard was placed near the window in nearly a vertical position to serve as a black background. The lens itself was illuminated by daylight passing through the window and falling obliquely on the back portion of the combination. Unfortunately it was found very difficult to eliminate the reflections of the window from appearing on the lens, but they are not sufficient to detract from the main object for which the photographs were taken. The photographs show, therefore, the peculiar markings on one or more of the inner surfaces of the lenses, as white on a dark background.

Fig. 1 (Plate 1) represents the 3-in. objective which has been employed in an instrument for 83 months. The markings, it will be noticed, are small, and near the middle give the lens a distinct mottled appearance. Nearer the edges their structure becomes more pronounced, and there is a greater distance between them.

Fig. 2 (Plate 1) represents the markings on another 3-in. lens which forms the camera objective of the same instrument. This lens was mounted and dismounted at exactly the same time as that shown in fig. 1 (Plate 1), and during this interval of 83 months it was situated within a foot of the other lens.

The different structure and number of the markings is very striking, and can be judged best by closely examining the photographs.

In fig. 1 (Plate 2) we have another example of quite a different appearance. This lens has an aperture of 3·5 inches, and has been in use as an objective to a small telescope for 40 months. The markings in this case are of a much finer type than those previously mentioned, and are far more elaborate in their structure.

This lens exhibits, in addition, a second set of markings, which appears on another of the interior surfaces of the combination, but which could not be photographed under the same illumination and distance, and therefore cannot be seen in fig. 1 (Plate 2). By

reversing the combination in the V-shaped mount, refocussing the camera and varying the direction of the daylight illumination, a very successful negative was secured, a reproduction of a positive from which is seen in fig. 2 (Plate 2). Around the centre of the lens will be observed a great number of small white spots which represent the markings in question. These are quite different in form and character from any of those which have been previously described.

In the case of two 4-in. lenses a recent examination has shown that after an interval of 35 months they are just beginning to show signs of development of markings.

In this initial stage one has observed, with the help of a watchmaker's glass only, that the markings take the form of very small single streaks, more or less homogeneously scattered over the whole lens, but a little more sparse nearer the edges than about the centre.

The large 12-in. objective has developed those markings at a much greater rate. After an interval of only 23 months very fine markings, of simple structure, but fairly closely packed together, extend over the whole surface of the lens, and, as before, their structure becomes more complex as the edge of the lens is approached.

So far, no mention has been made as to the particular surface of the combination on which the markings are to be found. As a matter of fact, although it is quite simple to take to pieces any of the lenses which have been examined, it is not easy to replace them properly centred in their cells. For this reason, therefore, only one of the above-mentioned lenses has been thoroughly examined by separating the component lenses forming the combination. The particular objective thus treated was that of 3·5 inches aperture, and it exhibited both kinds of markings to a very great degree.

When markings were first noticed to develop on one of the inside surfaces of the combination it was thought that they might be due to the possible lack of permanency of the boro-silicate flint glass, since there seemed more doubt connected with the permanency of this material * than with those used for the other lenses. It was therefore found, with some surprise, that when the 3·5-in. objective was taken to pieces, both surfaces of the negative lens were perfectly clean and unmarked, and therefore not deteriorated in any respect. This result was highly satisfactory from the point of view of the lasting power of the boro-silicate flint glass, and showed that, as mounted between the two other lenses, 40 months' use had not affected it. It was the inner surface of the back lens, of light silicate crown glass, which was affected to the greatest extent, and on which the markings illustrated in the above figures were scattered. This surface, it will be remembered, is separated from the back surface of the negative lens by a small air-space. It may be mentioned that no balsam or like

* Especially where sulphuretted hydrogen or sulphuric acid abounds, as in a town atmosphere.

material is employed in this combination to cement the lenses together, but the surfaces are kept separate by thin washers.

In order to investigate more thoroughly the nature of the two different kinds of markings, namely, those which appeared on the inner surfaces of the back and front lenses of the 3·5-in. objective, the combination was taken to pieces and the markings microscopically examined.

This inquiry was facilitated by the kindness of Professor J. B. Farmer, D.Sc., F.R.S., of the Royal College of Science, who not only allowed me to use one of his microscopes in his own room, but also examined the markings with me. It was at once obvious that we were not here dealing with any form of vegetable growth, as seemed at first probable by the general appearance of the markings, but that both kinds were of a crystalline nature.

In the case of the back lens the crystallisation seemed to have taken place on the surface of the lens, and had the appearance of a deposit on the surface. Each fungus-like figure seemed to have originated from a small area, not a single point, and developed radially in numerous directions, throwing out ramifications as the growth from this area progressed. Under the microscope it was found that the markings were really crystals, forming at first about the small area, and then gradually growing outwards, crystal by crystal, until they formed chains of crystals, giving the fungus-like growth described above. There seemed little doubt that if this process were allowed to continue, the individual groups of crystals would all eventually unite, and seriously interfere with the transparency of the combination. In the lens under examination several of these groups have already become interlaced and instances of this can be seen in the centre portion of the lens illustrated in fig. 1 (Plate 2).

In the case of the markings on the back surface of the front lens, the form of crystallisation is quite different. Here, under the microscope, the markings (seen in fig. 2, Plate 2, as white dots scattered about the centre of the lens) are composed each of a little irregular group of crystals, formed one above the other, and projecting from the surface of the lens, not embedded in it. There is little appearance of lateral growth to the extent described in the case of the markings on the back lens. Scattered over the surface of this lens are also small single crystals, which suggest the first stage in the building up of these heaps. In this case also it is probable that if this process of crystallisation were allowed to continue, more single crystals would be formed, the single ones already existing would each develop into a small pile of crystals, and the lens would gradually become less transparent.

By the kindness of Mr. W. H. Merrett, A.R.S.M., the Instructor in Assaying in the Royal College of Science, I was permitted the use of the photomicroscope set up in the metallurgical laboratory of that institution. With the help of this instrument and with his assistance I was able to take large scale photographs of the

crystalline formations on each of the two lenses, and some of these are reproduced on Plate 3.

Figs. 1, 2, 3 show three of the individual small groups of crystals formed on the back surface of the front lens, to which reference has just been made. The magnification employed for these photographs was 700. It will be seen that the formation is quite irregular, and, as a rule, there does not seem to be a point of origin, as if their existence was due in the first instance to the presence of a speck of dust. On the surface of this lens there appear also single crystals which are totally different from those just described. They do not show the lateral growth, as is here illustrated in figs. 1, 2, 3, but they are very much more raised from the surface on which they are formed.

The markings on the inner surface of the back lens of the combination are illustrated in figs. 4, 5, and 6 of Plate 3. Fig. 4 gives a general idea of the middle portion of one of the groups under a magnification of 45 diameters. It will be noticed that there does not seem to be a distinct single point of origin, but that the group seems to spring from a small area. An interesting point to which attention may be called is the fact that none of the lines of crystals cross each other: this can be observed in several instances in fig. 4 (Plate 3) where these lines approach each other. Further, the extremity of each line, where the crystallisation can proceed unhindered, is nearly always capped by a complete crystal.

Fig. 5 (Plate 3) is a more highly magnified (150 diameters) portion of fig. 4, and for identification purposes similar portions in figs. 4 and 5 have been lettered A, B, C. In this figure the building up of the lines by successive deposits of crystals can be more clearly seen.

Fig. 6 (Plate 3) is a portion of another group altogether. The magnification here is 700 diameters. It shows perhaps more clearly not only the individual crystals which help to build up the lines which compose the groupings previously mentioned, but the complete form of the last crystal deposited at their extremities.

Attention may here be drawn to the mottled appearance which can be seen in fig. 4 (Plate 3) in those parts of the lens where the crystals have not been formed. These markings, or "globules," as they are termed, evidently represent the stage previous to the actual crystal formation, and must have covered the whole surface of the lens before the crystals themselves were developed. As crystallisation set in they disappeared in the neighbourhood of each crystal, as if the crystals themselves required this material for their formation. The existence and behaviour of these globules are well known in crystallography, as I have since been informed by Professor J. W. Judd, of the Royal College of Science, South Kensington The whole phenomenon is clearly described and illustrated by Professor O. Lehmann in his magnificent work entitled *Flüssige Kristalle* (Wilhelm Engelmann, Leipsig, 1904). In this connection Professor Lehmann writes (p. 112), "Das die Globuliten in der Nähe von

Kristallen verschwinden, so dass um diese ein Hof ensteht, erklärt sich vor allen dadurch, dass überkühlte Schmelzen leichter löslich sind als die kristallinisch erstarrte Substanz, in einzelnen Fällen auch durch Strömungen, welche teils durch Dichte-Differenzen, teils, speziell bei Präparaten ohne Deckglas, durch die Oberflächenspannung veranlasst werden."

Although the microphotographs reproduced with this paper indicate in a general way the structure and crystalline nature of the markings, some difficulty was met with in attempting to secure satisfactory photographs with high powers, that is, with magnifications of 150 diameters or more.

The *thickness* of the crystals and their deposition on a *curved surface* rendered it impossible to focus the objects in their entirety in a satisfactory manner, so that one had to be contented with the somewhat blurred images when these magnifications were employed.

In order to test whether the markings on the two surfaces of the objective were responsible for any absorption of light passing through the combination, the objective was set up on the V-stand mentioned previously and photographed. On this occasion the background used was white instead of black, and light was only allowed to enter the lens by reflections from this surface. If the markings absorbed light, it was expected that the resulting photograph should give one the picture of the lens with the markings dark instead of light, as was shown in the previous illustrations (Plates 1 and 2). This is exactly what occurred, and both sets of markings were faithfully recorded. Those on the back lens, which covered the larger area, were not, fortunately however, so opaque as those on the front lens, but still a considerable absorption of light was indicated, a quantity quite sufficient to detract from the efficiency of the combination.

Inquiry was next made into the probable causes of the deposition of the crystals. The weathering of glass surfaces, on account of the dust, water-vapour, etc. in our atmosphere, has received considerable attention in Germany, and quite a mass of literature now exists on the subject. A very convenient summary of such investigations will be found in Dr. H. Hovestadt's book on *Jena glass, and its scientific and industrial applications*, an English edition of which appeared in 1902 (Macmillan & Co.).

In chapter x. of this volume the chemical behaviour of glass surfaces is dealt with, and attention is chiefly confined to glasses which are most closely related to the work of the Jena glass-making laboratory. Since the photo-visual lenses here discussed are made from material supplied from Jena, the information given is therefore specially serviceable on the present occasion. There seems little doubt that it is chiefly the moisture in the atmosphere, and not dust particles, which is responsible for the actions which occur on the surfaces of the lenses here described, for the surfaces attacked are all inside, and although protected to a great extent from dust, they are still liable possibly to action by water-vapour.

It has been shown that the durability of glass depends chiefly on its power of resisting attack by water. Glass being hygroscopic, absorbs moisture to a varying degree, and this moisture enters into chemical combination with its surface. As a result of this absorption, the alkaline components of the glass are gradually set free, and an opportunity is at once afforded for the carbonic acid of the air to combine with the alkalies and form carbonates, which are deposited in the form of crystals.

Dr. E. Zschimmer* made in 1901 an analysis of about 200 pieces of glass with polished plane surfaces, which had been stored, with a view of subsequent testing, in a dry place in Jena for several years, but so protected that they were only imperfectly excluded from air. He found that the behaviour of silicates without lead depended almost entirely on the amount of alkali contained in them. If the amount was as small as 10 per cent., dust particles initiated a deposit which he termed "dusty disintegration"; over 10 per cent., a "homogeneous decomposition" set in which attacked the glass uniformly. When the amount of alkali reached 20 per cent. the deposit became visible to the naked eye, and the greater the proportion of alkali, the coarser the phenomena, *i.e.* the formation of drops and the crystallisation of carbonates. The action of deliquescent carbonate of potash is to cover the surface with more or less minute drops, while carbonate of soda, being only slightly hygroscopic, covers it with assemblages of crystals.

In the case of the lenses dealt with in this paper, the formation of the single crystals took place on the baryta light flint glass, while the assemblages of crystals occurred on the light silicate crown.

In order to find out the actual composition of the latter glass, Messrs. Schott and Genossen of Jena were communicated with, and asked whether they would be good enough to supply this information. Needless to say, the requisite data were soon forthcoming, and I should like to take this opportunity of expressing to them my best thanks for so willingly sending me the facts desired. The following is an abstract of their reply, dated 12th April 1907, which they have kindly allowed me to publish:—

"We beg to say that the glass in question of our type O 374 constitutes a crown glass holding a high percentage of alkali, we had made at some previous time, of a similar composition as French glass. Owing, however, to its being wanting in permanency, we discontinued later on the manufacture, and hence this glass does not appear on our present catalogue.

"Its chemical composition is the following:—

B_2O_3 3%, K_2O 17%, Na_2O 5%, CaO 3·8%, PbO 1%, As_2O_3 0·4%, SiO_2 69·8%."

With these facts before us, one is led to conclude that the

* *Chemiker Zeitung*, 1901, 25, No. 69.

excess of the alkali has acted in the way suggested by Dr. Zschimmer, and produced the assemblages of crystals described above.

It may be here remarked that these assemblages of crystals are very similar in growth to those formed by the crystallisation of potassium permanganate, as described and illustrated by Professor Lehmann on page 124 of the volume on *Flüssige Kristalle* to which reference has previously been made.

In order to try and reproduce the forms of crystallisation deposited on the lenses, a series of experiments was made with partial success.

As potassium (17%) was the chief ingredient next to silicon (69·8%) in the composition of the glass, solutions of pure potash with distilled water were made and poured on to the cleaned surfaces of several cover-glass lantern plates, and then drained off. The moistened sides were then covered with another cover-glass, but separated by means of small wooden diaphragms in order to allow a slight circulation of air. The glasses were kept in an upright position in a cupboard, which was frequently opened.

In a similar way, several other glasses with solutions of potash and caustic soda, mixed in the proportion of 3 to 1, were also prepared and treated in the same way.

After 10 or 15 days crystallisation in most cases began to set in, and the plates were examined and occasionally photographed.

It was most interesting to observe, not only the different forms of crystallisation on the various plates, but also the different forms on the same plate.

In the few attempts made, the form of crystallisation shown in Plate 2, fig. 1, was not exactly reproduced, although in some instances a close approximation was reached.

In conclusion, it may be remarked that the deposit of crystals on the several lenses to which reference has here been made has in no instance been removed at the observatory. The objectives were always sent away to be cleaned, and the method of getting rid of the deposit was to rub the surfaces with material soaked in dilute sulphuric acid. It is, however, satisfactory to know that Messrs. Schott and Genossen no longer manufacture the particular kind of glass which has caused this trouble, and have erased it from their catalogues. The more recently made photo-visual lenses should not, therefore, in future be hampered with this defect.

Before submitting this paper to the Society, I sent it to Mr. H. Dennis Taylor, whose investigations on the theory and practice of photo-visual lenses are so well known. He has kindly written an "Addendum," which is here attached, and which will be found a valuable contribution to the subject of the present paper.

Addendum by H. Dennis Taylor.

Notes on Dr. Wm. Lockyer's paper on the Permanency of some Photo-visual Lenses.

Dr. Lockyer kindly submitted to me his paper before publication, with a view to my adding any remarks of my own on this interesting question.

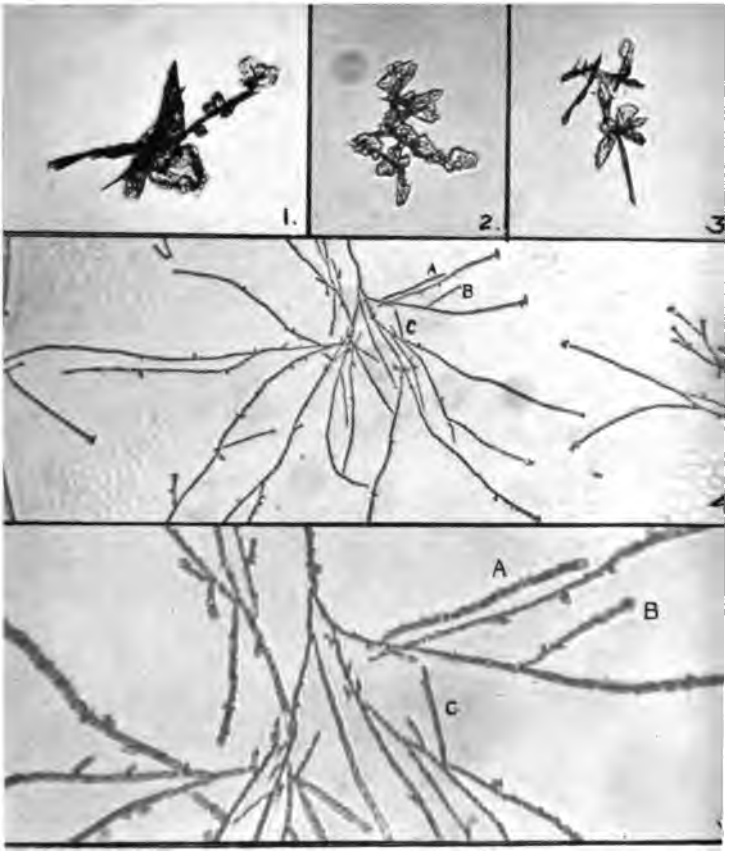
In the first place, I would like to point out that the dates of mounting the instrument given by Dr. Lockyer are not the dates on which the objectives were put together, so that, correctly, at least 12 months should be added to the periods given, on the average.

Dr. Lockyer says that he had not been able to notice any deterioration of the optical performance of any of these objectives after being cleaned up, but that no special tests were employed.

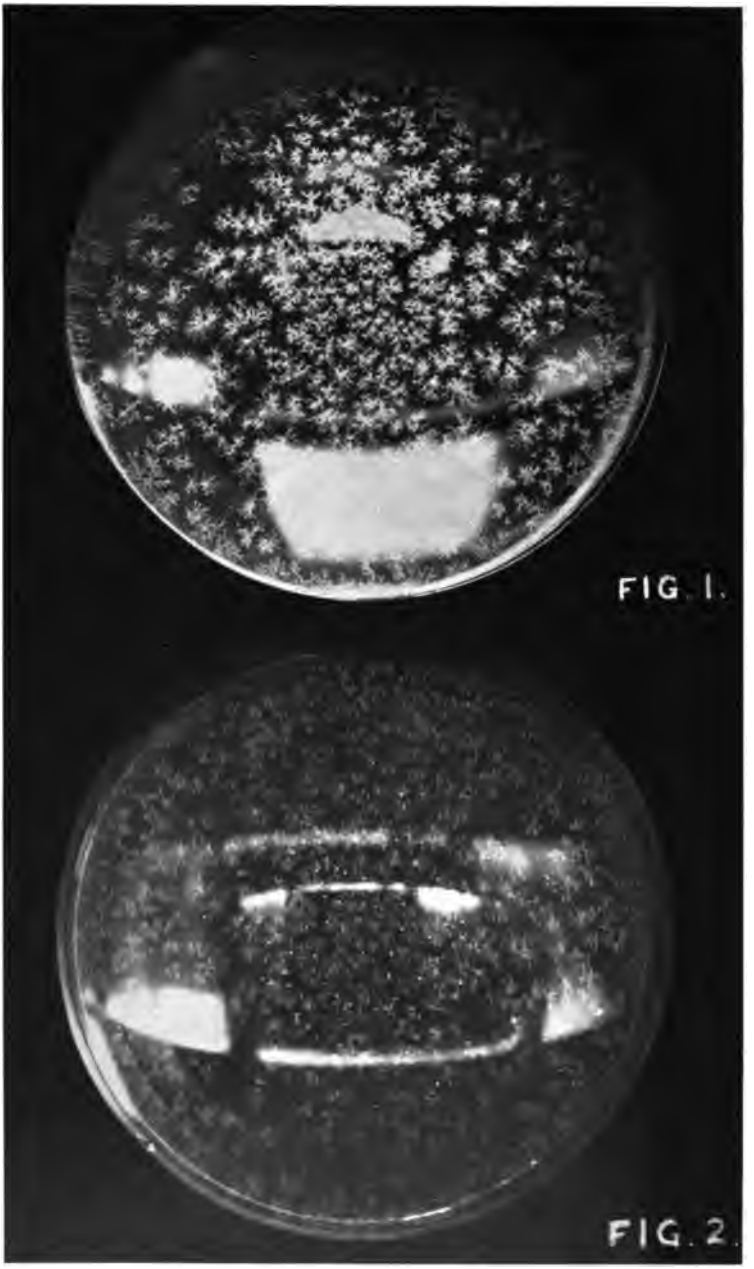
But after cleaning at Messrs. T. Cooke & Sons' works, the objectives were carefully examined and tested for optical deterioration and none whatever was noticed. One would have thought that the crystallising out of some of the alkaline constituents of the glass would have left a network of channels in the glass surfaces, and for the presence of such small inequalities, tests by reflected light should be more appropriate, but we could not see anything. Of course, it is quite possible that the repetition over many years of such crystallisation, supposing it does repeat itself, might deteriorate the surfaces of the lenses sufficiently to cause optical mischief, but so far it has not been apparent.

With regard to the small bundles of glistening crystals forming upon the inner surface of the front lens of baryta light flint, and shown in fig. 2, Plate 2, and figs. 1, 2, and 3, Plate 3, I would like to observe that these never seem to be at all fast to the surfaces, and can always either be blown off or brushed off with a delicate camel-hair brush; and after so brushing them off, the surfaces are just as clean as before, excepting for a little dust that may have found its way in.

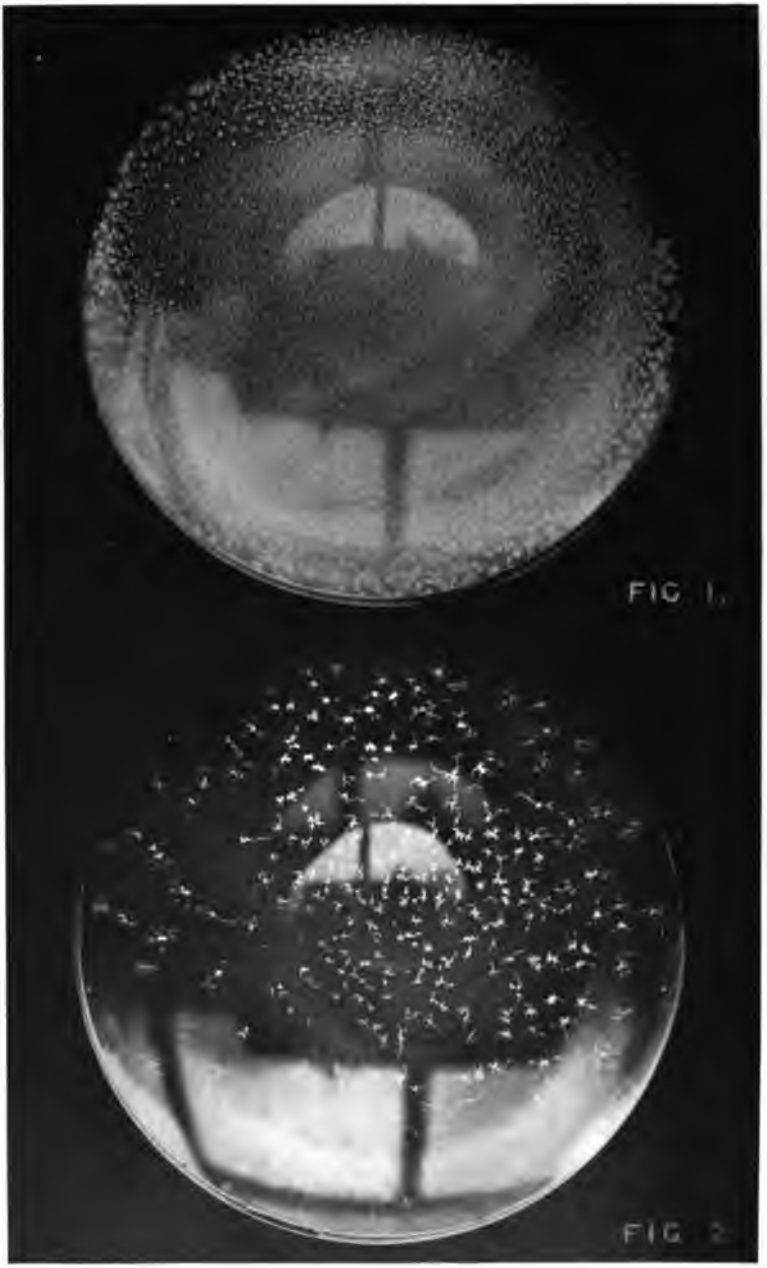
As regards the much more noticeable crystallisation on the inner surface of the back lens, I have never had any difficulty in rubbing it off with a soft rag dipped in very weak sulphuric acid, which, after thorough washing off, leaves the surface just like new. The amount of this crystallisation is largely dependent on the dampness or dryness of the situation in which the objective is used, and it would most probably never form at all could we keep the objective always in a dry atmosphere. Moreover, it depends also upon the particular type of crown glass used. I must here point out that the crown glass with a larger amount of alkali than usual, referred to by Dr. Lockyer as O 374, was employed in the 12-in. aperture, ·3½ aperture, and one of the two 3-in. aperture O. G.'s, while the other two, of 4-in. aperture, were made out of a borosilicate crown glass, type O 599, of Messrs. Schott's catalogue,



[To face page



To face page 29.



[*To face page 29.*

which latter glass is no more likely to develop hygroscopic crystallisation than is the ordinary crown glass used for older type double objective.

We have occasionally had cases, however, where ordinary double objectives that have been used in damp climates have developed the same crystallisation on the interior surface of the crown lens.

Indeed, in damp tropical climates such as that of Calcutta the crystallisation which so frequently develops on the interior surfaces of ordinary crown glass lenses is found, if left too long a time without wiping off, to leave an actual corrosion of the surface in the form of a fine grey veil. The appearance is just like that shown by an imperfectly polished surface, in which the fine pittings caused by the last emeries still survive.

Messrs. Cooke & Sons do not now employ the crown glass O 374, as it wants cleaning more often than is desirable, and use the boro-silicate crown of type O 599, or else a variety of it recently recommended by Messrs. Schott. As regards freedom from secondary spectrum, it is difficult to detect any difference between objectives made of these two different crown glasses.

DESCRIPTION OF PLATES.

Plate 1.

Figs. 1 and 2. Photographs of two 3-in. lenses after 83 months' use in a laboratory spectroscope. Fig. 1, collimating lens; fig. 2, camera lens.

Plate 2.

Fig. 1. Photograph of 3·5-in. lens after 40 months' use as a finder to a 3-ft. reflector.

Fig. 2 is a reproduction of a photograph of the same lens, but taken so as to show the markings (white dots near centre) on another surface of the combination.

Plate 3.

Figs. 1, 2, and 3. Enlarged views (700 diameters) of some of the white markings illustrated in Plate 2, fig. 2.

Figs. 4, 5, and 6. Enlarged views (45, 150, 700 diameters respectively) of parts of one of the groups of crystals shown on Plate 2, fig. 1.

The Nebula H IV 74 Cephei. By Dr. Max Wolf. (Plate 4.)

I have pointed out elsewhere that, as a rule, the nebulæ in the Milky Way are encircled by a ring which is void of faint stars, and that this lacuna is the end of a long starless hole, apparently showing the direction of some unknown cosmic motion. I have given a number of examples of this,* the best specimen being the nebula 2 degrees south of π^2 Cygni.

On photographing the Nebula H IV 74 Cephei (= N.G.C. 7023 : $\alpha = 21^h\ 0^m\ 30^s$ $\delta = +67^\circ\ 46'{\cdot}2$ (1900·0)) with the 28-inch reflector of the Königstuhl Astrophysical Observatory, I was surprised to find a striking specimen of this phenomenon, the nebula being surrounded by a lacuna absolutely empty of faint stars. In spite of the small field of the reflector-plate, there was evidence that the cavity was the end of a channel entering the picture almost exactly from the south.

This nebula was photographed by the late Dr. Roberts (vol. ii., plate 24). He says: "The nebula appears in a region almost void of stars" (p. 157).

I then endeavoured to photograph the channel with the large field of my 16-inch cameras. But the weather was very unfavourable during this spring and summer, and the sky always so hazy, that three nights were lost in these attempts. The last of these trials, on July 7 (exposure 3 hours 22 minutes), gave relatively the best picture, a reproduction of which is given on Plate 4, fig. 1. The scale is about 70 mm.† for 1 degree. As may be seen, the channel is directed in its first part almost exactly from south to north. At a distance of about 1 degree from the centre of the nebula the channel is divided into two branches. The eastern branch is perhaps shorter, and the western longer, ending at a distance of about $1\frac{3}{4}$ degrees from the nebula by becoming gradually filled up with stars. The somewhat round cavity encircling the nebula has a diameter of about half a degree.

We have therefore a further example of the rule that a nebula is encircled by a lacuna void of stars, which lacuna is the end of an empty channel.

The nebula itself is a very remarkable object. In many respects it resembles in form the π^2 Cygni nebula. On the other hand, the faint bands of nebulosity parallel with the brighter bands are very much like those in the Orion nebula. This parallelism gives an indication of some kind of wave formation in the nebulous material. These are especially visible at the north-west side, three parallel waves, each nearly half the brightness of the inner preceding wave. I am especially struck by the gradual fading of the light of the nebulous matter from the centre, producing the impression that nebulous matter is spread all over

* *Astron. Nachr.*, 3848; *M.N.*, *R.A.S.*, lxiv., No. 9; *Public. Astrophys. Inst.*, *Königstuhl*, *Heidelberg*, vols. i. and ii., etc.

† In the reproduction 48 mm. = 1°.

FIG. 2

FIG. 1. NEBULA H IV. 74 CEPHEI. MAX WOLF.

[*To face page* 30.

the lacuna, but has insufficient luminosity to be visible. I do not know any nebula in the sky which shows this more clearly.

I also give a somewhat enlarged reproduction of a picture of the nebula itself, taken with the 28-inch reflector (Plate 4, fig. 2). The photograph was taken April 18, 1907, with an exposure of 40 minutes. The scale on the reproduction is about 1 mm. to 21 seconds of arc.*

I may perhaps mention two peculiarities common to this nebula, to the π^2 Cygni nebula, and to perhaps all other similarly surrounded by lacunæ. First: the long channels are all nearly straight and clean in the vicinity of the nebula, and become curved and partly filled up with stars at a greater distance from it. Secondly: the channel is simple at the end nearest the nebula, and divided into two or three complicated arms at the end further removed.

* In the reproduction 1 mm. = about 30″·6.

Heidelberg, Astrophysical Observatory:
1907 July 27.

Occultation of the Hyades. By Walter Heath, M.A.

In order to determine the longitude of my observatory I have recently compared twenty-three occultation observations made by me with the corresponding Greenwich observations, and I find a discordance in the case of the reappearance of two stars of the Hyades on 19th September 1905. I have therefore compared the results with some observations published in *A.N.*, 4088; the following are the particulars. The results in brackets are derived from observations at the bright limb.

1905 Sept. 18	Utrecht	*f* Tauri	·95Δα + ·41Δδ =	(10·20)
,, ,,	,,	,,	·99 ,, + ·27 ,, =	8·44
,, ,,	Jena	,,	·95 ,, + ·39 ,, =	8·26
,, 19	,,	γ Tauri	·42 ,, + ·95 ,, =	(5·78)
,, ,,	,,	,,	·84 ,, − ·57 ,, =	3·97
,, ,,	Greenwich	θ_1 Tauri	·94 ,, + ·37 ,, =	1·71
,, ,,	,,	D.M. 15°, 633	·94 ,, − ·35 ,, =	1·36
,, ,,	Cobham	θ_1 Tauri	·94 ,, + ·38 ,, =	5·12
,, ,,	,,	D.M. 15°, 633	·95 ,, − ·35 ,, =	6·22

Note.—For Cobham the longitude assumed was 23′·6″ W., latitude 51°·19′ 39″, height above sea 180 feet. The sidereal times observed were $4^h\ 27^m\ 22^s{\cdot}49$ and $4^h\ 45^m\ 2^s{\cdot}02$ after correction.

Uplands, Cobham, Surrey:
1907 October 18.

Disappearance of Saturn's Ring System, 1907 October.

By R. T. A. Innes.

Thanks to the assistance of Sir David Gill and the Witwatersrand Council of Education (Mr. Th. Reunert, chairman), I have within the last few days erected a 9-in. refractor by Grubb.

The disappearance of Saturn's rings was watched with this instrument on the 3rd October 1907. An exact copy of the notes made reads as follows:

1907 Oct. 3, 6.45 p.m. twilight (4.45 p.m. G.M.T.). Saturn's ring distinctly seen by myself and several visitors. It was very faint and only seemed to be about half of its usual length. Definition medium, planet low.

8.45 p.m. (6.45 p.m. G.M.T.). The ring is invisible. The bright equatorial band was crossed by a sharp dark line which just lay to the N. of Saturn's equator. This dark line had been seen on preceding nights, but on this occasion it seemed to be sharper and darker. Two satellites, about 2″ or 3″ apart, followed Saturn.

9.30 p.m. (G.M.T. 7.30 p.m.). Ring distinctly seen, but like a ghost. Care taken to eliminate subjective effects. Definition very good.

10 p.m. (8.0 G.M.T.). Ring still glimpsed. Preceding part of ring modified by two beads, one twice as large as the other (Enceladus and Mimas). My son's description is "Ring visible on both sides, extending about three-quarters diameter of planet. Nearer and brighter satellite is half way along ring and touches the northern side. Fainter satellite precedes end of ring."—(E. A. I.)

11.45 p.m. (9.45 G.M.T.). Quite doubtful if ring really seen. Mimas and Enceladus very close together. My son, however, was quite certain that he held the ring steadily. His words are "Ring very faint, but ended off sharply. Fainter satellite is preceding."—(E. A. I.) Definition perfect.

12.30 p.m. (10.30 G.M.T.). No suspicion of ring; only one satellite seen prec. "Ring not seen with any certainty. The first of the satellites following Saturn getting too close."—(E. A. I.) Definition very good.

Thus at	3 Oct. 1907	4.45 G.M.T.	Ring quite easy.
		7.30 „	Ring difficult but distinct.
		8.0 „	Ring still seen.
		9.45 „	Ring still glimpsed.
		10.30 „	Ring invisible.

So that, as seen in a 9-in. refractor under circumstances of good definition, the ring system became invisible soon after 9.45 and before 10.30 G.M.T. As the ring was easy to see at 4.45, and got difficult as quickly, the Earth must have passed through its plane soon after invisibility.

Observations during the period that the dark sides of the rings are exposed to the Earth answer the question as to whether the rings are composed of solid particles or of vaporous matter. If the latter, the edges at least of the rings would be seen.

Johannesburg:
1907 *October* 5.

Observations of the Satellite of Neptune, from photographs taken at the Royal Observatory, Greenwich, between 1906 *December* 27 *and* 1907 *April* 24.

(*Communicated by the Astronomer Royal.*)

The following measures of position-angle and distance of Neptune's satellite were made from photographs taken with the 26-in. refractor of the Thompson equatorial. The occulting shutter was used as in previous years. The photographs were taken by Messrs. Davidson, Edney, or Melotte, and were measured in a position micrometer in direct and reversed position by Messrs. Davidson and Melotte. The tabular positions with which comparison is made were computed from the data given in the *Connaissance des Temps*, based on Dr. Hermann Struve's elements, the eccentricity of the orbit being neglected.

A discussion of these residuals gives the following differences from Dr. Hermann Struve's elements in the sense Tabular – Observed:

$$du = -1^{\circ}\cdot03 \quad dN = -0^{\circ}\cdot98 \quad dI = -0^{\circ}\cdot18 \quad da = -''\cdot008$$

giving for the epoch 1907·2

$$a = 16''\cdot279 \quad N = 188^{\circ}\cdot68 \quad I = 116^{\circ}\cdot69.$$

Neptune and Satellite.

Position-angle and Distance, from photographs taken with the 26-in. Refractor.

Date and G.M.T.	Position-angle.			Distance.		
	Obs.	Tab.	T – O.	Obs.	Tab.	T – O.
1906. d h m s	°	°	°	″	″	″
Dec. 27 10 57 45	305·13	305·07	–0·06	14·53	14·72	+0·19
1907.						
Jan. 17 10 14 43	105·43	106·11	+0·68	16·44	16·47	+0·03
17 10 42 11	104·02	105·30	+1·28	16·54	16·52	–0·02
17 11 7 30	102·39	104·57	+2·18	16·75	16·57	–0·18
30 10 30 31	44·20	45·99	+1·79	12·69	12·80	+0·11
30 10 56 6	43·18	44·76	+1·58	12·60	12·70	+0·10
30 11 22 51 (*a*)	43·40	43·43	+0·03	12·84	12·59	–0·25

Position-angle and Distance—continued.

Date and G.M.T.				Position-angle.			Distance.		
				Obs.	Tab.	T − O.	Obs.	Tab.	T − O.
1907.	d	h	m s	°	°	°	″	″	″
Feb.	1	12	46 8	269·00	269·10	+0·10	16·82	16·73	−0·09
	7	10	34 23 (*b*)	266·96	267·77	+0·81	16·53	16·65	+0·12
	21	11	2 29	115·59	117·16	+1·57	15·71	15·30	−0·41
	28	8	23 33	70·27	72·94	+2·67	15·39	15·42	+0·03
	28	8	54 19	71·73	71·93	+0·20	15·53	15·32	−0·21
	28	9	29 47 (*c*)	69·48	70·75	+1·27	14·96	15·20	+0·24
Mar.	1	9	40 41	356·64	357·01	+0·37	10·92	10·99	+0·07
	1	10	3 48	354·77	355·52	+0·75	10·73	11·01	+0·28
	1	10	25 37	352·86	354·13	+1·27	10·98	11·04	+0·06
	1	10	44 18	352·63	352·94	+0·31	11·10	11·06	−0·04
	23	9	6 19 (*b*)	93·73	93·30	−0·43	16·09	16·43	+0·34
	23	9	36 54 (*a*)	91·43	92·44	+1·01	16·68	16·42	−0·26
	23	10	6 38 (*a*)	90·23	91·61	+1·38	16·40	16·42	+0·02
	25	9	26 39 (*d*)	316·61	318·15	+1·54	12·88	12·79	−0·09
	27	9	1 44	218·08	218·66	+0·58	12·01	11·97	−0·04
	27	9	25 8	216·54	217·43	+0·89	12·10	11·89	−0·21
	27	9	56 16	214·39	215·76	+1·37	11·81	11·95	+0·14
	27	10	23 35 (*e*)	214·27	214·26	−0·01	11·74	11·69	−0·05
	28	8	58 37	133·22	135·42	+2·20	13·11	13·02	−0·09
	28	9	33 48 (*f*)	134·36	133·88	−0·48	13·01	13·16	+0·15
Apr.	6	9	50 47	301·44	302·67	+1·23	14·24	14·25	+0·01
	24	9	32 40	283·83	286·05	+2·22	15·51	15·69	+0·18

(*a*) Satellite faint and diffused.
(*b*) Satellite very faint and diffused.
(*c*) Photographic speck on Satellite.
(*d*) Very faint.
(*e*) Satellite diffused
(*f*) Very diffused. On fringe of secondary spectrum.

Royal Observatory, Greenwich:
1907 *November* 7.

Observations of Minor Planets, from photographs taken with the 30-in. Reflector of the Thompson Equatorial at the Royal Observatory, Greenwich, during the year 1906.

(*Communicated by the Astronomer Royal.*)

The following positions of minor planets were obtained from photographs taken with the 30-in. reflector during the year 1906.

The plates were measured with the astrographic micrometer. Six reference stars were, as a rule, measured with the planet, their positions being derived when possible from the Catalogues of the Astronomische Gesellschaft, or from the Radcliffe Catalogue, 1890.

The positions given are not corrected for Parallax.

Log Parallax Correction = log Parallax Factor − log Δ.

Date and G.M.T. 1906. d h m s	Apparent R.A. h m s	Apparent Dec. ° ′ ″	Log Parallax Factor. R.A.	Dec.
(324) Bamberga.				
Apr. 14 11 1 14	12 39 36·97	− 17 13 48·1	−8·223	+0·912
19 11 39 24	12 35 28·10	16 51 3·6	+8·943	0·909
25 10 33 53	12 30 54·16	16 22 50·6	+8·397	0·910
26 10 33 3	12 30 10·98	16 18 4·7	+8·500	0·909
(278) Paulina.				
Apr. 25 10 13 58	13 29 30·55	− 1 0 7·1	−9·004	+0·842
26 11 24 48	13 28 39·43	0 59 32·3	+8·283	0·842
27 10 30 35	13 27 51·53	0 59 8·2	−8·770	0·842
(191) Kolga.				
Apr. 28 11 0 8	14 21 35·91	+ 0 27 10·1	−8·955	+0·833
(146) Lucina.				
Apr. 28 11 42 9	14 22 38·29	+ 3 16 50·5	−8·408	+0·815
(443) Photographica.				
May 24 10 31 24	14 29 8·52	− 8 17 18·3	+8·162	+0·879
31 10 9 23	14 24 55·15	7 58 8·0	+8·463	0·878
(65) Cybele.				
May 24 11 16 15	14 45 18·23	− 11 0 58·8	+8·778	+0·890
31 10 37 31	14 41 19·64	10 45 10·0	+8·687	0·889
(148) Gallia.				
May 24 11 51 35	15 2 27·91	+18 46 52·8	+8·971	+0·679

Date and G.M.T. 1906.				Apparent R.A.			Apparent Dec.			Log Parallax Factor. R.A.	Dec.
d	h	m	s	h	m	s	°	′	″		
					(504) Cora.						
May 31	11	16	57	15	35	50·38	- 4	25	21·0	+8·381	+0·861
					(92) Undine.						
May 31	11	49	17	15	46	36·07	− 9	52	12·8	+8·776	+0·885
					(487) Venetia.						
June 26	11	0	51	17	29	23·80	15	39	33·1	−8·299	+0·907
					(374) Burgundia.						
June 22	11	47	58	18	3	21·22	−11	55	20·9	−8·359	+0·894
26	11	33	56	17	59	52·17	11	47	15·6	−8·167	0·894
					(409) Aspasia.						
June 22	12	18	7	18	35	22·41	−12	42	26·5	−8·360	+0·897
22	12	51	59	18	35	21·17	12	42	18·9	+8·515	0·897
25	11	0	28	18	32	34·52	12	30	22·0	−9·103	0·893
25	11	28	31	18	32	33·44	12	30	18·1	−8·918	0·895
26	12	1	21	18	31	34·61	12	26	19·1	−8·294	0·898
					(28) Bellona.						
June 22	12	18	7	18	33	10·10	−12	44	23·3	−8·418	+0·897
22	12	51	59	18	33	8·96	12	44	23·9	+8·469	0·897
25	11	6	2	18	30	37·38	12	50	4·3	−9·065	0·895
25	11	28	31	18	30	36·50	12	50	5·5	−8·901	0·896
26	12	1	21	18	29	43·15	12	52	13·3	−8·360	0·896
					(122) Gerda.						
June 26	12	33	45	18	34	29·04	−20	52	29·8	+8·437	+0·922
					(51) Nemausa.						
June 26	13	26	8	18	47	10·66	− 6	16	13·3	+8·950	+0·869
					(7) Iris.						
July 28	12	27	29	20	33	18·57	−11	26	56·1	+8·442	+0·892
31	12	44	57	20	30	10·29	11	31	25·5	+8·908	0·891
					(378) Holmia.						
Aug. 14	11	52	22	20	34	43·06	− 7	18	36·7	+8·885	+0·874
21	10	50	11	20	29	29·86	7	45	2·2	+8·462	0·877
22	10	25	22	20	28	49·23	7	48	55·8	−7·509	0·877

Date and G.M.T. 1906. d h m s	Apparent R.A. h m s	Apparent Dec. ° ′ ″	Log Parallax Factor. R.A.	Dec.
(434) Hungaria.				
July 25 13 3 36	21 31 19·97	+17 30 48·3	−8·428	+0·689
Aug. 7 11 37 33	21 22 6·04	14 37 43·9	−8·839	0·722
14 12 53 0	21 16 23·60	12 20 24·7	+9·033	0·746
15 12 56 5	21 15 35·25	11 58 45·8	+9·080	0·750
21 11 9 14	21 11 0·89	9 41 8·5	−7·812	0·766
(480) Hansa.				
Aug. 14 12 24 42	21 14 5·76	+18 43 19·0	+8·838	+0·677
15 12 34 9	21 13 13·47	18 40 17·7	+8·963	0·680
21 11 44 56	21 8 9·79	18 16 20·1	+8·756	0·682
22 10 46 34	21 7 22·50	18 11 33·0	−8·526	0·681
Sept. 7 9 18 9	20 56 11·42	16 20 30·8	−8·751	0·703
(136) Austria.				
Aug. 15 13 12 59	21 50 47·37	− 0 25 40·9	+8·947	+0·838
21 12 1 51	21 45 54·03	1 21 51·1	+8·349	0·844
22 11 24 48	21 45 6·31	1 31 37·2	−8·482	0·845
(386) Siegena.				
Aug. 7 11 51 38	22 1 18·45	+ 0 57 40·8	−9·023	+0·841
14 13 39 13	21 56 44·93	− 0 14 0·0	+9·059	0·837
15 13 32 2	21 56 4·35	− 0 24 54·9	+9·042	0·838
21 12 31 41	21 51 56·19	− 1 33 51·5	+8·770	0·845
22 12 6 4	21 51 14·90	− 1 45 44·3	+8·407	0·846
(308) Polyxo.				
Aug. 22 12 27 8	22 7 8·88	− 7 39 40·7	+8·549	+0·876
28 11 39 36	22 2 25·18	8 15 29·6	+7·509	0·879
(42) Isis.				
Aug. 28 14 46 31	23 44 1·69	−20 22 56·9	+9·165	+0·914
(26) Proserpina.				
Aug. 29 14 0 16	23 25 51·28	− 9 7 47·0	+9·010	+0·881
(478) Tergeste.				
Sept. 11 10 7 45	23 3 21·06	+12 44 57·6	−9·184	+0·746
25 11 40 33	22 53 35·71	11 10 17·6	+9·006	0·756
26 11 39 22	22 52 58·04	11 2 52·4	+9·025	0·757
27 11 21 1	22 52 21·54	10 55 29·7	+8·924	0·757
Oct. 10 10 27 36	22 45 43·85	9 16 16·1	+8·953	0·771

Date and G.M.T. 1906. d h m s	Apparent R.A. h m s	Apparent Dec. ° ′ ″	Log Parallax Factor. R.A.	Dec.
	(108) Hecuba.			
Sept. 26 12 16 15	0 29 18·35	+ 4 50 59·4	+7·983	+0·804
27 11 46 27	0 28 35·36	4 47 22·2	−8·483	0·804
Oct. 16 12 0 41	0 15 1·30	3 36 19·3	+9·119	0·816
	(47) Aglaia.			
Sept. 26 12 42 44	0 58 33·13	+ 7 21 20·3	+7·809	+0·785
27 12 6 35	0 57 44·66	7 18 29·5	−8·653	0·786
Oct. 16 12 19 46	0 41 33·92	6 16 6·2	+9·084	0·796
23 10 20 58	0 36 17·53	5 54 41·8	−8·206	0·796
	(175) Andromache.			
Sept. 26 13 22 52	1 18 13·42	+ 7 14 39·6	+8·586	+0·786
27 12 21 8	1 17 32·63	7 11 39·4	−8·724	0·787
Oct. 23 10 41 27	0 57 44·73	5 42 55·6	−8·247	0·797
	(153) Hilda.			
Oct. 16 12 52 59	1 19 45·76	+13 15 40·6	+9·065	+0·738
23 11 48 14	1 15 32·48	12 40 20·3	+8·803	0·740
	(500) Selinur.			
Dec. 6 10 38 13	5 14 52·34	+31 53 24·5	−9·240	+0·500
1907.				
Jan. 2 8 51 22	4 47 56·08	29 14 39·5	−9·107	0·535
	(19) Fortuna.			
1906.				
Dec. 14 11 24 9	5 41 7·53	+21 6 39·2	−8·894	+0·649

Royal Observatory, Greenwich:
1907 *November* 8.

Results of Micrometer Measures of Double Stars made with the 28-inch Refractor at the Royal Observatory, Greenwich, in the year 1906.

(*Communicated by the Astronomer Royal.*)

The measures were made with a bifilar position micrometer on the 28-inch refractor, focal length 28 feet. The power generally employed was 670. When bright stars were observed a blue glass shade was usually employed to diminish the light and irradiation. The observations were made in variously coloured fields or in a dark field with illuminated wires. The initials in the last column are those of the observers, viz. :—

L.	Mr. Lewis	B.	Mr. Bryant
W. B.	Mr. Bowyer	H. F.	Mr. Furner

The main portion of the measures are of pairs discovered by Hough, which usually consist of a bright star with a faint companion. The remaining measures are of stars selected from the catalogues of W. Struve, Otto Struve, Burnham, Hussey, and Aitken.

In general the present list of measures is confined to stars of which the separation is under 4″ or which show orbital motion.

The following stars, which are not included in the list of measures, have also been observed.

Hough Stars.

20	116	323	387	446 AC	501	573
26	117	327	394	448	519	576 AC
28	131	337 AB	400	452 AB	523	586
29	204	337 AC	406 AB	452 AC	534	597
30	236	343	406 AC	460	539	603
31	240	347	412	465	543	604
32	259	348	416	473	548	606
38	269	349	423	475 AC	550	611
39	281	362 AB	425	476	551	613
45	297	362 AC	426	477	552	615
64	302	365	433	478	553	616
91	305 AB	372	434	486	557	617
115	305 AC	377	440	487	570	

Struve Stars.

877	1934	2185 AB	2476 AC	2690 AB	2818 R
1297	1965	2185 AC	2585 AC	2690 AD	2824

Micrometric Observations of Double Stars at the Royal Observatory, Greenwich.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Dist.	Number of Nights.	Magnitudes.		Epoch 1906.	Obsr.
	h m	° ′	°	″					
β 1093	0 16	79 31	73·1	0·29	1	7·3	8·2	·882	B.
β 779	0 23	66 55	257·1	0·78	1	8·5	9·0	·882	B.
Ho 212	0 31	94 5	244·4	0·24	1	6·0	6·0	·882	B.
Hu 411	0 33	67 50	279·3	0·67	3	8·5	8·5	·884	W. B.
Ho 306	0 43	64 56	165·3	1·21	3	8·5	8·8	·884	W. B.
Σ 73	0 50	66 52	28·1	0·97	3	6·2	6·8	·860	B.
	...	...	27·9	0·99	3	...		·879	W. B.
Ho 307	0 52	58 17	88·8	3·01	1	9·5	9·7	·928	W. B.
Σ 113	1 15	91 0	357·5	1·56	1	6·2	7·2	·882	B.
Ho 310	1 21	61 55	359·0	1·52	2	9·0	9·0	·882	W. B.
Ho 9	1 24	68 45	97·2	3·12	2	9·0	11·0	·909	W. B.
Σ 138	1 31	82 50	38·7	1·62	2	7·3	7·3	·904	B.
Σ 183 AB	1 50	61 39	359·1	0·36	2	7·5	8·2	·904	B.
AC	...	...	162·9	5·19	2	7·5	8·7	·904	B.
Σ 186	1 51	88 37	36·8	0·68	1	7·2	7·2	·882	B.
Ho 12	1 57	55 46	97·2	3·75	1	8·0	10·7	·068	H. F.
Ho 315	2 34	91 58	359·7	1·38	2	8·0	8·2	·454	B.
Σ 305	2 42	71 1	315·0	3·30	2	7·3	8·2	·901	W. B.
	...	...	314·4	2·88	1	...		·925	B.
Ho 317	2 52	73 7	304·5	3·36	2	8·1	11·0	·456	W. B.
Ho 318	2 54	73 19	27·6	3·22	2	9·1	9·1	·454	B.
Σ 367	3 9	89 37	209·0	0·49	1	8·0	8·0	·882	B.
Ho 320	3 16	89 9	169·2	2·08	1	8·0	10·5	·025	B.
Aitken 979	3 20	59 35	274·0	1·45	1	9·2	10·0	·890	W. B.
Ho 14	3 28	62 1	201·0	1·88	1	8·2	8·7	·019	L.
Ho 324	3 44	75 18	162·8	0·68	1	8·1	8·3	·019	L.
β 1232	4 4	61 3	4·5	0·48	1	8·4	9·3	·019	L.
Ho 328	4 12	70 33	245·1	0·47	1	7·0	7·0	·068	H. F.
Ho 15	4 19	60 5	323·4	0·79	2	8·0	8·0	·041	W. B.
Hu 304	4 19	80 44	38·9	0·31	1	5·9	5·9	·025	B.
Ho 332	4 38	69 31	127·1	1·32	1	9·0	9·0	·025	B.
Ho 333	4 38	69 55	160·8	2·55	1	9·2	9·3	·025	B.
Ho 17	4 54	59 7	56·2	4·11	2	8·0	10·0	·065	W. B.
Ho 222	4 54	58 33	223·1	2·11	1	7·7	10·5	·022	W. B.
Ho 224 AB	4 59	61 24	297·2	1·46	2	9·0	10·7	·137	H. F.
Ho 334	5 12	67 16	183·7	2·12	2	8·1	10·2	·055	W. B.
Ho 225	5 17	77 25	115·8	0·51	2	8·0	8·1	·100	W. B.

UNIVERSITY OF CALIFORNIA

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Dist.	Number of Nights.	Magnitudes.		Epoch 1906.	Obsr.
	h m	° ′	°	″					
Ho 226 AB	5 21	62 28	239·0	0·62	4	7·0	7·0	·074	W. B.
$\frac{AB}{2}$ & C	...	...	273·6	23·57	3	7·0	10·5	·087	W. B.
Ho 335	5 27	63 17	114·1	3·19	2	9·0	10·5	·086	W. B.
Σ 749	5 31	63 6	169·3	0·81	1	7·0	7·1	·255	B.
Hu 825	5 34	54 3	339·5	0·30	1	7·8	8·0	·255	B.
Ho 510	5 39	56 19	241·2	1·31	1	9·0	9·2	·255	B.
Hu 38	5 40	67 8	138·0	0·77	1	8·6	8·8	·255	B.
Hu 39	5 42	68 10	65·5	0·43	1	8·4	8·5	·255	B.
Hu 40	5 46	69 53	8·0	3·29	1	8·5	9·5	·255	B.
Ho 228	6 1	77 30	270·2	2·38	2	8·0	11·0	·064	W. B.
Ho 22	6 8	79 43	198·1	0·72	2	8·0	8·0	·140	B.
Ho 230	6 14	76 11	60·0	1·89	4	8·3	10·5	·085	W. B.
Ho 232	6 17	75 17	356·5	2·58	1	9·5	11·0	·068	H. F.
Ho 233	6 18	73 26	38·3	2·24	2	8·2	11·0	·041	W. B.
Hu 702	6 19	55 33	318·3	0·93	3	8·5	9·0	·273	W. B.
Oζ 141	6 25	72 2	139·6	2·12	2	7·9	9·6	·130	W. B.
Σ 945	6 34	48 54	269·9	0·75	1	7·1	8·0	·255	B.
Σ 3117	6 36	80 10	94·2	0·66	1	8·9	9·4	·019	L.
Σ 3118	6 37	80 5	174·3	2·97	1	9·0	9·5	·019	L.
Ho 238	6 41	71 41	182·4	0·49	2	8·3	8·3	·155	W. B.
Oζ 156	6 42	71 42	298·1	0·56	3	6·5	7·0	·150	W. B.
Ho 239	6 45	75 12	139·9	0·59	1	8·3	8·3	·145	W. B.
Ho 27	6 50	69 47	125·7	3·27	2	9·0	9·0	·041	W. B.
Ho 342	6 58	76 46	84·9	1·02	3	8·0	8·8	·141	W. B.
Ho 518	7 5	59 29	143·6	2·97	2	8·0	10·0	·141	W. B.
Σ 1037 AB	7 7	62 37	297·9	0·60	1	7·1	7·1	·192	L.
AC	...	...	111·5	17·24	1	7·1	12·0	·192	L.
Ho 33	7 15	67 40	324·5	2·69	2	9·0	12·0	·099	W. B.
Ho 243	7 17	60 34	166·4	2·29	1	9·3	9·5	·164	W. B.
Ho 345 AB	7 17	67 45	284·0	1·16	1	9·0	10·0	·236	H. F.
Ho 346	7 21	71 40	60·2	13·18	2	7·0	11·8	·221	H. F.
Ho 34	7 26	68 44	10·5	2·45	2	9·5	9·5	·078	W. B.
Hu 707	7 29	68 11	27·2	2·23	2	8·6	12·0	·221	H. F.
Hu 457	7 35	66 33	143·6	2·30	1	8·5	12·3	·192	L.
Ho 247	7 41	68 39	106·9	0·54	1	7·5	8·0	·140	W. B.
Ho 36	7 42	64 19	123·1	0·65	2	8·5	8·5	·267	W. B.
Ho 250	7 52	68 47	168·3	0·52	2	7·0	9·0	248	H. F.
β 581 $\frac{AB}{2}$ & C	7 59	77 28	191·9	4·46	2	8·0	11·0	·217	B.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Dist.	Number of Nights.	Magnitudes.	Epoch 1906.	Obsr.
	h m	° ′	°	″				
Ho 351	8 2	68 51	228·9	2·32	1	7·0 11·7	·236	H. F.
Σ 1187	8 4	57 31	43·3	2·22	2	7·1 8·0	·261	W. B.
Σ 1196 AB	8 7	72 4	348·3	1·03	1	5·0 5·7	·101	B.
	...	...	347·0	1·04	2	...	·233	H. F.
	...	...	346·1	1·09	2	...	·251	W. B.
AC	...	...	111·2	5·19	1	5·0 5·5	·101	B.
	...	...	108·9	5·15	2	...	·233	H. F.
	...	...	109·3	5·36	2	...	·251	W. B.
BC	...	...	120·0	6·18	1	5·7 5·5	·101	B.
	...	...	122·6	6·22	2	...	·251	W. B.
Ho 524	8 11	71 2	337·1	4·10	2	8·0 11·0	·203	W. B.
Hu 626	8 14	57 23	155·3	2·88	1	8·0 12·0	·315	W. B.
Hu 855	8 17	76 33	225·0	1·42	1	9·2 10·8	·268	L.
Ho 525	8 18	69 42	334·4	0·49	1	8·5 8·5	·268	L.
Hu 714	8 21	57 30	344·3	0·53	1	8·6 9·2	·268	L.
Ait. 551	8 24	92 13	56·5	0·24	1	7·4 7·5	·312	B.
Hu 627	8 25	55 10	268·1	0·75	1	9·0 10·0	·315	W. B.
Hu 717	8 27	57 14	52·7	0·45	2	8·8 9·0	·307	L.
Hu 858	8 33	77 43	171·6	0·91	2	9·1 9·8	·248	H. F.
Ho 354	8 37	63 37	177·6	0·97	2	8·2 8·2	·307	L.
Ho 355	8 38	92 22	185·4	0·36	1	8·0 8·0	·312	B.
Σ 1273 AC	8 42	83 15	236·0	2·85	1	3·3 7·8	·270	W. B.
Ho 360	8 55	67 11	149·9	3·60	4	8·0 12·0	·243	W. B.
Ho 41	8 59	92 2	71·4	4·08	2	9·0 10·0	·217	B.
Ho 42	9 9	56 4	6·8	1·41	3	9·5 9·5	·228	W. B.
Σ 3121	9 13	61 2	216·8	0·42	2	7·5 7·8	·251	W. B.
	...	...	211·7	0·57	2	...	·284	B.
Ho 43	9 14	68 48	298·7	0·39	1	8·0 8·5	·315	W. B.
	...	...	288·9	0·27	1	...	·345	L.
Ho 364	9 14	66 42	334·8	3·97	2	8·2 11·2	·171	W. B.
Σ 1348	9 20	83 15	318·5	1·76	2	7·5 7·6	·274	W. B.
Σ 1355	9 22	83 21	335·1	2·45	2	7·2 7·2	·284	W. B.
Ho 366	9 23	58 8	14·8	0·69	2	8·5 8·7	·326	L.
Σ 1356	9 23	80 34	114·4	0·76	2	6·2 7·0	·284	B.
Ho 367	9 28	61 14	257·9	32·18	2	6·2 11·0	·214	W. B.
Hu 1128	9 30	53 47	36·8	5·40	1	5·5 14·0	·345	L.
Ho 253	9 43	79 30	293·6	1·13	2	7·0 12·0	·226	W. B.
Hu 631	9 59	56 55	271·0	0·73	1	7·0 8·6	·312	B.
	...	...	272·2	0·74	2	...	·326	L.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Dist.	Number of Nights.	Magnitudes.		Epoch 1906.	Obsr.
	h m	° ′	°	″					
Hu 634	10 11	56 24	161·3	1·69	2	8·4	9·8	·233	H. F.
Hu 875	10 13	52 2	62·3	0·96	1	7·0	9·8	·307	L.
	...	...	71·6	0·93.	1	...		·315	W. B.
Hu 879	10 23	52 50	234·6	0·42	1	4·0	6·5.	·312	B.
Σ 1439	10 25	68 44	112·8	1·86	3	8·0	8·5	·275	H. F.
Hu 880	10 26	52 23	124·9	0·80	1	9·0	9·7	·312	B.
Hu 636	10 27	56 42	204·1	1·86	1	9·0	10·5	·260	H. F.
Ho 374	10 45	66 41	271·0	3·71	3	8·4	12·0	·220	W. B.
Ho 376	10 48	66 19	218·4	3·17	2	8·8	10·0	·195	W. B.
Ho 48	10 59	66 22	8·7	1·93	3	8·0	11·2	·288	W. B.
Σ 1536	11 19	78 58	51·9	2·39	1	3·9	7·1	·255	B.
Ho 51	11 27	81 38	176·2	2·47	1	7·0	12·0	·318	W. B.
Hu 888	11 38	68 25	142·2	0·64	2	8·4	8·9	·266	W. B.
Lewis	11 54	59 2	9·2	0·84	1	7·0	11·5	·345	L.
Hu 890	11 57	78 10	64·2	1·38	3	9·0	10·0	·261	W. B.
Ho 535	11 58	67 44	139·6	2·18	3	8·0	12·0	·267	W. B.
Ho 53	12 19	75 35	296·4	2·11	3	8·0	11·7	·330	W. B.
Σ 1639	12 20	63 55	357·2	0·37	2	6·7	7·9	·370	B.
Σ 1643	12 23	62 23	39·1	2·29	1	8·7	9·2	·268	L.
Σ 1647	12 26	79 47	223·9	1·59	1	7·5	7·8	·268	L.
	...	...	224·2	1·45	3	...		·303	W. B.
Ho 54 BC	12 37	79 37	142·7	2·14	1	7·0	10·0	·318	W. B.
BD	...	...	64·6	10·44	1	10·0	13·5	·318	W. B.
Hu 892	12 38	74 59	160·7	0·78	2	9·3	9·3	·284	B.
Hu 893	12 43	77 13	33·8	1·50	2	9·1	9·1	·284	B.
Hu 894	12 49	76 20	144·7	1·08	2	9·1	9·1	·284	B.
Ho 257	13 1	63 17	154·6	1·75	2	8·8	8·9	·297	W. B.
Oz 261	13 8	57 26	344·8	1·37	1	6·9	7·4	·427	B.
Ho 260	13 19	60 8	321·2	0·70	3	8·3	8·5	·371	W. B.
Oz 269	13 29	54 38	217·3	0·37	1	6·5	7·0	·427	B.
Hu 645	13 35	68 5	30·4	0·88	1	9·2	9·6	·345	L.
Oz 278	14 9	45 24	76·4	0·28	1	7·5	7·7	·427	B.
Ho 541	14 16	77 25	91·6	2·12	5	9·3	10·2	·377	W. B.
Σ 1834	14 17	41 5	2·2	0·14	1	7·1	7·2	·427	B.
Aitken 1105	14 23	58 58	185·6	1·01	3	9·1	9·4	·417	W. B.
Ho 542	14 23	68 59	252·7	0·53	2	6·8	8·8	·370	B.
Σ 1865	14 37	75 54	145·9	0·47	3	3·5	3·9	·401	B.
Oz 285	14 42	47 14	126·1	0·31	1	7·1	7·6	·427	B.
Ho 263	14 44	65 32	175·4	1·07	2	7·0	10·0	·467	W. B.

LIBRARY OF THE

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Dist.	Number of Nights.	Magnitudes.	Epoch 1906.	Obsr.
	h m	° ′	°	″				
Ho 389	14 48	69 20	95·1	1·51	3	7·0 9·3	·372	W. B.
OΣ 287	14 48	44 42	321·6	1·04	1	7·5 7·6	·427	B.
OΣ 288	14 49	73 56	188·7	1·61	2	6·4 7·1	·362	W. B.
Σ 1909	15 1	41 57	244·6	4·60	2	5·2 6·1	·701	L.
Ho 60	15 10	54 47	35·9	0·41	2	7·5 7·6	·467	W. B.
Σ 1926	15 11	51 22	249·9	0·94	1	6·1 8·4	·671	L.
Σ 3091	15 11	94 33	25·5	0·25	1	7·7 7·7	·465	B.
Ho 547	15 12	72 52	297·1	5·55	2	7·9 12·0	·458	W. B.
Σ 1934	15 14	45 56	27·7	7·35	2	8·5 8·5	·701	L.
Σ 1944	15 23	83 36	325·3	1·22	2	7·5 8·1	·418	W. B.
OΣ 296	15 23	45 41	302·4	1·58	2	7·0 8·6	·701	L.
Lewis	15 24	43 32	334·9	3·06	2	9·5 9·8	·701	L.
Σ 1954	15 30	79 9	186·2	3·30	2	3·2 4·1	·458	W. B.
Ho 63	15 32	61 19	304·5	1·53	1	9·0 9·2	·312	B.
	...	...	307·6	1·24	2	...	·437	W. B.
OΣ 298	15 33	49 54	13·0	1·29	1	7·0 7·3	·671	L.
Hu 580	15 38	70 2	69·5	0·20	2	5·0 5·0	·446	B.
Ho 399	15 52	60 12	119·8	2·97	3	7·5 10·0	·448	W. B.
OΣ 303	15 57	76 28	144·9	0·81	3	7·4 7·9	·446	W. B.
Σ 1998	15 59	101 8	358·0	0·35	1	4·9 5·2	·465	B.
Σ 2021	16 9	76 14	336·2	4·08	2	6·7 6·9	·408	W. B.
Σ 2022	16 9	63 6	141·0	2·56	1	6·2 9·8	·460	L.
OΣ 309	16 16	48 7	69·8	0·49	1	7·5 7·8	·427	B.
Σ 2049	16 24	63 50	205·2	1·33	1	6·5 7·5	·460	L.
Σ 3105	16 27	96 50	30·7	0·35	1	7·7 7·7	·465	B.
Σ 2084	16 38	58 14	174·4	1·01	2	3·0 6·3	·446	B.
	...	...	175·4	1·09	2	...	·546	L.
	...	...	178·6	1·21	2	...	·563	W. B.
Σ 2091	16 39	48 38	125·6	1·03	2	7·5 8·0	·701	L.
Σ 2097	16 41	54 5	86·0	2·04	1	8·5 8·7	·441	L.
De 15	16 41	46 21	294·6	0·60	2	8·2 8·6	·701	L.
Σ 2101	16 42	54 12	53·3	4·56	1	6·3 9·0	·441	L.
Σ 2106	16 47	80 26	300·9	0·35	1	6·7 8·4	·482	W. B.
Σ 2107	16 48	61 11	358·8	0·80	1	6·5 8·0	·460	L.
Ho 409	16 52	66 30	19·5	8·19	2	8·1 13·0	·447	W. B.
Σ 2118	16 56	24 50	85·9	0·35	1	6·4 6·9	·427	B.
Σ 2114	16 57	81 25	161·4	1·36	1	6·2 7·4	·482	W. B.
Ho 411	16 59	66 10	262·6	2·09	2	8·3 12·0	·451	L.
Hu 167	17 4	80 3	63·4	0·61	1	9·5 9·8	·460	L.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Dist.	Number of Nights.	Magnitudes.	Epoch 1906.	Obsr.
	h m	°	°	″				
	...	...	62·9	0·63	1	...	·482	W. B.
Hu 170	17 8	80 8	274·5	1·48	1	8·5 10·8	·460	L.
	...	...	274·0	1·93	1	...	·482	W. B.
Ho 414	17 19	63 50	95·2	0·72	2	8·4 8·8	·555	W. B.
Ho 415	17 19	64 10	329·3	1·30	2	8·0 8·7	·555	W. B.
Σ 2171	17 24	99 56	75·6	1·51	1	7·5 7·6	·580	B.
Σ 2173	17 25	90 59	318·7	0·55	2	6·1 6·3	·589	B.
Hu 179	17 27	78 43	50·3	2·24	2	8·8 8·9	·555	W. B.
Σ 2205	17 40	72 15	303·3	2·11	2	8·3 8·7	·467	W. B.
Σ 2215	17 41	72 16	292·0	0·77	2	5·9 7·9	·467	W. B.
Ho 422	17 48	95 18	20·8	0·58	1	8·2 9·0	·580	B.
Ho 424	17 54	61 44	210·2	1·44	1	8·0 11·0	·460	L.
Σ 2272	18 1	87 28	170·6	2·32	11	4·5 6·0	·595	W. B.
Σ 2283	18 5	83 52	87·0	0·82	1	7·2 7·7	·580	B.
Hu 314	18 5	71 23	142·5	0·38	2	8·3 8·5	·631	W. B.
Hu 315	18 5	66 27	54·7	0·52	1	9·3 9·3	·627	W. B.
Hu 316	18 6	71 45	151·0	1·72	3	9·0 10·1	·620	W. B.
Hu 317	18 7	72 48	20·6	1·88	3	8·5 8·8	·620	W. B.
Hu 196	18 10	81 3	345·3	0·45	1	9·0 9·2	·663	H. F.
Hu 319	18 13	67 13	79·6	0·25	1	9·2 9·6	·627	W. B.
Hu 238	18 18	80 6	344·9	1·46	1	8·6 9·2	·663	H. F.
Ho 83	18 20	62 32	274·5	0·61	2	8·8 8·8	·631	W. B.
Ho 84	18 21	62 37	316·6	3·88	1	9·0 11·0	·731	L.
Σ 2315	18 21	62 40	199·3	0·31	1	7·0 8·0	·731	L.
Hu 583	18 26	76 16	307·5	0·80	1	9·0 9·5	·663	H. F.
Hu 245	18 29	78 17	232·9	2·24	1	8·2 9·2	·663	H. F.
Hu 247	18 33	79 49	42·4	0·51	1	9·0 9·3	·663	H. F.
Hu 248	18 35	80 56	295·0	2·00	1	9·5 9·6	·663	H. F.
Σ 2367 AB	18 37	59 49	254·4	0·40	1	7·0 7·5	·808	L.
$\frac{AB}{2}$ & C	...	...	195·9	14·13	1	7·0 8·2	·808	L.
Ho 437 AB	18 37	58 27	297·6	0·44	1	8·3 8·5	·808	L.
AC	...	...	291·3	22·72	1	8·3 13·0	·808	L.
Dd	...	...	344·9	3·81	1	11·2 11·7	·808	L.
Hu 254	18 46	81 59	168·7	1·03	1	8·9 13·5	·736	H. F.
Hu 256	18 46	81 24	186·4	3·78	2	8·5 12·8	·700	H. F.
Hu 258	18 49	78 30	213·8	2·58	3	8·9 9·4	·629	W. B.
Σ 2422	18 54	64 1	95·7	0·88	2	7·6 7·7	·680	W. B.
Hu 330	18 55	70 31	31·7	0·77	2	9·0 9·3	·700	H. F.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Dist.	Number of Nights.	Magnitudes.	Epoch 1906.	Obsr.
	h m	° ′	°	″				
Hu 678	18 56	77 54	363·1	3·04	3	8·0 10·5	·651	W. B.
Ho 92	18 57	57 37	34·0	1·33	1	9·0 9·1	·808	L.
Σ 2437	18 58	70 58	58·1	0·89	2	7·8 8·0	·681	W. B.
Ho 443	19 6	70 44	117·6	3·04	2	9·5 9·5	·700	H. F.
	...	...	117·9	2·77	1	...	·665	W. B.
β 1204 AB	19 7	87 32	8·7	0·32	2	7·7 8·5	·753	B.
Σ 2476 $\frac{AB}{2}$ & C		...	213·7	31·08	2	6·2 11·0	·753	B.
Ho 445	19 9	65 34	244·8	5·18	2	9·3 10·2	·660	W B.
Ho 446 AB	19 9	65 35	291·1	2·92	1	8·0 13·5	·736	H. F.
Ho 447	19 13	62 14	173·2	2·05	1	9·5 9·5	·665	W. B.
Ho 576 AB	19 16	83 21	181·2	3·59	1	7·0 10·7	·736	H. F.
Ho 105	19 18	73 31	187·5	2·56	3	8·5 10·0	·600	W. B.
Hu 339	19 21	71 32	47·0	0·50	2	8·6 8·6	·659	W. B.
Ho 451	19 23	62 18	297·0	4·14	2	9·3 11·0	·660	W. B.
Σ 2525	19 23	62 51	310·2	0·66	1	7·4 7·6	·682	W. B.
Hu 340	19 24	71 43	120·9	0·85	2	9·0 9·3	·659	W. B.
Σ 2536	19 28	72 25	79·5	1·78	1	8·0 11·0	·736	H. F.
Ho 274	19 31	73 45	74·1	3·75	3	8·3 11·0	·680	W. B.
Ho 110	19 35	71 31	78·9	2·50	2	9·5 9·5	·660	W. B.
Ho 112	19 37	71 35	79·4	2·65	3	9·0 9·0	·600	W. B.
Hu 1195	19 39	76 31	322·5	2·42	1	8·7 14·5	·736	H. F.
Hu 346	19 44	73 7	184·4	0·62	1	8·8 9·5	·736	H. F.
A.G.C. 11 AB	19 45	71 6	145·3	0·18	1	5·7 6·2	·810	B.
Σ 2596	19 50	74 58	320·6	2·12	1	7·2 8·6	·813	W. B.
Ho 583	19 55	68 8	252·6	1·59	1	9·0 10·7	·682	W. B.
Ho 584	19 57	64 3	228·7	2·05	1	6·5 12·0	·723	W. B.
Ho 118	19 59	56 35	26·7	2·73	1	9·1 11·0	·723	W. B.
Ho 591	20 14	62 23	292·1	2·14	1	9·5 10·0	·813	W. B.
Ho 457	20 21	60 55	60·8	2·21	2	8·1 8·1	·601	W. B.
Ho 130	20 23	53 6	106·3	2·51	1	8·5 8·7	·652	L.
Ho 131	20 24	71 32	326·1	4·82	2	7·8 11·2	·641	W. B.
Σ 2690 BC	20 27	79 3	234·5	0·27	1	7·5 7·5	·810	B.
β 151	20 33	75 43	100·3	0·15	2	4·7 6·1	·791	B.
Ho 458	20 35	61 10	279·8	1·98	2	9·0 9·1	·791	W. B.
Ho 144	20 48	70 13	167·7	0·42	1	7·0 7·0	·723	W. B.
Ho 150	21 4	71 31	135·1	2·69	2	9·0 11·5	·693	W. B.
Ho 151	21 6	86 31	192·0	1·78	1	8·5 8·5	·616	W. B.
Hu 767	21 9	74 24	157·5	0·25	2	7·0 7·0	·791	B.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Dist.	Number of Nights.	Magnitudes.	Epoch 1906.	Obsr.
	h m	° ′	°	″				
OΣ 535	21 10	80 24	212·2	0·16	3	4·1 4·1	·773	L.
	..	...	230·8	0·16	2	...	·791	B.
	...	...	227·7	0·24	2	...	·827	W. B.
Ho 284	21 12	74 24	89·0	3·57	3	9·0 9·3	·703	W. B.
Ho 154	21 14	59 48	208·6	3·61	1	7·8 11·0	·915	H. F.
Ho 155	21 16	57 8	35·8	2·37	2	8·0 9·5	·772	W. B.
Ho 157	21 19	58 22	23·1	3·80	2	7·7 7·7	·772	W. B.
Aitken 1219	21 21	60 8	124·3	1·33	1	9·3 9·4	·775	W. B.
Hu 276	21 24	82 40	26·7	0·85	1	9·3 9·7	·843	L.
Σ 2799	21 24	79 21	118·0	1·66	1	6·6 6·6	·843	L.
Σ 2804	21 29	69 41	155·4	2·97	2	7·3 8·0	·872	L.
Ho 161	21 28	50 20	359·5	2·50	1	7·0 11·0	·915	H. F.
Hu 371	21 31	65 57	169·9	0·22	1	7·0 7·5	·810	B.
Ho 164	21 37	55 20	66·6	3·75	1	8·0 8·1	·915	H. F.
Ho 166	21 40	62 34	73·9	0·31	3	7·5 7·5	·760	W. B.
	...	...	71·1	0·31	1	...	·862	L.
Σ 2822	21 40	61 39	124·1	2·22	2	4·0 5·0	·616	W. B.
β 989	21 41	64 46	104·5	0·27	5	3·9 4·4	·714	W. B.
	...	...	107·8	0·19	6	...	·821	L.
	...	...	108·1	0·49	1	...	·736	H. F.
	...	...	107·9	0·27	2	...	·791	B.
Ho 608	21 43	63 7	119·5	0·64	1	8·2 9·7	·862	L.
Ho 466	21 44	55 32	144·7	1·61	1	8·7 9·2	·915	H. F.
Ho 467	21 46	68 10	186·2	1·45	3	8·0 10·2	·689	W. B.
Ho 173	21 51	71 44	82·0	1·17	1	8·0 10·0	·843	L.
Ho 609	21 51	60 4	176·7	3·20	2	9·5 9·8	·794	W. B.
Ho 610	21 58	63 35	231·0	0·66	1	9·0 9·2	·862	L.
Hu 978	22 8	76 32	228·6	0·68	1	8·5 9·0	·819	W. B.
Ho 179	22 9	60 14	251·0	0·65	1	8·0 9·0	·819	W. B.
Ho 182	22 19	72 54	128·3	1·43	4	8·5 8·5	·809	W. B.
Ho 183	22 21	67 53	216·3	2·51	1	8·2 11·6	·843	L.
Ho 475 AB	22 29	64 3	317·6	0·89	2	8·0 8·2	·827	W. B.
Ho 294	22 33	63 2	62·9	2·10	1	8·0 10·0	·819	W. B.
Ho 296	22 36	75 56	68·5	0·32	2	5·5 5·5	·846	B.
Ho 190	22 45	62 28	153·7	2·03	3	9·2 9·2	·822	W. B.
Ho 482	22 47	64 5	73·0	0·14	1	6·8 6·8	·882	B.
Ho 484	22 54	69 44	108·5	3·19	1	8·0 12·0	·805	L.
Ho 192	22 55	60 24	29·7	1·82	2	8·5 8·5	·874	W. B.
Hu 398	22 58	71 53	323·8	0·37	1	8·7 9·0	·805	L.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Dist. ″	Number of Nights.	Magnitudes.		Epoch 1906.	Obsr.
Ho 299	23 10	66 15	77·0	1·48	1	8·0	10·2	·835	W. B.
Ho 198	23 13	73 38	33·7	2·26	4	8·0	9·0	·856	W. B.
Ho 300	23 19	78 11	304·1	0·15	1	5·0	5·0	·882	B.
Ho 301	23 20	76 1	359·7	1·09	1	7·2	11·0	·835	W. B.
Ho 489	23 22	62 47	240·0	0·42	1	7·8	7·8	·835	W. B.
Ho 201	23 33	55 51	340·9	4·11	1	8·0	9·3	·915	H. F.
Ho 303	23 36	70 8	187·4	1·15	1	8·0	11·0	·862	L.
Hu 300	23 41	84 1	120·3	1·29	1	8·7	9·0	·882	B.
Ho 206	23 54	56 14	184·4	2·11	1	8·0	10·0	·928	W. B.

Royal Observatory, Greenwich:
1907 *November* 6.

Proper motions of 1186 *Carrington Stars from a direct comparison between Carrington's Catalogue for* 1855 *and the Greenwich Second Nine-year Catalogue for* 1900.

(*Communicated by the Astronomer Royal.*)

The proper motions of the following stars, which have been observed with the transit circle at Greenwich during the years 1897–1905, for the purpose of determining their places for use as reference stars on the astrographic plates, have been derived from a direct comparison with Carrington's Circumpolar Catalogue for 1855. For this purpose the places in Carrington's Catalogue have been reduced to the epoch 1900 by the ordinary trigonometrical formula with the Struve-Peters' constants, and in addition the systematic corrections given on p. 321 of *Monthly Notices*, lxvi. 5, have been applied.

Full details will be published in the Introduction to the Greenwich Nine-year Catalogue, 1900. The proper motions for both R.A. and N.P.D. are given in arc, and arranged in Zones of N.P.D.; the magnitudes are taken from the Harvard Photometry (marked *) and the Bonn Durchmusterung.

ZONE 0°.

Carr. No.	Mag.	Approx. R.A. 1900. h m	Proper Motions. Unit ″·001. R.A. +	R.A. −	N.P.D. +	N.P.D. −
201	9·2	1 45	...	0	7	...
303	9·0	2 32	...	3	...	22
b	8·8	3 19	...	3	...	40
538	9·2	4 28	...	18	...	4
d	9·1	6 39	...	27	...	4
e	9·1	8 30	...	17	...	22
g	9·0	11 0	...	83	...	31
h	8·9	11 35	...	25	...	13
i	8·8	12 42	...	20	2	...
l	9·0	13 51	3	...	7	...
t	9·3	15 29	...	5	...	31
r	9·3	15 55	10	...	...	9
n	8·7	16 9	22	...	7	...
3601	9·0	23 11	2	...	...	9

Zone 1°.

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−
23	8·8	0 16	...	8	...	2
117	6·5*	0 56	68	...	31	...
137	9·2	1 0	...	24	9	...
183	8·0	1 18	24	...	18	...
181	2·1*	1 22	36	...	...	0
251	8·2	1 50	36	...	13	...
263	8·8	1 56	51	...	31	...
284	8·0	2 14	...	23	33	...
305	8·8	2 17	...	175	...	35
331	9·0	2 35	0	...	29	...
347	8·5	2 42	5	...	38	...
375	8·7	2 52	25	...	35	...
412	9·0	3 5	...	0	...	2
545	8·5	4 5	...	35	...	9
739	8·8	5 47	...	18	28	...
763	8·8	5 50	...	6	...	7
823	9·0	6 18	...	18	22	...
1035	7·0*	7 58	...	24	...	20
1089	9·2	8 8	...	31	39	...
1095	9·2	8 19	...	26	20	...
1336	9·1	9 27	...	25	17	...
1490	8·5	10 19	...	29	...	2
1639	7·4*	11 4	4	...	...	4
1834	6·3*	12 14	...	27	...	65
1922	9·0	12 46	...	37	...	35
1951	8·7	12 52	8	...	...	7
1972	8·0	13 4	...	0	...	24
2048	8·5	13 27	...	2	...	31
2068	9·2	13 27	0	...	...	28
2333	8·0	15 5	16	...	...	24
2494	9·0	16 6	20	...	...	2
2639	8·8	17 6	...	2	...	39
2740	9·0	17 32	8	...	...	7
2762	9·0	17 42	...	14	...	43
2793	8·5	17 52	9	...	...	24
2770	8·0	17 54	12	...	...	11
2942	8·8	18 49	...	9	...	17
2936	9·0	18 58	...	8	...	15
2976	8·8	19 11	4	...	...	11
3058	6·5	19 23	...	16	...	9
3082	8·0	19 43	...	8	...	11
3060	9·0	19 45	...	22	...	167
3106	8·8	19 54	9	...	...	4
3138	var.	19 59	147	...	...	81
3193	9·0	20 33	...	4	...	46
3241	9·0	20 51	9	...	...	4
3411	8·6	21 57	8	...	17	...
3465	8·7	22 12	...	18	18	...
3670	9·0	23 44	...	9	0	..

Zone 2°.

3709	8·8	0 0	...	17	...	22
11	9·0	0 7	...	31	...	4
54	8·8	0 27	29	...	...	28
140	8·8	1 0	1	...	26	...
191	9·1	1 21	24	...	60	...
267	9·5	1 54	...	3	18	...
369	9·3	2 47	...	17	...	19
394	9·0	2 50	...	31	...	19
393	8·9	2 50	...	49	...	39
406	8·7	2 59	21	...	39	...
533	9·2	3 54	7	...	...	26
619	9·0	4 36	...	21	15	...
699	9·4	5 2	...	1	26	...
707	9·3	5 9	...	16	0	...
758	9·0	5 31	...	1	50	...
783	7·9	5 46	...	14	65	...
956	5·2*	6 54	...	39	33	...
871	8·9	6 24	...	29	...	4
903	9·3	6 35	...	12	7	...
897	8·5	6 35	...	14	46	...
997	8·5	7 17	...	51	48	...
1049	9·0	7 34	...	6	...	50
1193	8·8	8 26	...	37	30	...
1194	9·1	8 27	...	20	...	2
1183	9·1	8 27	...	46	...	9
1241	9·0	8 50	...	31	...	11
1330	8·0	9 18	...	53	37	...
1361	8·5	9 28	10	...	...	28

ZONE 2°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−
1398	8·9	9 40	...	60	13	...
1418	7·7	9 44	...	47	33	...
1460	8·5	10 4	...	44	20	...
1617	9·4	10 56	...	11	...	20
1636	9·5	11 0	·8	...	...	2
1777	9·0	11 54	...	7	24	...
1778	8·0	11 55	...	29	...	2
1816	8·2	12 8	...	17	...	15
1896	9·1	12 41	...	7	...	7
1898	8·9	12 42	5	...	17	...
1943	8·4	12 58	...	41	...	69
2031	8·8	13 25	6	...	20	...
2058	9·0	13 35	...	10	0	..
2097	9·0	13 47	...	21	2	...
2121	8·6	13 54	...	0	0	...
2116	8·7	13 55	...	10	20	...
2184	8·5	14 18	...	21	...	24
2316	7·2*	15 9	3	...	...	28
2363	7·9	15 27	...	57	...	6
2398	9·1	15 37	...	16	2	...
2471	8·9	16 5	...	28	...	13
2564	9·1	16 48	...	16	...	50
2590	9·1	16 56	12	...	2	...
2600	9·1	17 1	...	10	21	...
2735	8·9	17 47	...	23	...	11
2769	8·1	18 4	29	...	...	2
2819	9·0	18 17	16	...	4	...
2959	8·0	19 15	21	...	15	...
2973	8·0	19 16	6	...	...	4
3076	8·9	19 54	23	...	15	...
3078	9·0	19 57	...	20	24	...
3160	8·0	20 25	6	...	20	...
3215	9·2	20 48	87	..	...	127
3267	9·1	21 12	7	...	...	4
3270	8·0	21 16	31	...	...	9
3376	8·8	21 49	44	...	...	7
3400	8·2	21 59	17	...	...	11
3450	9·0	22 22	...	25	22	...
3466	7·4*	22 24	30	...	2	...
3479	9·2	22 36	3	...	11	...
3660	8·5	23 43	20	...	2	...

ZONE 3°.

3714	9·0	0 2	290	...	32	...
13	8·9	0 9	152	...	...	4
15	8·8	0 9	2	...	...	32
18	9·2	0 10	...	30	...	24
56	9·0	0 27	...	12	...	6
85	8·6	0 37	...	28	9	...
113	8·8	0 49	318	...	...	108
145	6·4*	0 59	52	...	2	...
151	9·0	1 2	4	...	...	2
173	8·8	1 11	...	30	...	15
236	8·8	1 40	29	...	0	...
242	9·3	1 42	...	4	...	9
334	9·0	2 25	...	8	...	7
352	8·6	2 32	...	46	...	26
364	9·2	2 37	...	49	...	15
435	8·8	3 5	16	...	18	...
491	5·9*	3 34	140	...	70	...
497	9·3	3 37	...	8	...	4
537	8·9	3 53	...	27	...	11
548	9·0	3 54	6	...	65	...
635	9·2	4 35	8	...	...	13
640	8·3	4 38	...	47	...	20
662	8·0	4 46	...	4	24	...
670	8·7	4 51	...	16	...	2
797	8·8	5 47	18	...	...	4
842	6·6*	6 8	18	...	106	...
889	8·9	6 23	...	24	9	...
917	9·0	6 35	...	16	13	...
958	8·5	6 50	...	30	37	...
1002	9·2	7 10	...	10	...	35
1005	8·2	7 11	...	95	165	...
1097	8·7	7 47	...	39	2	...
1190	8·5	8 19	...	14	13	...
1290	9·0	8 59	...	12	...	7
1376	8·5	9 29	...	44	...	45
1458	8·6	9 58	...	4	...	17

Zone 3°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.		Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−			h m	+	−	+	−
1494	8·8	10 10	...	44	2	...	2484	9·0	16 20	12	...	...	30
1519	8·4	10 19	...	24	...	2	2517	8·7	16 35	4	...	...	26
1620	9·0	10 54	...	43	...	4	2541	9·1	16 45	...	18	30	...
1633	8·3	10 57	...	8	...	4	2579	9·4	16 59	...	14	...	28
1645	7·2*	11 2	...	32	...	4	2606	8·5	17 12	...	35	...	41
1669	8·9	11 13	...	10	68	...	2670	9·2	17 32	...	43	...	67
1683	9·0	11 17	...	4	...	4	2673	8·5	17 33	...	35	...	71
1712	7·4*	11 28	...	12	...	74	2755	4·5*	18 5	25	...	...	50
1749	8·2	11 40	...	16	...	2	2771	5·9*	18 8	69	...	11	...
1756	8·9	11 44	...	29	...	13	2811	8·2	18 26	0	...	...	17
1762	8·8	11 47	...	263	...	239	2852	8·8	18 41	5	...	...	15
1787	9·0	11 57	...	88	...	0	2882	6·8*	18 48	2	...	...	27
1793	6·4*	12 0	...	68	...	94	2881	9·0	18 49	2	...	0	...
1812	8·5	12 7	...	20	...	2	2906	8·8	18 58	...	6	30	...
1830	6·4*	12 14	262	...	2	...	2953	9·0	19 16	...	12	...	15
1875	7·1*	12 35	...	32	...	2	2966	8·8	19 20	15	...	...	37
1944	7·8*	13 0	...	18	...	11	3015	9·0	19 41	6	...	...	2
2015	8·9	13 20	...	24	...	2	3055	8·7	19 57	2	...	...	11
2027	9·0	13 26	...	10	11	...	3240	8·9	21 6	20	...	...	7
2075	9·0	13 40	47	...	...	177	3260	9·0	21 16	...	24	2	...
2106	8·5	13 51	28	...	20	...	3273	7·4*	21 20	24	...	...	11
2109	8·5	13 55	...	25	...	30	3347	8·2	21 46	6	...	...	20
2122	7·4*	13 59	...	28	...	15	3351	8·7	21 48	135	...	...	56
2145	8·8	14 7	15	...	2	...	3352	8·5	21 48	136	...	...	75
2198	8·5	14 30	25	...	33	...	3353	8·3	21 48	4	...	2	...
2231	9·0	14 41	...	25	0	...	3452	9·0	22 24	...	8	18	...
2253	8·6	14 48	...	25	2	...	3493	8·0	22 42	...	15	...	0
2258	7·0*	14 50	...	15	0	...	3494	8·7	22 43	20	...	24	...
2301	8·8	15 9	...	70	17	...	3619	8·0	23 28	...	45	...	31
2367	9·1	15 32	...	27	...	17	3621	5·6*	23 28	85	...	...	15
2400	9·2	15 44	29	...	...	54	3693	8·0	23 55	22	...	27	...
2463	9·0	16 12	...	19	...	15	3699	8·6	23 57	10	...	...	13

Zone 4°.

52	8·3	0 26	37	...	18	...	265	9·0	1 49	...	26	...	7
63	8·8	0 29	47	...	...	4	283	7·7	1 59	...	26	...	91
101	9·5	0 44	...	8	9	...	337	8·6	2 23	50	...	29	...
133	8·9	0 56	...	21	2	...	357	8·9	2 32	8	...	...	11
131	4·6*	0 55	100	...	20	...	374	8·8	2 39	...	13	33	...
218	9·0	1 32	...	26	...	22	377	8·8	2 41	...	32	...	16
235	9·2	1 39	...	13	...	14	441	9·2	3 5	18	...	15	...

ZONE 4°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−
505	8·5	3 38	13	...	...	22
572	9·0	4 5	...	5	9	...
575	6·7*	4 6	8	...	...	31
603	9·0	4 17	5	...	18	...
611	8·5	4 21	11	...	44	...
642	9·0	4 34	41	...	160	...
678	8·5	4 48	...	15	29	...
698	6·6*	4 56	39	...	66	...
704	8·5	5 0	...	8	128	...
724	9·0	5 6	...	21	7	...
727	6·7*	5 10	...	31	...	4
770	6·4*	5 30	5	...	...	9
782	8·0	5 35	...	15	...	18
787	8·9	5 39	...	93	122	...
788	9·2	5 41	...	18	7	...
810	9·0	5 48	...	8	...	4
876	8·9	6 15	...	11	22	...
927	8·6	6 36	...	13	...	13
933	9·0	6 36	2	...	11	...
941	8·9	6 40	...	18	20	...
957	9·0	6 46	8	...	13	...
968	8·2	6 51	...	8	81	...
1057	8·5	7 27	...	13	24	..
1064	9·0	7 30	...	26	...	24
1123	7·3*	7 52	...	44	26	...
1161	8·8	8 9	...	52	42	...
1167	9·0	8 9	...	13	...	18
1196	9·0	8 21	...	91	195	...
1205	7·4*	8 25	...	107	103	...
1213	8·2	8 28	...	19	9	...
1240	9·0	8 38	...	67	2	...
1259	8·0	8 47	...	59	...	15
1268	8·2	8 48	28	...	...	13
1277	9·0	8 55	...	35	...	0
1320	8·5	9 9	...	11	...	31
1357	8·5	9 20	...	2	22	...
1472	8·9	10 2	...	20	...	4
1478	8·7	10 4	...	28	...	24
1499	9·0	10 11	...	28	...	11
1533	8·5	10 20	0	...	...	22
1560	8·9	10 29	...	24	...	9
1566	8·4	10 31	6	...	...	22
1585	8·2	10 41	...	131	11	...
1613	9·2	10 50	...	24	7	...
1615	8·9	10 52	2	...	2	...
1700	7·4*	11 24	...	11	...	42
1745	8·6	11 38	19	...	...	11
1757	9·0	11 45	...	78	...	33
1759	8·4	11 45	...	18	...	22
1808	8·2	12 7	...	28	...	0
1809	8·9	12 7	...	31	...	13
1813	9·0	12 7	...	61	11	...
1849	8·4	12 20	...	8	...	2
1871	9·0	12 32	...	166	49	...
1905	8·8	12 45	...	148	...	17
1927	8·5	12 53	...	63	16	...
1928	8·9	12 53	...	106	0	...
1931	8·7	12 55	5	...	0	...
1942	8·5	13 0	...	65	...	9
1980	9·0	13 13	...	13	20	...
1999	9·0	13 18	...	28	2	...
2003	9·0	13 19	...	18	...	46
2001	7·4*	13 19	...	120	...	24
2013	8·8	13 22	...	0	20	...
2018	8·9	13 23	...	85	...	9
2038	9·0	13 30	78	...	77	...
2045	8·0*	13 32	..	41	...	31
2079	8·5	13 43	...	41	...	0
2092	8·0*	13 51	...	39	24	...
2112	8·8	13 56	15	...	...	6
2170	8·5	14 22	...	76	9	..
2183	9·0	14 26	...	44	...	7
2218	8·0	14 38	...	31	...	20
2247	8·5	14 47	...	31	...	22
2266	8·6	14 56	...	0	...	15
2290	8·1	15 6	...	64	...	30
2293	8·0	15 9	...	96	15	...
2338	9·0	15 26	...	15	...	13
2380	7·0*	15 43	...	8	55	...
2382	7·2	15 43	...	2	...	33
2395	8·8	15 47	...	33	18	...
2403	8·8	15 50	28	...	...	28

Zone 4°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.		Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−			h m	+	−	+	−
2423	7·1*	15 57	...	60	...	65	3141	8·0	20 27	...	58	34	...
2548	8·7	16 50	39	...	...	44	3200	8·0	20 50	40	...	...	38
2619	9·4	17 19	...	24	11	...	3218	8·8	20 59	247	...	73	...
2684	9·2	17 46	64	...	13	...	3232	8·1	21 7	13	...	...	18
2724	9·3	17 56	15	...	4	...	3277	9·0	21 23	...	13	...	13
2751	7·5*	18 7	...	2	...	9	3360	8·5	21 51	24	...	...	22
2804	8·7	18 28	11	...	18	...	3371	9·0	21 55	28	...	...	2
2809	9·3	18 30	...	11	33	...	3372	8·5	21 56	2	...	20	...
2907	8·9	19 1	...	8	0	...	3393	8·5	22 2	...	8	9	...
2926	9·0	19 10	...	13	...	11	3444	6·8*	22 22	39	...	...	7
2931	9·0	19 14	...	5	9	...	3441	5·4*	22 21	52	...	...	38
2989	8·4	19 34	50	...	...	78	3554	8·4	23 7	26	...	...	13
2990	8·9	19 36	79	...	...	2	3594	8·5	23 19	...	11	...	40
3067	8·5	20 3	...	32	38	...	3608	6·6*	23 24	...	24	...	31
3080	8·7	20 7	37	...	...	44	3615	8·0	23 26	...	19	22	...
3094	8·0*	20 14	...	28	...	213	3626	7·8	23 30	...	13	29	...
3099	9·0	20 16	15	...	9	...							

Zone 5°.

3711	8·2	0 1	10	...	...	18	371	9·0	2 38	40	...	55	...
19	8·0	0 10	48	...	9	...	422	8·7	2 56	...	3	...	9
57	9·0	0 27	48	...	17	...	426	8·5	2 58	13	...	...	11
62	8·5	0 29	23	...	7	...	451	5·7*	3 9	89	...	155	...
65	7·8	0 30	10	...	27	...	489	8·9	3 30	...	3	7	...
67	9·0	0 31	58	...	...	38	496	8·4	3 32	...	22	...	13
69	9·2	0 31	78	...	16	...	503	9·0	3 35	...	35	20	...
89	9·0	0 38	35	...	...	9	509	8·8	3 38	...	19	7	...
95	8·8	0 41	19	...	2	...	518	9·0	3 41	...	19	2	...
104	8·2	0 45	6	...	...	16	591	7·4*	4 9	...	3	...	35
122	9·0	0 52	50	...	59	...	622	9·0	4 24	...	22	21	...
124	6·7*	0 53	68	...	9	...	643	7·7	4 33	3	...	9	...
157	8·8	1 3	29	...	20	...	692	9·0	4 51	29	...	...	15
169	9·0	1 8	...	16	...	15	705	8·8	4 58	49	...	155	...
190	8·8	1 18	114	...	37	...	748	8·9	5 17	3	...	33	...
225	8·0	1 34	10	...	0	...	784	9·0	5 37	6	...	...	9
227	9·0	1 35	35	...	61	...	799	9·0	5 44	59	...	...	172
247	8·9	1 41	0	...	20	...	812	8·9	5 46	...	32	...	16
260	8·1	1 46	13	...	22	...	821	9·0	5 49	...	32	89	...
306	9·1	2 6	...	6	13	...	826	8·8	5 54	29	...	36	...
314	9·0	2 10	0	...	...	4	872	9·4	6 12	0	...	...	28
328	8·5	2 20	88	...	90	...	929	8·2	6 34	...	89	...	70

ZONE 5°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−
973	9·3	6 49	...	22	15	...
1012	9·3	7 6	...	3	7	...
1029	8·0	7 14	...	3	55	..
1065	9·0	7 28	...	42	...	20
1096	9·0	7 40	19	...	0	...
1098	8·9	7 42	...	10	59	...
1113	7·8	7 46	...	19	11	...
1127	6·3*	7 53	...	42	26	...
1153	8·8	8 2	...	22	13	...
1184	8·9	8 14	...	19	...	26
1188	8·3	8 15	...	10	28	...
1233	8·0	8 35	...	127	103	...
1278	9·0	8 53	...	26	9	...
1286	6·3*	8 55	6	...	...	20
1296	8·6	8 57	...	23	4	...
1347	8·0	9 15	10	...	18	...
1379	8·5	9 24	10	...	44	...
1393	8·3	9 31	...	22	22	...
1399	8·7	9 32	...	16	2	...
1397	8·8	9 32	6	...	...	31
1402	8·7	9 32	29	...	11	...
1410	8·0	9 35	...	16	7	...
1414	8·5	9 37	16	...	...	26
1442	8·8	9 48	...	29	...	2
1451	6·5*	9 53	6	...	...	13
1468	8·9	10 0	...	3	2	...
1516	5·7*	10 15	...	104	38	...
1529	8·5	10 17	...	68	40	...
1540	7·4*	10 21	3	...	...	56
1569	9·2	10 31	...	42	...	29
1605	8·6	10 47	...	10	...	4
1702	8·8	11 25	...	24	...	13
1709	8·6	11 26	...	107	4	...
1740	9·0	11 36	...	40	20	...
1818	8·1*	12 9	...	10	...	11
1829	8·5	12 14	...	48	2	...
1843	8·5	12 19	...	22	...	2
1856	8·7	12 23	...	10	...	4
1882	7·3*	12 38	...	220	...	20
1945	8·6	13 1	...	16	133	...
1956	8·6	13 5	...	45	20	...
1965	8·8	13 10	13	...	0	...
1975	9·0	13 11	16	...	...	11
1995	8·5	13 17	...	16	...	22
2006	8·0	13 20	...	96	...	37
2014	8·8	13 23	...	62	...	6
2093	8·8	13 53	...	10	...	20
2165	8·0	14 22	...	10	...	4
2187	9·0	14 30	...	19	...	45
2196	8·6	14 33	6	...	26	...
2223	9·0	14 42	...	22	9	...
2271	7·1*	15 2	...	19	...	11
2283	8·4	15 7	13	...	0	...
2288	8·9	15 8	19	...	73	...
2342	8·3	15 29	16	...	20	...
2354	9·0	15 33	37	...	15	...
2373	8·8	15 40	107	...	50	...
2426	8·4	16 0	...	66	...	116
2491	8·7	16 29	6	...	...	28
2498	8·5	16 30	16	...	...	15
2515	8·5	16 41	13	...	2	...
2558	9·0	16 57	...	13	7	...
2565	8·5	16 59	...	37	0	...
2571	9·3	17 1	...	3	...	19
2598	8·2	17 12	...	3	...	46
2642	7·7	17 29	35	...	31	...
2655	8·8	17 34	...	10	18	...
2668	8·6	17 41	...	16	...	7
2679	8·7	17 44	13	...	11	...
2693	8·9	17 49	29	...	...	35
2698	9·2	17 51	6	...	...	4
2743	9·0	18 6	32	...	...	50
2746	8·2	18 7	22	...	...	28
2744	8·4	18 7	22	...	24	...
2778	9·0	18 20	...	48	2	...
2782	8·5	18 21	23	...	9	..
2790	7·4*	18 25	10	...	...	7
2810	8·5	18 32	26	...	...	80
2817	8·5	18 34	16	...	9	...
2824	8·8	18 36	...	16	...	18
2859	8·6	18 47	37	...	...	26
2895	9·2	19 1	22	...	44	...

ZONE 5°— *continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.		Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−			h m	+	−	+	−
2949	9·2	19 21	35	...	...	24	3262	9·3	21 20	...	3	32	...
2964	9·0	19 26	32	...	0	...	3285	9·0	21 27	...	22	50	...
2983	8·7	19 34	13	...	...	22	3382	9·0	22 0	...	29	0	...
3019	9·3	19 49	19	...	...	33	3422	8·0	22 14	39	...	...	65
3034	8·5	19 54	35	...	...	9	3436	7·5*	22 21	...	13	7	...
3043	8·5	19 57	88	...	...	57	3458	7·2*	22 27	45	...	...	14
3068	8·8	20 5	35	...	18	...	3509	7·1*	22 50	92	...	35	...
3091	6·6*	20 14	10	...	64	...	3520	6·2*	22 53	35	...	...	94
3120	8·5	20 23	20	...	...	22	3519	8·0	22 53	53	...	...	15
3123	7·2*	20 24	45	...	...	60	3634	8·7	23 34	42	...	4	...
3128	7·1*	20 25	...	81	0	...	3646	9·0	23 38	13	...	20	...
3188	9·3	20 47	...	19	15	...	3647	8·2	23 39	...	26	20	...
3205	8·4	20 53	3	...	48	...	3653	9·0	23 41	...	3	18	...
3237	9·3	21 9	...	10	4	...	3661	8·1	23 44	3	...	...	11

ZONE 6°.

2	8·5	0 4	58	...	2	...	485	8·2	3 23	9	...	18	...
10	9·0	0 7	156	...	28	...	504	7·5*	3 34	...	3	16	...
60	8·3	0 28	22	...	...	17	530	9·0	3 43	...	0	24	...
78	9·2	0 34	78	...	...	11	532	8·7	3 44	...	3	7	...
109	5·6*	0 45	63	...	28	...	565	8·8	3 58	...	22	...	24
115	9·1	0 48	49	...	...	11	571	8·7	4 1	...	19	...	4
174	9·2	1 9	15	...	...	2	580	5·4*	4 5	3	...	...	7
179	9·3	1 11	219	...	...	4	593	5·7*	4 8	...	49	...	95
206	8·9	1 24	22	...	...	4	596	8·7	4 12	19	...	26	...
233	8·9	1 37	...	85	...	2	618	7·4*	4 22	0	...	54	...
243	8·7	1 39	31	...	...	9	633	8·9	4 28	...	3	44	...
255	9·0	1 44	26	...	...	11	647	9·0	4 34	...	7	...	13
257	9·0	1 44	...	7	13	...	648	8·5	4 34	22	...	33	...
299	6·9*	2 1	49	...	38	...	654	8·5	4 37	...	27	...	13
316	8·9	2 10	12	...	18	...	657	8·8	4 38	...	19	9	...
332	8·0	2 20	...	19	...	11	669	8·5	4 42	12	...	18	...
342	9·0	2 23	...	26	...	2	691	9·0	4 50	...	19	...	22
354	9·0	2 28	...	56	...	37	725	8·7	5 5	0	...	50	...
363	9·0	2 31	...	22	7	...	726	9·0	5 5	22	...	...	4
411	9·2	2 51	51	...	...	28	732	9·0	5 9	...	48	24	...
427	9·2	2 57	19	...	...	4	738	7·3*	5 12	...	15	0	...
456	7·5	3 8	15	...	15	...	747	9·0	5 15	3	...	...	9
457	9·0	3 9	...	29	...	9	776	8·7	5 30	22	...	19	...
458	8·2	3 9	41	...	46	...	836	9·2	5 55	...	32	...	15
477	8·8	3 20	9	...	...	29	849	9·2	6 1	15	...	26	...

ZONE 6°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−
852	9·1	6 5		7	11	...
866	8·9	6 10	...	7	32	...
908	9·3	6 25	...	12	9	...
950	9·0	6 40	...	12	...	13
953	8·5	6 41	13	...	245	...
974	9·0	6 48	12	...	...	24
979	8·9	6 50	...	7	24	...
990	9·0	6 53	3	...	4	...
1000	8·2	6 59	...	19	22	...
1066	8·0	7 26	12	...	13	...
1074	9·2	7 29	63	...	...	41
1109	9·4	7 42	...	12	37	...
1159	8·2	8 4	...	56	57	...
1177	9·3	8 10	...	7	55	...
1207	9·2	8 23	...	60	96	...
1235	9·2	8 35	...	15	...	7
1254	7·5*	8 42	...	3	...	2
1262	7·1*	8 45	15	...	...	17
1272	8·5	8 49	...	3	4	...
1289	9·0	8 54	...	15	2	...
1298	9·0	8 57	...	26	...	17
1297	9·0	8 57	...	19	...	4
1316	9·2	9 3	...	71	51	...
1322	9·5	9 7	...	3	...	13
1339	9·5	9 12	...	15	20	...
1369	7·1*	9 20	...	12	11	...
1407	8·0	9 33	...	37	...	4
1408	8·3	9 33	...	0	...	4
1419	8·8	9 37	...	12	...	11
1427	8·8	9 40	7	...	0	...
1434	8·5	9 43	...	15	26	...
1435	8·0	9 44	...	139	115	...
1441	9·0	9 47	...	15	...	13
1470	8·9	9 59	...	7	9	...
1475	8·5	10 0	...	22	24	...
1503	6·9*	10 12	15	...	11	...
1518	8·6	10 15	3	...	...	20
1523	9·0	10 15	26	...	4	...
1537	8·4	10 18	...	7	26	...
1538	5·4*	10 19	...	65	...	33
1561	9·0	10 28	...	41	0	...
1581	9·5	10 36	12	...	...	11
1583	8·6	10 38	...	7	...	4
1619	8·5	10 52	...	31	...	29
1641	8·2	10 59	...	87	20	...
1643	8·8	11 0	...	97	33	...
1665	9·0	11 9	...	44	...	15
1668	8·4	11 11	...	22	26	...
1704	8·8	11 25	...	34	...	4
1746	8·4	11 38	...	15	4	...
1761	8·0	11 45	...	15	15	...
1763	9·0	11 48	...	92	20	...
1766	9·0	11 48	...	10	...	28
1771	8·5	11 52	49	...	37	...
1780	8·7	11 54	..	34	2	...
1781	8·8	11 54	0	...	33	...
1790	8·4	11 58	15	...	...	15
1831	9·0	12 15	...	10	...	6
1833	8·7	12 16	...	26	...	9
1836	8·1*	12 17	7	...	...	15
1841	8·4	12 19	...	63	...	7
1847	7·9	12 21	12	...	4	...
1852	8·6	12 21	...	15	...	2
1913	6·0*	12 48	...	27	...	49
1914	5·1	12 48	...	27	...	42
1916	8·5	12 49	..	15	...	17
1921	7·2*	12 53	26	...	...	35
1925	8·8	12 53	...	15	...	6
1937	9·0	12 59	...	41	...	2
1938	8·3	12 59	...	27	0	...
1970	8·7	13 11	...	12	29	...
1969	8·9	13 11	7	...	6	...
2022	8·5	13 26	...	22	...	17
2023	7·3*	13 27	29	...	...	98
2080	6·1*	13 45	46	...	44	...
2105	9·2	13 56	...	26	...	49
2137	8·9	14 9	...	22	20	...
2141	9·0	14 10	...	19	...	30
2177	8·5	14 27	...	12	4	...
2207	8·6	14 36	...	75	9	...
2206	8·5	14 36	...	26	...	93
2244	9·0	14 48	22	...	...	11

ZONE 6°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A. +	R.A. −	N.P.D. +	N.P.D. −
		h m				
2281	8·8	15 7	12	...	...	4
2286	9·0	15 8	...	48	0	...
2291	8·5	15 12	22	...	11	...
2296	8·8	15 12	...	77	...	43
2381	8·8	15 46	3	...	...	35
2387	8·9	15 47	19	...	0	...
2401	8·4	15 53	...	44	...	48
2404	7·3	15 54	26	...	2	...
2416	8·5	15 59	19	...	...	26
2418	8·5	16 0	...	3	...	46
2424	7·7	16 1	12	...	18	...
2448	8·2	16 12	3	...	...	9
2501	7·1	16 34	3	...	...	13
2524	8·2	16 46	26	...	4	..
2557	9·0	16 58	...	12	...	11
2599	8·8	17 15	0	...	...	17
2631	8·7	17 27	...	7	13	...
2633	9·0	17 28	7	...	2	...
2632	9·0	17 28	41	...	...	30
2647	8·3	17 32	19	...	...	11
2648	7·5*	17 33	88	...	...	103
2729	8·8	18 2	...	7	...	35
2747	7·7	18 9	...	15	...	4
2779	7·8	18 21	...	9	31	...
2786	8·2	18 25	15	...	...	2
2788	9·0	18 26	...	15	...	7
2796	8·8	18 28	27	...	0	...
2802	8·5	18 30	...	3	...	27
2820	7·3*	18 36	19	...	...	38
2822	6·2*	18 37	19	...	28	...
2870	8·8	18 53	53	...	44	...
2876	8·8	18 54	22	...	20	...
2884	8·8	18 56	34	...	...	11
2908	6·8*	19 4	31	...	24	...
2935	9·3	19 18	63	...	...	13
2965	6·4*	19 28	37	...	...	36
2971	9·0	19 30	31	...	...	4
3005	9·0	19 45	26	...	...	26
3004	8·3	19 46	3	...	...	0
3071	9·0	20 6	44	...	...	55
3072	8·6	20 8	61	...	40	...
3073	8·6	20 8	34	...	52	...
3077	8·5	20 10	31	...	...	9
3103	8·5	20 18	31	...	...	22
3132	9·0	20 27	15	...	20	...
3163	6·2*	20 39	38	...	15	...
3202	8·8	20 54	12	...	...	5
3216	7·7	20 59	43	...	...	94
3264	7·1*	21 22	41	...	13	...
3297	8·5	21 31	37	...	62	...
3309	8·6	21 35	19	...	2	...
3318	8·4	21 40	41	...	...	20
3334	8·8	21 46	...	12	22	...
3337	8·9	21 47	401	...	...	42
3346	7·0*	21 50	102	...	...	20
3367	9·0	21 55	41	...	20	...
3386	8·2	22 2	7	...	...	0
3395	9·0	22 4	...	12	22	...
3416	9·0	22 12	3	...	...	24
3418	8·5	22 12	46	...	...	29
3438	8·3	22 22	298	...	...	5
3469	9·3	22 36	44	...	4	...
3505	8·0	22 49	...	151	...	132
3525	5·0*	22 55	129	...	...	29
3573	8·0	23 13	41	...	...	13
3628	9·2	23 33	194	...	...	22
3630	9·0	23 33	60	...	37	...
3656	9·1	23 42	56	...	...	15

ZONE 7°.

7	8·9	0 7	57	...	...	20
16	9·0	0 10	...	4	...	16
17	8·8	0 10	...	35	18	...
22	8·6	0 12	31	...	...	18
61	9·1	0 28	22	...	...	27
108	5·6*	0 45	14	...	...	14
116	8·5*	0 49	4	...	...	74
119	8·4	0 50	98	...	109	...
138	8·8*	0 55	61	...	0	...
155	9·0	1 1	0	...	2	..

ZONE 7°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−
189	9·0	1 15	18	...	...	11
203	8·8	1 23	43	...	...	13
205	8·6	1 23	0	...	2	...
248	8·8	1 40	53	...	9	...
274	8·4	1 50	0	...	...	11
310	9·2	2 7	...	8	...	2
333	8·7	2 20	47	...	35	...
344	9·2	2 23	8	...	...	31
359	9·2	2 30	82	...	...	11
423	8·0	2 54	43	...	...	7
438	9·0	3 0	31	...	42	...
445	9·4	3 4	4	...	...	18
459	9·3	3 11	22	...	...	7
521	8·4	3 39	...	18	...	13
541	9·0	3 46	8	...	27	...
556	9·0	3 54	...	14	...	11
601	9·3	4 12	18	...	...	4
609	9·3	4 16	...	14	11	...
614	8·6	4 18	...	18	...	13
687	8·6	4 48	...	43	4	...
695	9·0	4 50	...	14	...	27
711	9·2	4 57	...	8	46	...
742	8·9	5 12	8	...	...	2
745	9·2	5 14	51	...	2	...
774	9·0	5 28	22	...	...	7
802	9·2	5 40	4	...	...	18
800	7·6	5 40	49	...	...	42
820	8·7	5 47	18	...	22	...
879	8·5	6 14	4	...	13	...
880	9·0	6 14	14	...	...	44
898	9·0	6 20	69	...	137	...
910	6·4*	6 23	8	...	28	...
909	8·8	6 24	18	...	...	24
944	8·7	6 37	31	...	...	44
954	8·7	6 40	...	25	...	2
972	9·0	6 46	0	...	2	...
992	8·0	6 55	14	...	...	4
1017	9·0	7 5	...	35	2	...
1027	5·0*	7 10	27	...	36	...
1040	8·5	7 15	...	4	...	4
1050	8·9	7 20	...	22	35	...
1078	9·0	7 30	4	...	0	...
1108	9·3	7 41	...	4	...	22
1125	9·1	7 49	...	4	...	9
1144	8·3	7 55	...	61	7	...
1165	6·2*	8 5	18	...	25	...
1197	8·7	8 18	...	4	13	...
1211	8·5	8 23	...	14	7	...
1215	9·3	8 24	...	14	...	13
1220	8·7	8 27	...	22	24	...
1222	6·7*	8 28	...	8	7	..
1257	9·1	8 41	...	8	38	...
1274	9·2	8 49	4	...	7	...
1318	8·8	9 4	...	25	38	...
1341	8·5	9 11	25	...	58	...
1342	8·5	9 12	...	0	...	13
1360	8·7	9 17	...	25	36	...
1377	8·9	9 22	0	...	41	...
1387	7·5*	9 26	8	...	...	52
1392	8·8	9 29	...	53	18	...
1404	8·5	9 31	...	53	...	2
1424	9·2	9 39	...	8	...	2
1432	9·0	9 42	...	57	16	...
1474	7·6*	10 0	22	...	29	...
1500	9·0	10 9	...	59	...	4
1524	8·1	10 15	...	49	22	...
1521	9·0	10 15	...	4	4	...
1545	9·0	10 21	14	...	...	20
1547	8·9	10 21	...	78	29	...
1556	8·2	10 24	14	...	...	7
1562	7·7	10 28	...	39	34	...
1567	9·0	10 29	...	53	27	...
1578	8·9	10 33	10	...	...	20
1623	8·8	10 52	14	...	...	16
1628	8·5	10 53	57	...	11	...
1650	9·0	11 2	...	18	...	20
1651	7·1*	11 2	...	120	168	...
1657	8·6	11 5	...	25	...	11
1693	8·3	11 23	76	...	41	...
1731	8·0	11 33	18	...	24	...
1742	8·6	11 37	...	53	...	16
1747	7·7	11 38	22	...	...	9

ZONE 7°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ".001. R.A.		N.P.D.	
		h m	+	−	+	−
1765	8·8	11 48	18	...	14	...
1768	8·3	11 48	...	49	...	23
1773	8·8	11 53	...	14	...	9
1791	7·6	11 58	4	...	...	7
1807	6·3*	12 7	...	14	7	...
1810	8·9	12 7	22	...	...	38
1854	8·9	12 23	18	...	13	...
1859	8·0	12 25	...	67	...	16
1869	8·8	12 31	18	...	...	63
1902	8·5	12 45	14	...	...	2
1912	8·6	12 47	4	...	..	24
1923	8·5	12 53	...	86	...	31
1971	8·7	13 12	...	39	2	...
1982	8·5	13 15	...	63	7	...
1983	9·9	13 16	4	...	...	20
1987	8·4	13 17	...	63	...	18
2009	8·5	13 22	...	14	9	...
2036	8·6	13 32	...	14	25	...
2052	8·9	13 38	...	8	...	2
2066	9·0	13 42	...	4	...	13
2103	8·7	13 57	...	29	...	11
2111	8·2	13 59	...	4	24	...
2132	8·5	14 9	10	...	0	...
2152	9·0	14 18	10	...	4	...
2157	8·5	14 19	20	...	36	...
2186	9·0	14 31	...	22	...	11
2193	8·7	14 33	...	29	...	82
2222	8·5	14 43	14	...	20	...
2232	9·0	14 46	...	25	22	...
2261	5·7*	14 57	153	...	232	...
2263	9·1	14 59	...	4	...	9
2289	8·8	15 11	8	...	...	33
2328	8·8	15 26	...	39	...	7
2343	8·2	15 32	204	...	...	25
2360	6·9*	15 38	47	...	18	...
2376	8·5	15 44	...	43	...	51
2399	9·0	15 53	25	...	...	44
2406	8·2	15 55	49	...	...	9
2410	8·6	15 58	...	22	...	0
2437	9·0	16 6	14	...	...	132
2442	8·6	16 7	84	...	34	...
2451	9·0	16 14	25	...	38	...
2461	8·5	16 19	25	...	9	...
2475	8·7	16 23	0	...	...	18
2479	8·0	16 24	...	8	...	31
2502	8·8	16 36	...	0	...	55
2537	8·5	16 52	10	...	...	14
2542	8·3	16 54	10	...	...	38
2550	4·4*	16 56	...	10	...	0
2552	9·0	16 57	31	...	27	...
2575	9·0	17 6	8	...	2	...
2587	8·3	17 10	0	...	18	...
2592	9·0	17 13	8	...	...	4
2595	9·0	17 15	22	...	...	9
2624	8·5	17 26	31	...	...	7
2638	8·5	17 29	18	...	...	27
2650	9·0	17 34	...	18	...	44
2651	9·1	17 36	8	...	9	...
2662	9·0	17 39	61	...	31	...
2713	9·0	17 58	31	...	...	9
2717	8·8	17 59	0	...	22	...
2741	8·3	18 9	43	...	4	.9.
2772	8·7	18 20	31	...	18	...
2780	7·3*	18 23	22	...	...	20
2792	8·4	18 28	22	...	...	11
2795	8·5	18 30	...	45	...	0
2800	8·5	18 31	...	4	...	7
2813	8·0	18 35	...	10	2	...
2865	8·5	18 53	25	...	...	73
2903	6·8*	19 5	25	...	...	30
2920	7·7	19 12	10	...	14	...
2937	9·2	19 21	8	...	...	7
2940	9·3	19 21	31	...	11	...
2985	8·5	19 37	18	...	...	34
3006	9·3	19 47	8	...	...	18
3036	9·3	19 57	0	...	...	4
3041	7·6	19 59	10	...	...	34
3081	8·5	20 13	...	35	...	79
3133	8·1*	20 29	18	...	46	...
3134	8·5	20 29	10	...	...	2
3152	6·8*	20 34	14	...	...	5
3186	8·6	20 48	39	...	...	32

ZONE 7°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−
3189	5·7*	20 50	55	...	...	29
3195	9·2	20 52	...	4	25	...
3225	8·5	21 5	27	...	...	2
3266	7·8*	21 23	49	...	...	100
3283	8·0	21 28	10	...	11	...
3291	8·6	21 30	22	...	2	...
3301	8·8	21 32	...	10	2	...
3323	9·0	21 42	...	14	39	...
3343	8·5	21 49	22	...	...	9
3363	8·7	21 55	0	...	2	...
3387	7·5	22 2	...	120	27	...
3389	8·0	22 2	...	133	9	...
3408	7·7	22 9	49	...	...	27
3433	9·2	22 21	43	...	9	...
3437	9·0	22 22	184	...	...	36
3490	8·0	22 44	31	...	...	14
3502	5·0*	22 48	53	...	...	36
3527	8·5	22 57	22	...	...	7
3552	8·1	23 7	31	..	...	25
3564	9·2	23 11	...	49	...	11
3574	9·0	23 13	43	...	11	...
3576	9·0	23 14	31	...	...	5
3575	8·7	23 14	...	53	34	...
3584	8·8	23 16	27	...	...	34
3625	9·0	23 31	...	14	16	...
3629	7·7*	23 33	18	...	7	...
3639	9·0	23 37	14	...	...	2
3641	8·5	23 37	...	4	14	...
3652	8·2	23 41	...	4	...	16
3659	8·8	23 44	157	...	7	...
3662	8·5	23 45	18	...	...	2
3673	9·0	23 47	22	...	7	...
3681	6·6*	23 52	49	...	...	25
3700	7·1	23 58	...	4	...	5

ZONE 8°.

24	9·5	0 14	49	...	...	4
• 76	6·4*	0 32	...	100	...	91
83	8·6*	0 35	...	9	16	...
92	9·2	0 40	60	...	...	29
98	7·6*	0 42	...	29	...	4
130	7·5	0 53	9	...	...	9
136	8·5*	0 55	...	175	66	...
141	8·6	0 55	...	60	...	13
144	7·9*	0 56	20	...	...	13
161	8·6*	1 3	24	...	...	11
207	9·0	1 24	...	9	7	...
219	8·8	1 30	40	...	16	...
229	8·7	1 34	36	...	...	22
234	9·0	1 36	4	...	15	..
245	8·6	1 38	20	...	16	...
256	7·2*	1 44	44	...	...	4
272	8·9	1 49	4	...	...	7
277	9·0	1 52	9	...	...	20
282	8·9	1 54	44	...	...	31
320	8·8	2 15	0	...	...	4
322	9·0	2 15	49	...	...	67
330	9·0	2 19	64	...	24	...
338	9·0	2 21	0	...	20	...
346	8·4	2 23	20	...	...	4
367	9·4	2 33	4	...	0	...
387	8·6	2 40	9	...	...	20
392	8·5	2 41	60	...	20	...
429	5·9*	2 56	...	75	...	5
453	7·3*	3 7	20	...	2	...
474	8·4	3 18	...	16	...	11
480	9·0	3 21	16	...	...	26
484	8·8	3 22	16	...	15	...
511	7·9	3 36	16	...	...	16
526	9·0	3 41	...	69	...	59
528	9·0	3 41	...	49	...	61
539	7·8	3 45	80	...	67	...
543	7·8	3 46	...	24	33	...
577	7·3*	4 2	0	...	...	2
581	8·3	4 3	33	...	20	...
584	7·9	4 4	16	...	15	...
621	8·8	4 20	...	4	...	24
659	8·9	4 37	33	...	...	41
665	8·5	4 39	...	9	13	...
675	5·3*	4 42	...	16	...	42

ZONE 8°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit ″·001. R.A.		N.P.D.	
		h m	+	−	+	−
681	8·4	4 44	...	4	...	9
685	9·2	4 47	9	...	125	...
720	9·2	4 59	40	...	0	...
722	8·7	5 0	4	...	4	...
750	8·6	5 14	102	...	134	...
762	8·6	5 20	...	4	...	18
790	8·5	5 35	...	4	...	31
798	8·0	5 38	11	...	27	...
819	9·2	5 46	16	...	9	...
824	8·8	5 50	...	53	347	...
844	8·7	5 59	20	...	2	...
854	9·0	6 4	0	...	...	35
858	8·9	6 4	53	...	...	44
892	9·0	6 16	104	...	...	127
928	8·4	6 29	0	...	...	7
955	8·8	6 39	...	11	13	...
976	9·0	6 48	24	...	9	...
980	8·9	6 49	...	24	20	...
1008	7·8	7 2	...	11	88	...
1018	9·2	7 5	...	4	...	4
1021	6·2*	7 6	9	...	26	...
1024	9·0	7 8	...	24	27	...
1045	6·5*	7 16	...	9	11	...
1077	7·6*	7 28	...	9	0	...
1105	7·1*	7 39	...	24	26	...
1128	8·7	7 50	...	9	26	...
1131	9·2	7 51	...	4	...	13
1143	8·1	7 54	...	20	...	9
1209	9·5	8 22	...	9	7	...
1229	8·9	8 31	...	44	...	11
1261	7·6	8 43	...	4	...	32
1265	8·6	8 44	...	20	7	...
1291	7·9	8 53	...	31	43	...
1295	8·4	8 53	20	...	26	...
1301	6·5*	8 56	...	20	24	...
1311	8·7	9 1	...	24	9	...
1325	9·0	9 6	20	...	24	...
1352	9·0	9 14	20	...	460	...
1365	9·0	9 19	...	24	16	...
1381	4·6*	9 23	...	29	...	2
1394	8·9	9 29	...	49	...	29
1421	9·0	9 37	...	20	...	2
1430	9·3	9 41	...	16	...	0
1483	9·2	10 3	9	...	...	44
1491	9·1	10 5	...	16	...	13
1525	8·6	10 15	...	40	27	...
1536	7·9	10 17	24	...	9	...
1558	6·6*	10 26	...	24	2	...
1577	8·4	10 32	...	60	...	25
1622	8·8	10 51	...	11	4	...
1637	8·2	10.57	...	206	62	...
1652	8·2	11 3	...	40	7	...
1663	9·0	11 7	...	16	...	33
1671	8·3	11 11	...	40	27	...
1691	9·0	11 20	4	...	...	16
1698	8·0	11 23	56	...	...	47
1707	6·1*	11 25	...	155	...	46
1717	9·0	11 28	...	5	...	18
1719	9·0	11 29	...	33	4	...
1721	9·0	11 29	...	133	73	...
1728	9·0	11 32	4	...	16	...
1732	9·0	11 34	...	36	...	44
1741	8·5	11 36	...	40	7	...
1769	8·9	11 50	...	16	...	7
1772	8·8	11 53	...	36	2	...
1774	8·0	11 53	...	71	50	...
1784	6·4*	11 55	...	75	32	...
1838	8·8	12 17	40	...	18	...
1845	8·5	12 20	...	24	...	9
1866	8·1	12 30	56	...	16	...
1889	6·3*	12 42	9	...	35	...
1915	9·2	12 49	...	93	...	55
1936	7·3*	12 59	...	69	2	...
1968	6·3*	13 12	0	...	...	11
1988	9·0	13 17	...	138	46	...
1997	8·8	13 20	...	36	...	4
2025	8·5	13 28	...	4	...	24
2061	8·5	13 41	29	...	0	...
2064	9·0	13 42	...	53	9	...
2073	8·9	13 44	4	...	...	13
2088	6·8*	13 52	51	...	2	...
2108	9·2	13 58	...	44	0	...

ZONE 8°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit "·001. R.A.		N.P.D.	
		h m	+	−	+	−
2135	8·8	14 10	16	...	...	2
2171	8·6	14 27	...	64	...	22
2176	9·0	14 28	...	4	...	31
2188	7·1*	14 33	...	95	127	...
2194	8·7	14 34	...	24	0	...
2200	8·8	14 35	...	16	...	2
2208	8·7	14 38	...	36	...	49
2230	8·9	14 46	49	...	...	00
2252	8·6	14 53	4	...	42	...
2256	7·2*	14 55	...	24	...	27
2303	8·9	15 17	...	16	...	22
2324	8·7	15 26	...	36	...	58
2327	7·8	15 27	4	...	...	91
2352	7·0*	15 36	0	...	...	34
2358	8·5	15 39	4	...	4	...
2361	9·0	15 39	4	...	4	...
2392	8·4	15 51	...	80	...	74
2393	7·4	15 51	...	16	7	...
2421	8·0	16 3	...	29	...	7
2447	9·0	16 12	...	29	...	86
2458	8·2	16 18	29	...	18	...
2478	8·2	16 25	24	...	...	4
2505	9·2	16 39	...	4	...	33
2554	8·7	16 58	...	0	...	24
2569	8·5	17 5	...	0	...	13
2568	6·7*	17 5	36	...	...	29
2581	9·0	17 9	...	11	...	38
2591	8·3	17 14	...	0	...	69
2629	8·6	17 29	20	...	...	29
2641	8·8	17 31	36	...	...	20
2725	8·9	18 2	0	...	...	94
2726	8·7	18 3	24	...	9	...
2730	8·9	18 5	29	...	...	32
2757	8·5	18 15	44	...	...	31
2768	8·7	18 19	36	...	5	...
2775	7·5*	18 22	33	...	...	18
2808	9·2	18 36	24	...	40	...
2840	9·2	18 45	0	...	14	...
2854	9·1	18 50	4	...	...	18
2863	8·2	18 54	9	...	27	...
2868	9·0	18 55	16	...	2	...
2911	9·2	19 8	...	36	18	...
2923	8·7	19 13	...	20	...	29
2934	8·3	19 30	20	...	...	23
2947	7·9	19 24	67	...	...	43
2948	8·0	19 24	20	...	...	18
2960	8·8	19 27	20	...	5	...
2962	8·0*	19 29	11	...	...	11
2969	8·3	19 30	29	...	87	...
2980	9·0	19 36	0	...	20	...
2995	8·8	19 44	...	20	14	...
3031	7·7	19 56	31	...	...	70
3049	8·6	20 3	44	...	...	22
3054	8·9	20 4	16	...	...	27
3086	8·6	20 16	16	...	...	13
3087	7·4*	20 16	31	...	...	41
3140	6·9*	20 33	47	...	...	14
3149	5·6*	20 35	31	...	...	14
3161	8·5	20 39	0	...	...	16
3162	9·0	20 39	4	...	...	9
3166	7·2*	20 42	71	...	...	41
3203	8·9	20 56	16	...	18	...
3217	8·0	21 1	20	...	...	14
3235	8·8	21 11	51	...	...	38
3236	8·7	21 11	16	...	5	...
3261	7·8	21 22	4	...	...	38
3263	7·9	21 23	24	...	7	...
3290	8·0	21 31	260	...	...	88
3304	9·2	21 34	4	...	...	7
3324	9·2	21 43	36	...	...	0
3326	9·2	21 43	36	...	...	7
3383	9·2	22 2	20	...	...	11
3426	8·9	22 17	9	...	...	2
3440	7·0*	22 24	84	...	...	72
3459	8·5	22 29	16	...	2	...
3472	8·8	22 37	...	4	2	...
3483	7·7	22 43	0	...	2	...
3486	8·0	22 43	36	...	...	2
3489	9·0	22 44	...	24	7	...
3503	9·5	22 48	...	4	0	...
3504	8·9	22 49	...	0	...	5
3507	9·0	22 50	...	20	27	...

Zone 8°—*continued.*

Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit "·001. R.A.		N.P.D.		Carr. No.	Mag.	Approx. R.A. 1900.	Proper Motions. Unit "·001. R.A.		N.P.D.	
		h m	+	−	+	−			h m	+	−	+	−
3514	8·0	22 54	36	...	9	...	3579	8·8	23 16	11	...	14	...
3538	8·5	23 2	...	11	27	...	3580	8·1	23 16	11	...	7	...
3553	8·7	23 8	20	...	18	..	3587	8·5	23 18	0	...	23	...
3555	8·3	23 9	109	...	4	...	3637	8·3	23 37	...	16	...	2
3561	8·0	23 10	95	...	...	90	3668	9·0	23 46	124	...	...	7
3562	8·5	23 11	51	...	...	9	3674	8·0	23 47	20	...	...	0

Zone 9°.

1186	9·4	8 11	5	...	...	7	2029	8·9	13 30	9	...	...	36
1187	9·3	8 11	17	...	9	...	2039	8·9	13 32	...	5	...	16
1238	8·0	8 33	...	21	15	...	2050	8·7	13 37	...	69	...	29
1243	8·3	8 35	...	21	43	...	2065	7·2*	13 42	5	...	...	22
1349	8·6	9 12	9	...	...	9	2115	9·2	14 1	...	26	...	11
1374	8·5	9 20	...	5	...	13	2169	9·0	14 25	...	0	11	...
1386	9·0	9 24	17	...	7	...	2197	8·5	14 35	5	...	...	11
1444	8·1*	9 46	...	5	...	7	2199	8·5	14 35	12	...	14	...
1447	8·4	9 48	...	64	56	...	2214	8·9	14 40	...	21	...	13
1580	6·7*	10 33	21	...	...	11	2312	8·7	15 21	17	...	...	38
1586	9·0	10 38	5	...	4	...	2326	8·6	15 26	5	...	...	16
1606	8·9	10 45	...	21	7	...	2348	6·5*	15 34	...	199	...	125
1607	8·5	10 46	21	...	...	45	2350	8·3	15 35	...	190	...	114
1616	8·8	10 50	...	38	17	...	2370	7·0*	15 42	...	47	...	53
1646	8·6	11 0	...	64	...	22	2415	8·7	16 1	...	9	...	66
1733	9·0	11 33	...	31	5	...	2438	7·6*	16 7	...	31	...	40
1824	8·4	12 11	5	...	10	...	2452	8·5	16 15	...	17	50	...
1825	8·3	12 11	0	...	...	9	2489	8·5	16 29	21	...	...	22
1835	9·0	12 16	...	43	9	...	2644	8·5	17 33	21	...	...	33
1868	7·4*	12 31	...	111	9	...	2695	8·9	17 53	...	5	5	...
1907	7·6	12 46	...	12	23	...	2716	9·0	18 0	0	...	17	...
2030	9·0	13 30	...	73	22	.							

SOME OPPORTUNITIES FOR ASTRONOMICAL WORK WITH INEXPENSIVE APPARATUS.

A Lecture delivered by Professor George E. Hale, Director of the Mount Wilson Solar Observatory of the Carnegie Institution of Washington, at the Royal Astronomical Society, Burlington House, London, W., on Wednesday evening, June 26, 1907. (Plate 5.)

I have sometimes heard it said that the great cost of modern observatories tends to discourage workers with small instruments—observers who are no less interested in the pursuit of astronomical research than the astronomers in the large institutions. It seems to me that if there is any serious discouragement, due to this cause, of men who are engaged in original research with small telescopes and inexpensive apparatus, it is a question whether large observatories should be established. For at any period in the progress of observational astronomy there are two most important subjects for consideration. One relates to the accomplishment of a great amount of routine observation and the discussion of results, and the other relates to the introduction of new ideas and to the beginnings of the new methods which will make the astronomy of the future. I think we will all admit that the introduction of new ideas is quite as important as the prosecution of routine research; and that if any cause whatsoever tends to discourage the men from whom the new ideas might be likely to proceed, that cause of discouragement should be set aside if possible. And therefore I say, with all seriousness, that it is a fair question whether large observatories, with powerful instrumental equipment, should be established if they tend to keep back the man who is pursuing the subject with less expensive appliances, and is introducing, through his careful consideration of the possibilities of research, the new methods which in the process of time will take the place of the old ones. I think it can be shown, however, that the large observatories should be a help rather than a hindrance, at least by suggesting new possibilities of research, in which most valuable results can be obtained by simple means.

I am talking to-night, in purpose at least, to the amateur; but my definition of the amateur is perhaps a broader one than is generally accepted. According to my view, the amateur is the man who works in astronomy because he cannot help it, because he would rather do such work than anything else in the world, and who therefore cares little for hampering traditions or for difficulties of any kind. The "amateur," then, is the person to whom I wish to address my remarks, whether he be connected with a small observatory in the capacity of professional astronomer, or working by himself with very simple instrumental means. But in speaking to the amateur I do not wish to deal with work that shall be satisfactory merely from the standpoint of instruction or

amusement. That is not my purpose. If it is possible to carry on research by simple means that shall really be important and useful, it is my hope to point out some such possibilities. But I do not wish to speak of any work except that of the first class, nor to recommend that any investigations should be undertaken with simple instruments that are not quite as important as other investigations which can be better undertaken with more expensive instruments.

The problem then becomes one of this character—to determine the relative advantages of large and small telescopes for different classes of research, and the possibility of constructing really powerful instruments at moderate expense. I cannot pretend to discuss all phases of this large problem; I shall mention only a few of them, and approach it from a single direction. But before taking up the details of this discussion, perhaps I may be permitted to say that the conception that is sometimes formed of the newer observatories, the idea that vast sums of money are expended, perhaps without the fullest sense of economy, is not always well-founded. For I am quite sure that if you would visit us (to take a single concrete case) in California, you would agree that we have considered the economical side of the question, that we have perhaps in some instances gone almost too far in our desire to save money for instruments of research, and to economise in certain directions where money can be saved. For example, you would find that our offices, our buildings, are of the simplest and least expensive character, while our instruments and machinery are as effective as we can make them. The great expense of such an observatory as the Solar Observatory on Mount Wilson does not depend in large degree on the cost of the instruments used for investigations of the Sun, but in surmounting the difficulties encountered in utilising a mountain site, deprived of the ordinary means of transportation, and in the construction of large equatorial reflecting telescopes for stellar work, which cannot be built cheaply if they are to be really efficient.

I wish now to come to the question before us, and to illustrate some of the advantages and some of the disadvantages of large and small instruments. Perhaps you will permit me, in showing the first slide on the screen, to say that I have some right to undertake a discussion of this sort, because I have viewed the subject from the standpoint of the man using small and inexpensive apparatus. In my first spectroscopic work, which was done in a room in my father's house, the instruments were of the simplest character, and largely of my own construction. Later, a small building was constructed for a concave grating of 10 feet focal length, and the apparatus, although powerful, was not expensive. Subsequently a tower and dome were added, and a 12-inch telescope was erected for photographic work upon the Sun. After the preliminary experiments had been completed, and the spectroheliograph had begun to take form, the possibility that its results could be greatly improved through the use of a larger telescope suggested itself, and

for this reason I made many efforts to acquire a large instrument for these solar investigations. The result, through the generosity of Mr. Yerkes, was the 40-inch Yerkes telescope, which proved to be very useful for the extension of the spectroheliograph work. The next slide shows the instrument, which you will see is a large and expensive machine. The question, then, comes right down to this point: What are the advantages of such a telescope compared with, let us say, a 6-inch equatorial or possibly a 4-inch equatorial? Is it possible with a 6-inch equatorial to do work comparable in importance with the work that can be done with a 40-inch equatorial?

The next slide will show that there was an advantage in passing from the Kenwood 12-inch to the Yerkes 40-inch, at least for the photography of the Sun. Very minute details of the flocculi were brought out which had not previously been known. But it may easily be shown that the advantages of the 40-inch telescope for most classes of solar work are due more particularly to its great focal length than to its large aperture.*

Let us take another illustration. Here we have a picture of the Moon made by Professor Ritchey with the 12-inch Kenwood telescope. You will notice that near the terminator is the crater Theophilus, which you will see again in the next slide as photographed with the 40-inch telescope. This photograph taken by Professor Ritchey is probably as good a photograph of the Moon's surface as has yet been made, and in this case the advantage of the 40-inch telescope is apparent.† But if we take another case, as illustrated in the next slide, it becomes obvious enough that for certain classes of work the Yerkes telescope is not well suited. Here is a picture made with the 40-inch of the Andromeda Nebula. You see how little it shows, since a long-focus telescope, unless of very great aperture, is not well adapted for the photography of faint nebulæ. When we compare this picture with the next one, made by Professor Ritchey, with the 2-foot reflector (of 8 feet focal length), we appreciate immediately that the 40-inch, in spite of its great advantages for certain classes of work, is wholly unadapted for other investigations. As you know, a refractor of much smaller aperture and of shorter focal length would also give a photograph of the Andromeda Nebula far superior to anything that could be taken with the 40-inch.

If we look at the next slide, which shows Professor Barnard's 10-inch Bruce telescope when it was mounted on Mount Wilson, where he was using it to photograph the Milky Way, you will see an instrument that is very small and inexpensive as compared with the Yerkes telescope. It has a 10-inch Brashear lens of 50 inches

* So far as resolving power is concerned, an aperture of 8 inches would be sufficient to permit the smallest known details of the flocculi to be photographed.

† Here, again, the full visual resolving power is not utilised, but the great aperture is of advantage in permitting the large image to be photographed with very short exposures.

focal length and certain smaller cameras attached to the side of the tube. With such an instrument as this, superb photographs of the Milky Way, like the one illustrated in the next slide, can be taken, which are indispensable for investigations on the distribution of stars in this part of the heavens. Excellent work can also be done with a much smaller lens, provided with a very simple mounting.* A fine instance of systematic work with a portrait-lens is afforded by Mr. Franklin-Adams's photographic map of the northern and southern heavens.

It is hardly necessary to recall the fact that the 40-inch could not do this work at all. If we attempted to photograph the Milky Way with it, we might get a very small region on a very great scale, but to give us any notion as to the general distribution of stars in the Milky Way the 40-inch would be a total failure. However, if it were a question of studying some star cluster like the one shown in this slide, which would occupy a very small region indeed of the Milky Way, the 40-inch would enable us to pick out the separate stars, to study their individual phenomena, their changes in light and position, while such work could not be done on photographs taken with a portrait-lens.

I have shown these miscellaneous illustrations for the purpose of emphasising, what is perfectly well known to all of you, that each instrument has its particular fields of work, in which it can accomplish, or permit to be accomplished, various investigations which are not within the reach of other kinds of telescopes. But I now wish to discuss the question somewhat more specifically, and in doing so I shall confine myself almost entirely to observations of the Sun, although one might attack the subject from many other directions. The first point is this. Suppose one has a small telescope of 4 inches or 6 inches aperture and wishes to observe the Sun with it; and let us assume at the outset that he has no attachments whatever in the form of spectroscopes, but that he wishes simply to make direct observations of the Sun: Is there work for such an instrument at the present time? If you will examine the literature of the subject you may perhaps be surprised to find that many years have elapsed since very careful and extensive investigations have been made similar to those of Langley, which may be almost forgotten by many astronomers, but certainly are not forgotten by those of us who follow the Sun and are accustomed to the appearance of the spots when the definition is good. The next slide shows the well-known drawing of Langley's typical sun-spot. You will remember, if you have systematically observed the Sun, that every time the conditions become extremely good, the structure of sun-spots more and more closely resembles this drawing. This is a typical drawing; it does not represent any particular spot; it brings together observations of various spots; but in general the details of a sun-spot look very much

* Professor Barnard has illustrated in the *Astrophysical Journal* some of the admirable results he has himself obtained with a cheap "lantern lens" belonging to an ordinary stereopticon.

indeed like that drawing when the definition is good enough to show them properly. This subject has been greatly neglected for a long time, and it would well repay observers with large or small instruments to observe sun-spots, and to study many of the details of their structure which still remain obscure and difficult to understand.* Of course the question of the resolving power of the instrument must then be considered. A 4-inch telescope, capable of separating objects one second of arc apart, would not do for the very finest details in a sun-spot. According to Langley, the penumbral filaments sometimes exhibit structure considerably smaller than such a telescope would show; but a 10-inch or 12-inch telescope would show everything that has ever been recorded in a sun-spot, and there are many instruments of that size available for such observations.† Even a much smaller telescope, if carefully and systematically used, would contribute largely to our knowledge of sun-spots and of the structure of the solar surface. One might enlarge upon this subject, but time is hardly sufficient to permit me to do so.

Now let us consider the case of the prominences. If we have available a small spectroscope like that admirable little instrument designed by Evershed, or the one made by Thorp,‡ or a still simpler home-made instrument, and attach such a spectroscope to a 4-inch or 6-inch telescope, we have an almost ideal equipment for the observation of the solar prominences. As a matter of fact, an instrument like the 40-inch is wholly unsuited for work of this kind. You will easily see why. If you wish to observe the entire prominence, its image in the focal plane of the 40-inch telescope is usually so large that the slit cannot be opened wide enough to include the prominence without admitting too much light of the sky. Therefore, for a study of the general characteristics of prominences, the small instrument has a great advantage over the large one. It was practically out of the question with the 40-inch for us to do systematic visual work on prominences. When the conditions were peculiarly fine we could study the structure of certain prominences, and I never saw anything more remarkable than such details when they came out under the best seeing. But with the spectroscope available, and under ordinary atmospheric conditions, we could not make records of the general form and distribution of prominences that would compare in value with the records obtainable with small telescopes.

* For example, it would be of great interest to study the structure of the umbra, as seen through a minute pin-hole in the focal plane of a positive eyepiece, as Dawes did many years ago.

† It must not be forgotten that photography is still far behind visual observations in revealing the minute structure of sun-spots. It can hardly be doubted, however, that if only the umbra and penumbra were permitted to fall on the plate, and the exposure properly regulated, new and valuable results would be obtained. The amateur will readily find many opportunities for work in this field.

‡ I wish to call special attention to the solar spectroscopes and other inexpensive instruments made by Mr. Thomas Thorp of Manchester. One of these, a polarising helioscope, has done excellent service on Mount Wilson.

It has remained for certain amateurs here in England very recently to show that objects upon the surface of the Sun which escaped many of the earlier solar observers can be observed at any time when the conditions are favourable with a very small instrument indeed. For example, Mr. Buss and Captain Daunt, and, I believe, some others, have been observing the Sun with such instruments, and have been able to see upon the disk dark regions in which the D_3 line is strengthened, which I think have never been recorded before in a systematic way. Observations of the dark D_3 line upon the face of the Sun were formerly mentioned as unusual and rather remarkable phenomena, and certainly, so far as I have ever seen in the literature of the subject, the dark hydrogen flocculi were never recognised upon the Sun by the earlier spectroscopists; but they are seen, at times at least, by those gentlemen to whom I have referred. This I can make quite certain from my own knowledge, because on one occasion, when Mr. Buss had described one of the very peculiar dark hydrogen flocculi—flocculi of this type appear very much darker than the ordinary ones photographed daily with the spectroheliograph—I looked up our photographs of that date, and there was the image recorded by the spectroheliograph precisely as it had been described. So that if I had previously been a little doubtful as to the possibility of seeing these objects with such an equipment, I gave up all doubt after having made that comparison.* One might say that it would hardly be practicable to observe such phenomena in any satisfactory way with a large telescope. A small one is very much more advantageous for work of this kind. As soon as possible we are going to set up a small equatorial for the purpose of seeing these objects and comparing them with our photographs, after having derived the knowledge of the possibility of observing them from the work done by these men in England. But we will not undertake systematic work in this field, as I hope the valuable observations now in progress here will be continued. No records are made with the spectroheliograph of the D_3 image of the Sun at present. We have tried experiments, but so far they have not been successful. We ought to be able to photograph the Sun through the D_3 line, but we have not done it yet. The only existing records are those made by the members of the British Astronomical Association. These observations should be made in conjunction with other solar observations, as in fact is being done at the present time. The characteristics of the hydrogen lines are being observed at the same time that these D_3 images are being recorded, so that any relationship between the two may be discovered. I cannot dwell upon this very interesting subject. There is a great opportunity here for further work of high importance.

I must now pass to the question of sun-spot spectra. I need hardly tell those who are present that observations of sun-spot

* As I understand the matter, only the more conspicuous dark flocculi can be observed visually.

spectra made visually are sometimes far more valuable than those which can be made by photographic methods. Take, for example, the lines in the green region of the spectrum. This photograph will suffice to show them. Here is the *b* group in the spectrum of a sun-spot and also in the spectrum of the photosphere. We see in the spot a large number of fine lines, long ago observed by Young and Maunder, and now being studied with great care. All of these fine lines shown by a powerful instrument photographically can be seen visually with a small spectroscope attached to a 6-inch or probably a 4-inch telescope, and many other phenomena which cannot be photographed at all can be seen with a similar equipment. There is a certain advantage in observing such spectra with a larger telescope, provided that the spot under consideration is a small one. But if the spot is a fairly large one (and hitherto no one has had time to observe the spectra of small spots systematically), I think there is no advantage whatever in having a large telescope to form the image of the Sun on the slit of the spectroscope; it is merely a question of having an image of moderate dimensions upon the slit, and after that the spectroscope does the work. So that, so far as the spots actually under observation are concerned, a small telescope is quite as satisfactory as a large one for *visual* work on their spectra.

I will return in a moment to the question of the relative advantages of the photographic and the visual method of observing spot spectra; but I want to point out in passing that the 40-inch telescope has certain very definite advantages for work on the Sun. If one wishes to observe the spectrum of the chromosphere, for example, the advantages of great focal length immediately become apparent. The width of the spectroscope slit is essentially constant; the chromospheric arc must have a certain linear width on the slit in order to permit the base of the chromosphere to be observed; and consequently the spectrum of the chromosphere, as seen with the 40-inch telescope, is a remarkable sight, showing thousands of lines which do not come out with a small focal image of the Sun.

Here we have, then, an illustration of the advantages for certain purposes of considerable focal length. I think it is not so much a question of the telescope's aperture here, because we must not forget, in thinking of the optics of this question, that the brightness of the spectrum (for constant purity) is quite independent of the linear or the angular aperture of the object-glass that forms the image of the Sun on the slit of the spectroscope.* Perhaps it is well to bear in mind that the brightest solar spectrum one can get is obtained without any telescope whatever to form an

* When the focal length of the collimator is limited (as is usually the case in a spectroscope attached to an equatorial telescope), an increase in the angular aperture of the telescope permits the linear aperture of the spectroscope, and consequently the resolving power and the brightness of the spectrum, to be increased up to a limit fixed by the size of the grating available. With a coelostat telescope, however, the same conditions do not obtain, since the aperture of the spectroscope can be increased by merely increasing the focal length of the collimator.

image on the slit, but merely with a collimator of suitable angular aperture. But a large solar image is frequently advantageous, and an equatorial telescope of great focal length is necessarily an expensive instrument. The aperture in the case just mentioned is less important than the focal length; but even if the aperture were only 6 inches and the focal length unchanged, the tube must still be 64 feet long, and the mounting would cost no less than the mounting of the Yerkes telescope. So if we wish to have an instrument of great focal length, and yet keep down the expense to a reasonable figure, we must use a telescope of a different type. There are many other reasons why we should wish to use a fixed telescope for certain kinds of solar work, although I should be the last to admit that the 40-inch telescope is not an almost perfectly satisfactory machine of its kind. It has, as we have seen, inconveniences and disadvantages for some classes of work, but in other fields its superior qualities become more and more striking day after day as the observer learns to appreciate them. I only wish we could afford to have such a telescope (or even a much smaller equatorial refractor) on Mount Wilson, as it would be of great service for many purposes.

Now let us consider some of the possibilities of the fixed telescope; and let me show, for purposes of comparison, a picture on the screen of the Snow telescope which is now employed at Mount Wilson. Here is a cœlostat, with mirror 30 inches in diameter. After passing to a second mirror the light is reflected to a concave mirror of 60 feet focal length, which sends it back and forms a large image of the Sun within a laboratory. This is a very simple instrument indeed. The first cœlostat we set up on Mount Wilson was a small one used by the Yerkes Observatory party at the eclipse of 1900, and it was not originally arranged for work of this kind; so we simply built a wooden support for a second mirror, and with the aid of a 6-inch objective of 60 feet focal length we made a telescope which served admirably for our solar work until this one was put up on the mountain.

The next photograph shows the spectrograph used with the Snow telescope. It is of the Littrow or auto-collimating type, with slit and plate-holder at one end of a long tube and lens and grating at the other. Light from the solar image, after passing through the slit, falls on the lens 18 feet (its focal length) distant. The rays, thus rendered parallel, then strike the grating and are returned to the lens, which forms an image of the spectrum on the photographic plate, just above the slit (the grating being tipped back a little). Such an outfit (fixed telescope and spectrograph) is an extremely simple thing to build in inexpensive form. Cœlostats, for example, are common nowadays for eclipse work. One might have a cœlostat with a mirror only 6 inches in diameter and a second mirror about 4 inches in diameter, and then perhaps a telescope lens of 4 inches aperture and 40 feet focal length. Such an instrument as that, which could be very cheaply built indeed, would give a large solar image, adapted for many kinds of solar work.

Let me show you in the next slide how we build our spectrographs in actual practice. This is the most powerful spectrograph in use in the laboratories of the Solar Observatory. Here is a little slit I bought from Hilger, the last time I was in London, for a few shillings. All other parts of the spectrograph, except a lens and grating, are of wood, built in a few hours by a carpenter.* The wooden support for slit and plate-holder stand on a concrete pier, and close an opening through a partition which forms one end of a narrow dark room. Eighteen feet from the slit, within the dark room, is another concrete pier. A sliding wooden support, carrying a lens, and a simple wooden mounting for the grating, stand on this pier, and complete the spectrograph. Owing to the scarcity of gratings, we are fortunate in being able to use one loaned by Professor Ames, of Johns Hopkins University. If we had no reflecting grating, we could buy a replica very cheaply from Thorp, or Wallace, or Ives,† which would give quite as good photographs as we obtain now (though the exposures would be longer, because of the smaller aperture). They might even be better, because our photographs of spot-spectra (made with the similar spectrograph of the Snow telescope) are not what they ought to be, or what I hope they will subsequently become. They would not stand comparison for a moment, so far as perfection of definition is concerned, with those magnificent photographs of the solar spectrum made by Mr. Higgs in the centre of Liverpool, under conditions which would ordinarily be called very bad even for a crowded city, with tram-cars constantly passing in front of the house. With a spectrograph of his own construction (except the grating), Higgs made the finest photographs of the solar spectrum ever produced; superior, as Rowland would have said, to the best photographs made by himself at the Johns Hopkins University. It is obvious that something other than an expensive instrument is required to make a good photograph. Mr. Higgs has the ability, which others may acquire, to obtain superb definition and exquisite photographs with very simple apparatus indeed.

With a spectrograph of 1 inch aperture and 10 feet focal length, used with a fixed telescope of 4 inches aperture and 40 feet focal length, one would be in a position to make good photographs of the spectra of sun-spots.

What, then, are the relative advantages of visual and of photographic work? The next slide shows some photographs. The upper one is the spectrum of the Sun and the lower one is that of a spot. These photographs are better than visual observations for the determination of the wave-lengths of unknown lines in spot spectra, simply because you can measure the position of a line on

* Except the plate-holder, which is of a standard make.

† As these are not reflecting gratings, the auto-collimating spectrograph might in this case give way to one in which a separate camera lens is used. With the angular aperture here considered, well-made simple lenses would obviously serve perfectly well for collimator and camera, the photographic plate being set at the angle required to bring a sufficient range of spectrum into focus.

the photograph to much better advantage than you can do it visually at the telescope. They are also better for the determination of the relative intensities of the lines, especially the fainter ones. But when you have said that, you have said almost everything that can be said for the photographs, and you have left out of account many of the very important advantages of visual observation. These photographs represent the integrated spot spectrum, as it were. Even with a large image of the spot on the slit of the spectrograph (and you realise here that the principal point of our great focal length is to have a large image of the spot on the slit), we cannot as yet satisfactorily record minute differences in the spectrum corresponding to small details in the spot. If we wish to study these very important differences in the spot, we must do so, at present at any rate, by visual means. For example, Mr. Newall, your President, told me the other day that he had found the spectrum of the outer edge of the penumbra of a spot to have the same characteristic strengthening of the lines that is observed in the umbra, which is a very difficult thing to explain from the standpoint of the hypothesis I have been favouring of late, viz. that the principal cause of the change of the relative intensities of lines in a spot is reduced temperature of the vapours in the umbra. I knew nothing about that; I had not been observing the spot spectrum visually for many years, and in our photographs this phenomenon is not recorded. You see, then, in such a case the decided advantage of visual observations. I might go on to speak of other advantages. For example, suppose there were a sudden change in the spectrum due to an eruption; the chances that one would get a photograph just at that time are small, whereas visual observations necessarily occupy a considerable period of time, during which eruptions might be detected.* Even a few results might be of extreme importance, and would probably be wholly missed in the photographs. Again, the extension of certain lines outside of the spot upon the photosphere is not recorded at all in our photographs, because of the method we usually employ of excluding from the plate all light except that which comes from the umbra, and perhaps part of the penumbra. We ordinarily get no trace of these extensions, but perhaps the conclusions drawn from the study of such phenomena may have much to do with the final views as to the nature of the spots themselves.

To mention only one other thing, the reversals of spot lines which have been seen by some observers have not been photographed with our present apparatus. Whether they can be photographed in the future remains to be seen. But, without going into this subject of spot spectra any more in detail, you will certainly agree that the visual observer has a superb opportunity, which the photographic observer cannot by any possibility take away from him.

* It is, of course, desirable to take photographs as often as possible, since a photographic record of a marked change in the spectrum, if fortunately obtained, may be much more valuable than the results of a few visual observations made hastily.

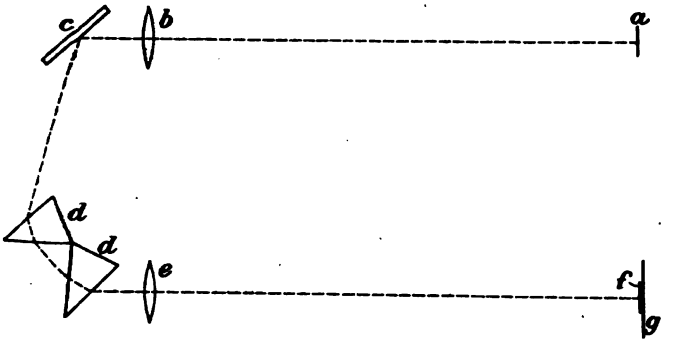
I now wish to speak rather more particularly of another phenomenon mentioned here the other night, which is peculiarly adapted for investigation with a small solar image. I refer to the differences between the spectrum of the centre of the Sun and the spectrum of the Sun's disk near the limb, as shown in the next photograph. Here is the spectrum of the centre of the Sun, and here is the spectrum of the Sun at a point a short distance inside of the limb. You will see at once the remarkable changes that take place. The broad H_1 and K_1 lines (or bands) are greatly reduced in width; and the same thing occurs, I think, in the case of all lines that are accompanied by wings. In this region of the ultra-violet many of these lines have wings, which are lost or greatly reduced near the edge of the Sun. This causes a remarkable change in the appearance of the spectrum. Several other curious things occur. Not only do these wings change in intensity very much, but the central part of the line, which seems to be sharply distinguished from the wings, undergoes a decided change of intensity also, so that we find from a preliminary examination of the plates that the lines that are strengthened in sun-spots are generally strengthened near the edge of the Sun, while the lines that are weakened in sun-spots are generally weakened near the edge of the Sun. This is true, I think, in the great majority of cases. Again, we find another curious thing: almost all of the lines derived from points near the Sun's limb are shifted towards the red in the spectrum with reference to lines from the centre of the disk. But there are some striking exceptions, and one of them is most significant: the lines in this fluting of cyanogen are not appreciably displaced. As we know from laboratory experiments that flutings are not displaced by pressure, whereas lines are thus displaced, we seem to have an interesting confirmation of the conclusion previously reached by Halm from his visual observations of two lines in the red—that the displacement of these lines is to be ascribed to pressure.*

This investigation is a many-sided one, with applications to both solar and stellar phenomena. There is room here for many investigators, who can obtain results quite equal, and very likely superior, in value to any we can get at Mount Wilson. A large image of the Sun is not required, because the effect is very appreciable at some distance from the limb. It is also a matter of no importance whether the definition of the solar image be good or bad. The one essential point is that the spectrograph be fairly powerful, and this is a very simple thing to realise at moderate expense. I hope to see this subject taken up by several observers, who will determine the shifts and the relative intensities of the Fraunhofer lines, seek for evidence of periodic changes, and work out an explanation of these remarkable phenomena which will harmonise with some

* This conclusion is further confirmed by the fact that lines of a given element, which exhibit unequal displacements at a certain pressure in the laboratory, in general show corresponding displacements near the Sun's limb. It remains to be seen, however, whether some other hypothesis may not be equally capable of accounting for the observed phenomena.

explanation of the relative intensities of the same lines in sun-spots and in the spectra of stars.

I may now touch upon another field of solar research, and consider the possibility of doing useful new work with the spectroheliograph, which is by no means so expensive and formidable an instrument as one might suppose. The slide shows the first spectroheliograph used on Mount Wilson, before we built the more permanent one now employed; and since the fact that we did substitute a permanent instrument for the temporary one might lead to the inference that the former did not give good results, I may add that the photographs made with the wooden instrument are even better than the later ones. They show only narrow zones of the solar surface, but for sharpness they have never been surpassed.* In the illustration the spectroheliograph is partly hidden under this spectrograph, and you can only get a rough notion of it. There is a rectangular wooden platform here mounted on a pier.



At each corner of the platform was screwed a small cast-iron block, in which a V-shaped groove had been planed. In each groove was a steel ball. A moving platform, also built of wood, carried the optical parts of the spectroheliograph and rested on these balls, so that it could be moved across the image of the Sun (formed by a coelostat telescope). The motion was produced by a small electric motor, belted with a piece of fish-line to this large wooden pulley, which drove a screw passing through a lead nut fastened to the movable platform. The screw was cut on a foot lathe and the nut cast on it. This simple mechanism provided the means of producing a slow uniform motion of this upper platform across the image of the Sun. The arrangement of the optical parts was precisely the same as in the Rumford spectroheliograph.

Looking at the instrument in plan, we have a slit here (*a*) through which the light passes. A very simple slit will do. This was an old one; I think it came from a portion of the old Kenwood

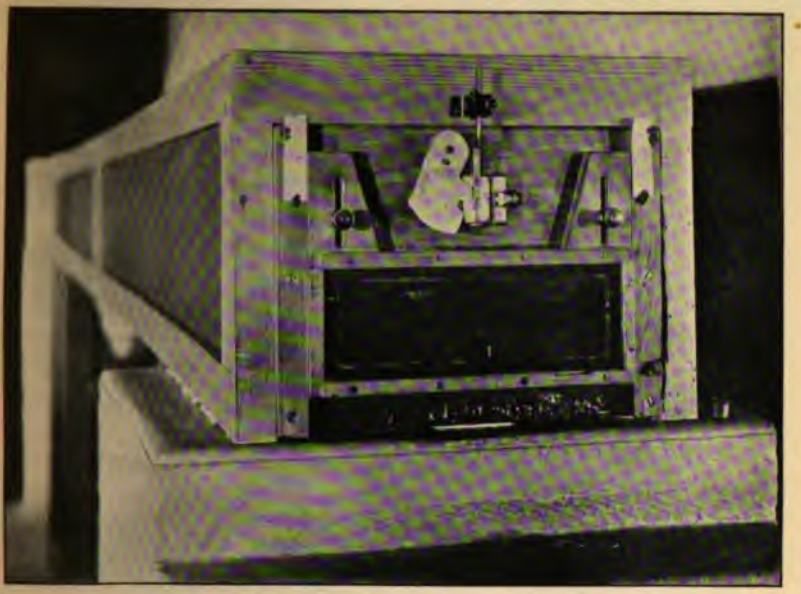
* In the 5-foot spectroheliograph now employed, the dispersion is great enough for photography with the hydrogen as well as the calcium lines. For this reason the exposures are longer, and the definition somewhat less perfect, though quite satisfactory for practical purposes.

spectroheliograph. The light passed through this slit and fell on a collimating lens (*b*), which may be an ordinary uncorrected lens if the focal length is sufficient. We happened to have some achromatics which we used, but they were no better than a simple lens would be. The parallel rays fell on a plane mirror here (*c*), and were reflected to these prisms (*d*, *d*). We used two prisms, but one will do perfectly well, unless hydrogen as well as calcium flocculi are to be photographed. These prisms had been discarded; they were originally made for the Bruce spectrograph, but they were so poor they could not be advantageously used, so we borrowed them from the Yerkes Observatory and put them in here. The two prisms, with the mirror, gave a total deviation of 180°. The light then passed through the camera lens (*e*)—here, also, a simple lens will serve very well—which formed an image of the spectrum on a second slit (*f*), close to the fixed photographic plate (*g*). By setting this slit on the H_2 line of calcium, and moving the instrument slowly across the solar image with the motor, excellent photographs of the calcium flocculi were obtained.

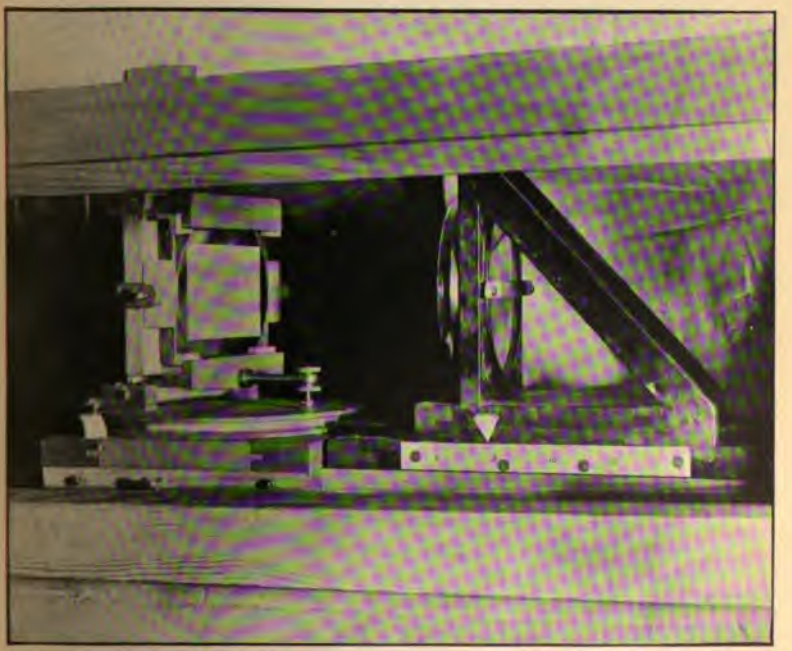
The next slide shows some photographs taken with the permanent instrument. Such photographs as these, made with the calcium and hydrogen lines, open up for investigation a large field, which anyone can enter with just such an equipment as I have described—a very simple instrument, with small prisms and lenses, and built almost entirely of wood.

I will show you in the next photograph some pictures obtained with the wooden instrument. You will notice that in this case the motion was not absolutely uniform; you can detect the slight irregularity of motion, but it did not affect the usefulness of the negatives. This is a direct photograph of the Sun; this is made with the H_1 line of calcium, and this is the same region as photographed with the H_2 line of calcium. If somebody would go to work with such an instrument and let us know exactly what such photographs as these mean, they would at least confer a very great favour upon me, because hitherto I have been unable to determine with certainty the relative parts played by the continuous spectrum of the faculæ and the light of the H_1 line of calcium in producing the photographs. That question is still open, and many investigations will be required to settle it beyond doubt.

In this H_2 photograph we probably have a picture of the calcium vapour at a higher level than the level represented by the H_1 plates. You see, for example, this bridge of calcium vapour across the spot, which is not shown by H_1. Many investigations of great interest could be carried on with such a spectroheliograph as I have described. I wish I had time to go into them; there is only one I may mention, and that is the comparison of the calcium and the hydrogen images. Mr. Butler has asked me to explain to-night a point which I unfortunately failed to make clear in my talk here at the last meeting of the Society. In speaking of the relative level of the calcium and hydrogen flocculi, I said we found that the dark hydrogen flocculi are shifted somewhat towards the



Slit and plate-holder end of simple wooden Spectrograph of Littrow or auto-collimating type (18 feet focal length), used in the spectroscopic laboratory on Mt. Wilson [not the instrument referred to in the lecture, but a similar one, suitable for use in an open room].



Grating and lens supports for modern Spectrograph (18 feet focal length), used in the spectroscopic laboratory on Mt. Wilson. The bar for cutting off reflections from the lens is shown.

[*To face page 76.*

limb of the Sun as compared with the corresponding bright calcium flocculi. The natural conclusion to which I came was that the hydrogen absorption shown in this photograph is produced at a somewhat higher level, amounting to something like 1500 miles, than the calcium radiation which gives us this photograph. Mr. Butler pointed out to me that the photographs of the flash spectrum show the calcium vapour to rise to a higher level than the hydrogen gas, and that the difference is about 1500 miles. There is no question about the validity of this result, and the point is to show that it is compatible with my conclusion. I think the reason is simple enough, and lies in this fact: the flocculi photographed with the H_2 line do not represent the highest calcium vapour, but a level considerably below that; whereas the *absorption* phenomena known as hydrogen flocculi apparently represent the upper hydrogen in the chromosphere, or in some cases the prominences themselves. The level of the hydrogen absorption seems to be about 1500 miles higher than the region from which the H_2 light of calcium proceeds. If, as occasionally happens, the highest calcium vapour in the chromosphere is recorded photographically, it acts as hydrogen does, and gives dark absorption phenomena, due to the high level H_3 line, and not to be confused with the bright calcium flocculi due to H_2. This point is perhaps a minor one, but it illustrates some of the results that can be obtained with a spectroheliograph.

I see that I must rapidly draw to a close. I might mention various other methods of employing spectroheliographs, and if anyone present should be interested at some future time to take them up I shall be delighted to discuss them in detail. I may remark in passing that with a Littrow spectrograph, or any long focus spectrograph, and a fixed solar image, one can undertake other work of various kinds, such as a determination of the solar rotation, along some such plan as Dunér or Halm followed, but using different lines in the spectrum, and benefiting from the advantages of photographic methods. In all such work, co-operation with other investigators is greatly to be desired, because it might otherwise frequently happen that two men would be doing the same thing, whereas it would be just as easy for them to supplement each other's work instead of duplicating it.

One other phase of the subject which I should like to have time to discuss, but cannot, is that of stellar spectroscopy. You will see that for stellar spectroscopy a large telescope in general does have an advantage. The more light one can collect and concentrate in a stellar image the more dispersion can be employed in the spectroscope, and the users of large apertures therefore do have an advantage in stellar spectroscopic work. But the fact remains that small instruments can be used to very great effect in this field also, provided that one intelligently plans his investigations. I know of no better example of this than one which I am permitted, by the kindness of Father Sidgreaves, to illustrate. Here is a photograph of the spectrum of o Ceti, made with a

refractor of 4 inches aperture, with a prism of $22\frac{1}{2}^{\circ}$ angle placed over the object-glass. The focal length of the telescope is 4 feet.

The slide shows the spectrum of Omicron Ceti on the 29th November 1905 and on the 1st December 1906, and brings out with great clearness the remarkable changes which occurred during that period. If this spectrum had been photographed with such an instrument, let us say, as the Bruce spectrograph of the Yerkes Observatory attached to the 40-inch telescope, there would have been some advantages, but there would also have been some disadvantages, because the entire region covered by the photographs made with that instrument (when three prisms are used) is a limited one here in the blue. All of these remarkable flutings in the less refrangible region would not have appeared in the photographs, and nothing would have been known, if one had been confined with such an instrument to a short region of the spectrum, about the very interesting changes shown in this particular case. The next slide shows another photograph taken by Father Sidgreaves, in this case with a somewhat different instrumental arrangement—a direct vision prism at the focus of a 15-inch equatorial. But you will see the great range of spectrum included on the plate, and remember again that almost all the spectrum, except a very small region, would be missing on photographs taken with such instruments as the Bruce or Mills spectrographs, or other three-prism instruments employed for the investigation of stellar motions in the line of sight. You will notice the remarkably interesting and important fact that the Hϵ line of hydrogen is absent from the picture, probably, as Mr. Newall suggested, cut out by the absorption of the H line of calcium—the broad H_1 band; perhaps in another star lying nearer to us than the star which gives the bright lines of hydrogen. This serves to illustrate the great importance of the work that can be done with an instrument of very small size indeed, even in this field of stellar spectroscopy, which seems peculiarly to belong to telescopes of large aperture. As I said before, in general the investigator with a telescope of large aperture does have an advantage in stellar spectroscopic work; but there are various investigations of this sort—and of the kind Professor Pickering has taken up in his very extensive surveys of the whole sky with objective prisms—which are of extreme importance, and which cannot be replaced by work done with large instruments.

I might go on to speak of the possibilities of work on variable stars, but they are familiar to most of you. The observation of many wide double stars, my friend Burnham tells me, has been neglected since the time of Herschel, because the large instruments, and even the small ones, have been devoted to closer objects, so that in revising his great catalogue Burnham had to measure with the 40-inch a great many wide doubles which had not been looked at perhaps since Herschel discovered them more than a century before. Important double-star work is always open to men with small instruments, if a micrometer is available.

Then I might go on to the case where a man has no telescope at all, and still wants to make contributions to astrophysics. I do not now speak of such splendid work as Anderson did when he discovered Nova Persei with the naked eye; but if one were convinced that the overcast sky of London would never open again, he could still work in his laboratory and make important contributions by identifying lines and bands in spot spectra, as Professor Fowler has been doing of late, or by researches in a score of other fields.

I will close with a few practical suggestions. One reference to the matter of atmosphere. Perhaps some of us feel that if we could only ascend into the upper regions we could get results very much better than are obtainable in London. But if we stop to think of the men who work in London and what they have done, we must recognise the fact that even here the conditions are not so bad as we sometimes imagine. I have often been strongly impressed (since my work in Chicago) with the belief that a smoky atmosphere has some advantages in astronomical work, for it seems that the seeing is frequently improved in solar observations when the sky is smoky. Here is a fine chance to test that question, and I think it has been tested at Greenwich, and that some of the photographs taken there (both solar and stellar) prove that London smoke does not prevent excellent definition. I examined rather carefully some plates there yesterday, and the star images are surprisingly good in many instances. It seems to me that definition by night as well as by day at Greenwich must be of an order much higher than one might suppose when one thinks of Greenwich as being within the boundaries of London. But it is perfectly possible to get good results anywhere, provided sufficient care is taken. One must consider, for example, the best time of day for solar work. It usually happens that the best definition of the Sun occurs in the early morning and the late afternoon. Mr. Newall tells me that this is as true at Cambridge as it is at Mount Wilson. This is worth looking into if one takes up work on the Sun. Further, one must have a definite plan of work. This is of prime importance. Devote your entire attention to a single investigation, involving, if possible, two or three parallel series of observations, so devised as to throw light on one another. Frequently the value of a given series of observations may be enormously enhanced if other observations are available to aid in their interpretation. For example, in studying the spectra of sun-spots, the character of the spots, their motions, and changes of form, and the distribution of the flocculi in their neighbourhood, may be vital factors in interpreting the spectroscopic phenomena. Then, again, there is the great possibility that new methods and new instruments can be applied. Up to the present time I think the interferometers of Michelson or of Pérot and Fabry have never been systematically employed for work on the Sun: that admirable method which Fabry is using at the present time in the determination of absolute wavelengths would perhaps be very useful indeed if applied to the

measurement of the displacement of solar lines at the centre and at the limb. I also believe that the echelon spectroscope has never been used for the observation of the narrow bright lines in the chromosphere. Furthermore, we are always confronted by the possibility of perfecting our optical apparatus. I have been trying for years to get good prisms of large size, but cannot get homogeneous glass, and therefore it now seems necessary to attack the problem of fluid prisms. If someone could take that question up and show us how to make very large prisms that would be essentially perfect, they would accomplish a great advance. Lord Rayleigh told me the other day how he made some large fluid prisms that gave nearly theoretical resolution. By an extension of the same methods it seems likely that still larger prisms, suitable for the exacting requirements of photographic work, could be obtained.

And so I might go on pointing out opportunities of various kinds, but I should tire you if I ventured to do so. We must not forget, however, that the possibility always exists of getting some entirely new method that will be quite as important as any application of the interferometer, or the échelon, or other instruments to which I have called attention.

In concluding, I may add that we have made at the Solar Observatory a few drawings of some of these simple wooden instruments, which I shall be very glad to place at the disposal of anyone who might care to build instruments in a similar way.* They may serve a useful purpose by saving a certain amount of time.

I hope I have shown that it is possible not merely to do work of an inferior quality, but to do work of the first quality, with small or inexpensive instruments; work that cannot be duplicated or will not be duplicated with large instruments; in other words, that there is a splendid field for any man who wishes to accomplish results, wherever he may be situated, and however simple his means of research may be. I feel so strongly on this subject that I hope the suggestions I have made will not be entirely without effect. We need the ideas of men from all parts of the world; we need the contributions they can make; and we need them even more than we need larger instrumental means than we now possess.

* At Mr. Maw's request, a number of blue prints will be sent to the Royal Astronomical Society for convenient reference.

Errata in the Rev. T. E. R. Phillips's paper, Monthly Notices, vol. lxvii.

Page 524, line 8, *for* 1895–6 *read* 1905–6.
Page 526, line 31, *for* 1907 *read* 1906.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVIII. DECEMBER 13, 1907. No. 2

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Burdett Mason, Larondouette, Bayonne, France; and
Arthur William Meers, F.R.G.S., Lugano, Wickham Road, Beckenham,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Henry Hermann Gruning, M.Sc., Patent Agent and Engineer, 3 Blakesley Avenue, Ealing, W. (proposed by Rev. F. B. Allison);

Jan B. Hubrecht, Christ's College, Cambridge (proposed by Bryan Cookson);

Victor A. Lowinger, Trigonometrical Survey Department, Taiping, Federated Malay States (proposed by Sir David Gill);

Rev. Malcolm Parker Miller McLean, M.A., The Rectory, West Raynham, Norfolk (proposed by Henry T. Gerrans);

Alfred W. Porter, B.Sc., Assistant Professor of Physics, University College, London, W.C. (proposed by L. N. G. Filon);

Harold Knox Shaw, B.A., Trinity College, Cambridge (proposed by S. A. Saunder);

Capt. Eldred Weston White, Brockley Villa, Upton Road, Southville, Bristol (proposed by E. Fawcett White).

Mr. A. E. Conrady, Mr. R. Inwards, and Mr. G. J. Newbegin were appointed auditors of the Treasurer's accounts for 1906.

Eighty presents were announced as having been received since the last meeting, including, amongst others :—

Abbadia Observatory, Observations, tome 5; and Bordeaux Observatory, Catalogue photographique du ciel, Coordonnées rectilignes, tome 2, presented by the Observatories; Gronnigen Astrographical Laboratory, Libration of the three inner large satellites of Jupiter (W. de Sitter), presented by the Laboratory; Ziegler Polar Expedition, Scientific results, presented by the estate of Wm. Ziegler.

Astrographic Chart; 16 charts presented by the Royal Observatory, Greenwich; and 19 charts presented by the French Minister of Public Instruction.

Solar Parallax Papers. No. 6.

Construction of a Standard Catalogue of Photographic Star Places. By Arthur R. Hinks, M.A.

§ 1. The construction of a standard catalogue of photographic star places along the track of the planet Eros was undertaken in the first instance to provide for the reduction of those series of photographs of the planet which, having a field smaller than 2° square, could not be reduced directly with the *étoiles de repère*; and also to provide exact places of the stars which had been used in visual micrometric comparisons with the planet.

It now seems probable that the standard catalogue may have an extended use; for, as will be shown later, it will be possible by its aid to effect very simply the equivalent of a re-reduction of all the published places of the planet to this new system of stars, lying close to the track of the planet, and more nearly equal to it in magnitude than are the *étoiles de repère*. This should provide a powerful method of searching for systematic errors in the photographic places of the planet and the value of the Solar Parallax derived from them.

§ 2. The published material for the construction of the catalogue was found in the Paris Eros Circulars, Nos. 10 and 11.* The results of an inter-comparison of the work of the different observatories were published in *Solar Parallax Papers* Nos. 4 and 5 (*M.N.*, vol. lxvi. p. 481, vol. lxvii. p. 70, 1906 June and November). In those papers may be found the following conclusions :—

(1) That several series of photographic star places are affected with a magnitude equation of serious amount.

(2) That other series are affected by progressive discordances,

* Circular No. 12 appeared after the principal part of the work was finished. But see later, §§ 11 and 12.

which are sometimes dependent on variations in the adopted places of the fundamental stars, and sometimes not.

(3) That, in general, the star places published in the Paris Circulars are by no means homogeneous.

(4) And that the adopted form of publication did not permit of a complete inquiry into these matters.

§ 3. In 1906 March I ventured to send to the Directors of the observatories concerned a circular letter, submitting for their consideration the following proposals:—

That I should print and distribute a large number of lists of the *étoiles de repère*.

That they should kindly undertake to fill in upon those sheets, in the spaces provided for the purpose, the deduced photographic places of the *repère* stars, using a separate list for each plate, so that the results from each individual plate were kept quite distinct.

That they should also fill in, upon other sheets to be provided for the purpose, the lists for each separate plate of the deduced places of the *étoiles de comparaison* (given in Paris Circulars, Tableau III., either in ledgers or as means, without indication of the plate from which each was derived).

These proposals were very kindly received by the Directors to whom they were addressed. The printed sheets were sent out in 1906 May, and were quickly filled up and returned. My sincere acknowledgments are due to the Directors of the Observatories of Bordeaux, Helsingfors, Northfield, San Fernando, and Toulouse, for the cordiality with which they undertook the troublesome task of filling in upon the printed sheets all the desired information; to the Astronomer Royal, who sent advance sheets of the Greenwich volume containing equivalent information; to the Director of the Pulkowa Observatory, who had previously given me similar material in advance of publication; and to the Director of the Paris Observatory, who furnished advance proofs of the Paris original measures.

The information thus obtained, added to that already published in full in the Paris Circulars for the *étoiles du carré de 20′*, gave me the completely separated results from a very large number of individual plates covering the path of the planet to 1900 December 31. (The discussion of the material for 1901 is postponed to a later date.)

§ 4. To make this considerable mass of material (about 60,000 star positions) homogeneous, I had first to find a way of reducing all the series to one fundamental system,* and to decide what this system should be.

In paper No. 4 I have given reasons for thinking that no one of those systems of fundamental star places based directly upon the meridian observations is satisfactory for our purpose. I decided to form a provisional system from photographic series which had

* See *S.P.P.* No. 4, § 4, for the system to which each was originally reduced.

been reduced to Loewy's system of meridian places. These series are compared in *S.P.P.* No. 5, p. 80, Table VI. It is there shown that the results from Greenwich and Paris are in excellent agreement; that the Toulouse places are nearly accordant with them; but that the Catania places differ from Paris in a large and irregular manner. I decided to omit the Catania results for the time, and to base the provisional photographic system of *repère* stars on a mean of Greenwich, Paris, and Toulouse.

The three observatories were given equal weight. A photographic place depending on 1 or 2 plates was given weight 1; on 3 to 6 plates, weight 2; and on 7 or more, weight 3. There would have been little advantage in adopting a more elaborate system of weights, since the real comparative weight of a result from one and from two plates depends very much upon the consideration whether or not the two plates are upon the same centre, and reduced with the same selection of fundamental stars.

We will call this system the System P.Ph. (provisional photographic).

The rôle of system P.Ph. is temporary. It serves to detect and eliminate systematic deviations between different observatories. It will be judged by the eventual degree of accordance between the reduced results when they are brought together for formation of the mean standard places. If this is satisfactory, we may consider that the system P.Ph. has played its part, and it may be allowed to disappear. We need not give, therefore, in this paper, the adopted places of the *repère* stars in the P.Ph. system, but may proceed at once to examine its use, and the effects of its use.

§ 5. The system P.Ph. is used as follows:—

We take differences between the places of the *repère* stars *derived from a single plate*, and their places in the system P.Ph. The mean of these differences, in the sense P.Ph.—plate, applied as a correction to the plate places of the *repère* stars, makes them conform on the average to the system P.Ph.

The same correction applied to all other star places deduced from this plate should reduce them equally to the system P.Ph., provided that there is no systematic discontinuity between *repère* stars and other stars.

The application of this correction produces in a general kind of way the same effect upon the star places as a re-reduction of the plate to the system P.Ph.; it leaves the results rougher in detail than a complete re-reduction, but nearly without those systematic errors which have been called Progressive discordances in *S.P.P.* No. 5, Tables VI. and VII.

§ 6. Corrections to be applied to individual plates were obtained in this way for the whole of the plates made at Bordeaux, Helsingfors, Northfield, San Fernando, and Toulouse. The following summary will give an idea of the size and distribution of these corrections.

Table I.

Plate Corrections.

R.A. Corr.	Bord.	Hels.	North.	San Fern. Ord.	San Fern. S.S.	Toulouse. Ord.	Toulouse. S.S.
	s	s	s	s	s	s	s
Greatest	+·015	+·027	+·021	+·023	+·034	+·017	+·017
Least	−·010	+·002	−·005	−·017	−·014	−·009	−·021
Mean	+·003	+·014	+·007	+·007	+·009	+·001	+·001
No. greater than +·015	0	16	4	8	6	1	2
between +·015 and +·006	10	15	26	30	18	4	7
+·005 and −·005	21	5	17	19	15	5	22
−·006 and −·015	2	0	0	7	2	6	7
less than −·015	0	0	0	1	0	0	1
Decl. Corr.							
Greatest	+″·18	+″·06	+″·30	+″·30	+″·20	+″·10	+″·19
Least	−·10	−·08	−·12	−·02	−·06	−·06	−·13
Mean	+·04	−·01	·00	+·09	+·07	+·02	+·04
No. greater than +″·15	1	0	1	7	3	0	3
between +·15 and +·06	13	1	8	41	23	5	10
+·05 and −·05	17	33	27	17	14	10	25
−·06 and −·15	2	2	11	0	1	1	1
less than −·15	0	0	0	0	0	0	0

The above figures show that the reduction to a system P.Ph. is by no means superfluous.

The fact that the mean correction for an entire series often differs considerably from zero may no doubt be attributed to divergence in the methods of combining and weighting the meridian results for the adopted *repère* star places.

The variations from plate to plate must be due partly to these divergences, and partly to the roughness of the meridian places compared with the smoothed system P.Ph.

Plate corrections for Catania, Greenwich, and Paris were not derived, because the separated results for each plate were not at the time available.* It seems certain, however, that it would have been superfluous to apply the process to the Greenwich and Paris series, which agree exceedingly well, and have preponderating weight in the formation of the system P.Ph. The absence of plate corrections for Catania is to be regretted, since large divergences occur in this series. (See *S.P.P.* No. 5, p. 84.)

§ 7. *Corrections for Magnitude Equation in the San Fernando and Toulouse Series.*—It was shown in *S.P.P.* No. 5, p. 78, that the Toulouse results have a large and nearly linear magnitude

* The separated results for the Greenwich plates have since been communicated to me in proof, by the kindness of the Astronomer Royal.

equation in declination (but none in right ascension). This result has been confirmed by M. Baillaud, Director of the Toulouse Observatory, but he has not succeeded in discovering the cause of it. (For this information I am indebted to M. Baillaud.)

It was shown in the same place that the San Fernando right ascensions and declinations were both affected by a considerable non-linear magnitude equation. No further light has been thrown on the cause of this error. Corrections were obtained by plotting the results for successive magnitude groups (*loc. cit.*), and drawing the best possible smooth curves through them. The quantities derived from these curves were adjusted to allow for the fact that a general mean correction for stars of all magnitudes resulted from the application of the plate corrections of Table I.: the quantity $0''{\cdot}03$ was subtracted from the Toulouse corrections, and $0^s{\cdot}007$, $0''{\cdot}08$ from the respective San Fernando corrections. The final magnitude corrections applicable to Toulouse and San Fernando are given in condensed form in the following table:—

Table II.

Adopted Corrections for Magnitude Equation.

	Toulouse.	San Fernando.	
	Decl.	R.A.	Decl.
Mag.	$''$	^{s}	$''$
7·0	−0·23	+0·011	−0·21
7·5	− 15	+ 6	− 12
8·0	− 8	+ 2	− 6
8·5	0	− 2	+ 2
9·0	+ 8	− 2	+ 8
9·5	+ 15	+ 2	+ 12
10·0	+ 23	+ 6	+ 15
10·5	+ 31	+ 12	+ 15
11·0	+ 38	+ 18	+ 15
11·5	+ 46	+ 24	+ 14
12·0	+ 54	+ 32	+ 10

§ 8. *Collection of material for formation of catalogue.*—The individual star places published in Paris Circulars 10–12, or communicated to me in manuscript upon the printed forms sent out from Cambridge, were entered on catalogue cards, differently coloured for each observatory. These will be referred to as "detail cards."

The plate corrections and the magnitude equation corrections were applied on the detail cards, and "observatory means" were formed.

Whenever a star had been observed at more than one observatory, a "summary card" was added to the card catalogue on which the observatory means were collected, and the final weighted mean taken.

§ 9. *The System of Weights.*—The summary cards showed at once that the observatories were not entitled to equal weight. At the same time it was not possible to derive a rigorous system of weights from a discussion of the divergences between one observatory and another, or between the individual results of an observatory. These divergences were necessarily much affected by the fact that five or six separate determinations of a star place made at one observatory would depend on the same set of *repère* stars, and be on the same part of the plate, whereas the same star would be determined elsewhere on a different centre.

After some trial it appeared that substantial justice would be done if Greenwich and Paris were given a weight double that of the other observatories, and if the weight of each observatory's contribution, so far as it depended upon the number of observations contributed by each, were on the same scale as was used in making the system P.Ph. of § 4.

Two Catania plates and one San Fernando were rejected for unexplained systematic discordance of large amount; a certain number of obvious misprints in the Circulars were corrected, and a few discordant observations, probably misprints, were excluded.

Finally, stars whose weighted mean had weight 4 or more were passed into a first class, and these form the photographic standard system discussed in what follows. In the region covered by the planet between 1900 Oct. 1 and Dec. 31 the catalogue contains 1300 first class stars. And in addition there are about 2500 stars in a second class, whose weight is less than 4, which are not for the present included in the standard system.

§ 10. *Test of the Standard System.*—As a test of the homogeneity of the contributions which the different observatories make to the standard system, I have discussed the differences, Standard minus Individual contribution to it, from several points of view.

A. The differences, taken with regard to sign, have been grouped (1) in magnitude groups, to see if any residual magnitude equation remained; and (2) in date groups, to see if there were any discontinuity along the path of the planet which might gradually distort the standard system, and damage an ultimate determination of the mass of the Moon.

B. The differences, taken without regard to sign, have been grouped likewise (1) in magnitude groups, to see how far the interagreement of the photographic places is dependent on magnitude; and (2) in date groups, to see if the same standard of internal agreement is maintained along the whole system.

The magnitude groups are those used in *S.P.P.* No. 5, Tables III., IV., and V.

The date groups are broader than those used in Table VII. (*loc. cit.*).

Group	I.	covers	Sept. 15–Oct. 23
,,	II.	,,	Oct. 24–Nov. 24
,,	III.	,,	Nov. 25–Dec. 15
,,	IV.	,,	Dec. 15–Dec. 31

There is no special significance in this particular division, which arose more or less accidentally, and has been retained for convenience.

§ 11. *Search for Magnitude Equation in R.A.*—The division into magnitude groups of the differences Standard minus Individual contribution was made separately for the four date groups. But there is no evidence in the results of any definite change of magnitude equation with the date; if any such change exists, it is obscured by the accidental discordances. Consequently I give only the mean results for the whole period Oct.–Dec., except in the case of San Fernando, to which a correction for magnitude equation has been applied. The separate results for each San Fernando date group are given.

The figures in brackets are the numbers of observatory means contributing to the mean discordances which follow.

Table III.

Comparison of Right Ascensions in Magnitude Groups.

Standard System minus—

Mag.	Bordeaux.	Catania.	Greenwich.	Helsingfors.	Northfield.
	s	s	s	s	s
– 7·4	(14) – ·007	(78) – ·004	(32) ·000	(12) + ·015	(24) + ·012
7·5 – 8·4	(43) – 1	(66) + 3	(73) – 1	(57) + 4	(59) + 3
8·5 – 9·2	(99) 0	(55) + 6	(125) – 1	(75) – 5	(89) – 4
9·3 – 9·8	(48) – 3	(50) + 8	(32) + 2	(89) – 3	(10) – 3
9·9 – 10·5	(131) – 2	(72) + 7	(27) – 5	(117) – 8	...
10·6 – 11·2	(53) – 4	(118) + 3	(5) – 1	(99) – 5	...
11·3 – 12·1	(6) – 15	(167) + 1	(3) – 8	(7) + 6	...
12·2 –	...	(21) – 4	...	...	...

Mag.	Paris.	Pulkowa.	Toulouse.	Upsala.	Minneapolis.
	s	s	s	s	s
– 7·4	(47) + ·003	(7) + ·002	(33) – ·003	(15) + ·016	(8) + ·011
7·5 – 8·4	(70) – 1	(22) + 2	(124) + 3	(43) + 6	(17) + 3
8·5 – 9·2	(71) 0	(27) + 1	(229) + 4	(111) + 1	(38) + 2
9·3 – 9·8	(60) – 3	(5) + 10	(99) + 3	(68) – 1	(38) – 13
9·9 – 10·5	(103) + 1	...	(179) + 5	(84) – 1	(49) – 1
10·6 – 11·2	(121) + 2	...	(147) + 8	(33) – 6	(25) – 7
11·3 – 12·1	(193) + 2	...	(49) + 7	(5) – 3	(9) – 15
12·2 –	(267) + 2	...	...	...	(2) – 16

Mag.	S. Fern. Group I.	S. F. II.	S. F. III.	S. F. IV.	S. F. Mean.
	s	s	s	s	s
– 7·4	(19) – ·002	(16) – ·012	(4) – ·002	(15) – ·008	(54) – ·007
7·5 – 8·4	(35) + 3	(31) + 4	(9) + 9	(28) + 1	(103) + 3
8·5 – 9·2	(67) + 4	(72) + 2	(42) + 4	(52) + 1	(233) + 2
9·3 – 9·8	(21) + 11	(33) + 2	(21) – 4	(14) + 8	(89) + 4
9·9 – 10·5	(40) + 3	(63) + 11	(29) + 5	(14) – 2	(146) + 6
10·6 – 11·2	(46) 0	(79) – 1	(12) – 3	(8) – 10	(145) – 1
11·3 – 12·1	(8) – 6	(40) + 2	...	(3) – 13	(52) – 1
12·2 –	(7) – 2	...	...	...	(8) 0

The results of this table confirm the conclusion of *S.P.P.* No. 5, that the photographic series which contribute to the standard system are free from relative magnitude equation, with the exception of San Fernando.

The latter observations have been corrected, and we can now see how far this correction has been successful. The correction applied was decidedly non-linear; the above comparison gives some indication that a slight non-linear correction of opposite curvature is now required; that is to say, that the curvature of the original determination was a little excessive. (The material used was not altogether the same.) The magnitude equation results from the separate date groups are not, however, accordant enough to make it probable that any magnitude equation correction for San Fernando is sharply determinable; and we may conclude that the attempt to correct these R.A.'s has been as successful as the material allows.

The Upsala and Minneapolis results were not published in time to be included in the first edition of the standard system. But they show no magnitude equation when compared with it, and may therefore be included safely on a revision.

§ 12. *Search for Magnitude Equation in Declination.*—The declinations have been treated exactly as the right ascensions. Again there is no trace of any definite change of magnitude equation with the date, and the general means are free from it. The corrections applied to the declinations of San Fernando and Toulouse seem to have been successful in eliminating the large errors originally found in them. Table IV. gives the results for the date groups in which the elimination has been least successful, and for the mean.

The lately published declinations of Upsala and Minneapolis show no magnitude equation relative to the standard system.

Table IV.

Comparisons of Declinations in Magnitude Groups.

Mag.	Standard System minus— Bordeaux.	Catania.	Greenwich.	Helsingfors.	Northfield.
- 7·4	(14) +″·06	(78) +″·06	(32) +″·01	(12) +″·02	(23) -″·03
7·5 - 8·4	(43) - 1	(67) 0	(73) - 1	(57) + 5	(59) + 1
8·5 - 9·2	(99) + 1	(57) + 4	(125) + 2	(73) - 1	(89) 0
9·3 - 9·8	(48) + 1	(50) - 3	(32) + 4	(89) 0	(10) - 5
9·9 - 10·5	(131) 0	(74) + 2	(27) + 3	(117) - 2	...
10·6 - 11·2	(53) - 4	(118) + 3	(5) - 3	(99) - 3	...
11·3 - 12·1	(6) - 3	(177) - 6	(3) 0	(7) - 6	...
12·2 -	...	(22) - 11	...	...	...

Table IV.—*continued.*

Comparisons of Declinations in Magnitude Groups.

	Standard System minus— Paris.	Pulkowa.	S. Fern. III.	S. Fern. IV.	S.F. Mean.
– 7·4	(47) – ″·06	(7) – ″·02	(4) + ″·03	(15) + ″·07	(54) + ″·05
7·5 – 8·4	(70) – 3	(22) + 2	(9) + 6	(28) + 1	(103) + 4
8·5 – 9·2	(71) + 4	(27) 0	(42) + 10	(52) + 5	(233) + 1
9·3 – 9·8	(60) – 3	(5) – 5	(21) + 8	(14) + 9	(89) + 2
9·9 – 10·5	(103) – 2	...	(29) + 5	(14) + 8	(147) + 3
10·6 – 11·2	(123) – 3	...	(12) + 37	(8) + 19	(146) + 8
11·3 – 12·1	(193) – 2	...	...	(3) + 24	(54) + 4
12·2 –	(267) + ·1	...	...	...	(8) – 2

	Toul. II.	Toul. III.	Toul. Mean.	Upsala.	Minneapolis.
– 7·4	(9) + ″·13	(9) + ″·07	(33) + ″·05	(15) + ″·06	(8) – ″·05
7·5 – 8·4	(40) – 1	(18) + 3	(124) + 1	(43) + 3	(17) + 9
8·5 – 9·2	(71) – 1	(41) – 9	(229) – 2	(111) + 2	(38) – 1
9·3 – 9·8	(30) + 5	(19) – 7	(97) + 1	(68) + 6	(38) + 3
9·9 – 10·5	(76) + 1	(39) – 1	(179) + 2	(84) + 1	(48) + 9
10·6 – 11·2	(51) – 5	(20) – 17	(147) – 3	(33) + 1	(25) – 5
11·3 – 12·1	(17) – 9	(9) – 21	(49) – 7	(5) + 41	(9) + 9
12·2 –	...	...	...	...	(2) + 10

§ 13. *Conclusion as to Photographic Magnitude Equation.*—As the result of our operations, we have a system of stars derived from photographs made at nine different observatories; and the comparison of each series with the mean shows no relative magnitude equation.

Further, two additional series, not originally included, show no magnitude equation relative to the system.

It is extremely unlikely that these eleven series should be affected by magnitude equation of like sign and amount.

I conclude that our photographic standard system is sensibly free, not only from relative, but from absolute magnitude equation.

§ 14. *Search for Progressive or Uniform Discordances irrespective of Magnitude.*—With the fear of magnitude equation removed, we may examine the mean divergences in the date groups, to see if there is any evidence of progressive or general discordance. The tables which follow are self-explanatory.

Table V.

Comparison of Right Ascensions, in Date Groups.

Standard System minus—

Group	Bordeaux.	Catania.	Greenwich.	Helsingfors.	Northfield.
	s	s	s	s	s
I. Sept. 15–Oct. 23	(117) − ·004	(176) + ·001	(70) − ·002	(76) − ·003	(35) + ·001
II. Oct. 24–Nov. 24	(163) − 3	(290) + 2	(104) 0	(154) − 4	(57) − 1
III. Nov. 25–Dec. 15	(42) − 2	(100) + 9	(52) + 1	(145) − 3	(36) 0
IV. Dec. 16–Dec. 31	(73) − 1	(61) − 1	(71) − 3	(81) − 3	(54) + 1
Mean	(395) − 2	(627) + 3	(297) − 1	(456) − 3	(182) 0

	Paris.	Pulkowa.	San Fern.	Toulouse.	Upsala.	Minneapolis.
	s	s	s	s	s	s
I.	(243) + ·003	...	(243) + ·002	(216) + ·002	(39) − ·002	(42) − ·004
II.	(390) 0	(14) + ·004	(334) + 2	(295) + 9	(149) − 3	(69) − 4
III.	(129) 0	(32) + 4	(118) + 2	(156) + 2	(97) + 18	(33) − 2
IV.	(173) + 2	(15) − 4	(135) − 1	(194) + 2	(74) − 13	(42) − 4
Mean	(935) + 1	(61) + 2	(830) + 2	(861) + 4	(359) + 1	(186) − 4

Table VI.

Comparison of Declinations, in Date Groups.

Standard System minus—

Group	Bordeaux.	Catania.	Greenwich.	Helsingfors.	Northfield.
	"	"	"	"	"
I. Sept. 15–Oct. 23	(117) ·00	(176) − ·05	(70) + ·01	(74) + ·01	(35) + ·02
II. Oct. 24–Nov. 24	(163) − 2	(307) − 2	(104) + 1	(154) − 3	(57) + 1
III. Nov. 25–Dec. 15	(42) + 1	(100) + 1	(52) + 2	(145) − 1	(35) 0
IV. Dec. 16–Dec. 31	(73) + 3	(60) + 16	(71) 0	(81) + 1	(54) − 2
Mean	(395) 0	(643) 0	(297) + 1	(454) − 1	(181) 0

	Paris.	Pulkowa.	San Fern.	Toulouse.	Upsala.	Minneapolis.
	"	"	"	"	"	"
I.	(243) − ·02	...	(243) + ·03	(216) + ·01	(39) + ·09	(42) + ·08
II.	(390) + 1	(14) + ·01	(338) + 1	(294) − 1	(149) − 1	(69) − 2
III.	(129) − 2	(32) 0	(118) + 11	(155) − 6	(97) + 3	(32) + 6
IV.	(172) − 5	(15) − 3	(135) + 6	(194) + 2	(74) + 8	(42) + 4
Mean	(934) − 1	(61) 0	(834) + 4	(859) − 1	(359) + 3	(185) + 3

It is clear that no series has now any sensible uniform discordance from the mean. And the results from the separate date groups are nearly concordant, so that it seems out of the question that the standard system has been seriously distorted in any part by the inclusion of discordant material. The only marked cases of discordance are in Catania group III. and Toulouse group II. We shall have occasion to examine these in a later paper.

§ 15. *Effect of the application of Plate Corrections.*—We may look in the above table for the effect of the application of the plate corrections of § 6. The largest corrections were found for Helsingfors, Northfield, and San Fernando plates.

The Helsingfors plates received on the average corrections of $+^{s}\cdot014$, $-''\cdot01$ (derived from *repère* stars); the results, including all stars, are now discordant from the standard system by $-^{s}\cdot003$, $-''\cdot01$.

The Northfield plates received on the average corrections of $+^{s}\cdot007$, $''\cdot00$; the results are now discordant from the standard system by $^{s}\cdot000$, $''\cdot00$.

The San Fernando plates received on the average corrections of $+^{s}\cdot008$, $+''\cdot08$; they are now discordant from the standard system by $+^{s}\cdot002$, $+''\cdot04$.

In these cases the application of plate corrections has very much reduced the general discordance.

For Bordeaux and Toulouse the mean plate correction was small, and the mean discordance from the standard system remains small.

The discordances of the series to which no plate corrections were applied are also, fortunately, small. We may conclude that the application of the plate corrections has been successful in eliminating such mean discordances between the different series as were sensible.

§ 16. *Influence of Magnitude on the Accuracy of the Photographic Star Places.*

Many of the star images measured upon these plates are at the limit of measurability; and in many cases, by reason of thick skies or unduly short exposures, the images of the planet also are only just measurable. It is therefore very necessary to determine whether the accuracy of the photographic places is seriously impaired when the images are very faint.

To examine this point I have tabulated the average discordances (without regard to sign) of each observatory's series from the standard system. Since the mean discordances are very small, the average discordances will give a fair idea of the relative accuracy of the star places for different magnitudes and for different series.

Table VII.

Average Discordance of Right Ascensions, in Magnitude Groups.

Standard System minus—

Mag.	Bordeaux.		Catania.		Greenwich.		Helsingfors.		Northfield.	
		s		s		s		s		s
– 7·4	(14)	·014	(78)	·026	(32)	·011	(12)	·019	(24)	·021
7·5 – 8·4	(43)	12	(66)	16	(73)	9	(57)	18	(59)	13
8·5 – 9·2	(99)	11	(55)	22	(125)	7	(75)	17	(89)	14
9·3 – 9·8	(48)	9	(50)	17	(32)	8	(89)	16	(10)	25
9·9 – 10·5	(131)	13	(72)	23	(27)	7	(117)	23	...	
10·6 – 11·2	(53)	9	(118)	21	(5)	6	(99)	23	...	
11·3 – 12·1	(6)	15	(167)	23	(3)	8	(7)	27	...	
12·2 –	...		(21)	22	...		...		...	

Paris.		Pulkowa.		San Fernando.		Toulouse.		Upsala.		Minneapolis.	
			s		s		s		s		s
(47)	·012	(7)	·010	(54)	·015	(33)	·015	(15)	·030	(8)	·030
(70)	9	(22)	14	(103)	17	(124)	18	(43)	15	(17)	26
(71)	9	(27)	13	(233)	15	(229)	16	(111)	17	(38)	21
(60)	9	(5)	17	(89)	21	(99)	14	(68)	20	(38)	19
(103)	8	...		(146)	24	(179)	18	(84)	21	(49)	21
(124)	12	...		(145)	26	(147)	18	(33)	27	(25)	22
(193)	12	...		(52)	24	(49)	19	(5)	27	(9)	34
(267)	11	...		(8)	19	...		...		(2)	16

Table VIII.

Average Discordance of Declinations, in Magnitude Groups.

Standard System minus—

Mag.	Bordeaux.		Catania.		Greenwich.		Helsingfors.		Northfield.	
		"		"		"		"		"
– 7·4	(14)	·17	(78)	·27	(32)	·09	(12)	·19	(23)	·21
7·5 – 8·4	(43)	10	(67)	22	(73)	8	(57)	19	(59)	10
8·5 – 9·2	(99)	9	(57)	21	(125)	7	(73)	16	(89)	13
9·3 – 9·8	(48)	10	(50)	17	(32)	6	(89)	12	(10)	16
9·9 – 10·5	(131)	11	(74)	20	(27)	8	(117)	16	...	
10·6 – 11·2	(53)	11	(118)	18	(5)	3	(99)	20	...	
11·3 – 12·1	(6)	9	(177)	20	(3)	9	(7)	29	...	
12·2 –	...		(22)	27	...		...		...	

Paris.		Pulkowa.		San Fernando.		Toulouse.		Upsala.		Minneapolis.	
			"		"		"		"		"
(47)	·09	(7)	·07	(54)	·17	(33)	·21	(15)	·18	(8)	·33
(70)	10	(22)	9	(103)	14	(124)	16	(43)	13	(17)	19
(71)	10	(27)	8	(233)	15	(229)	16	(111)	13	(38)	20
(60)	10	(5)	17	(89)	17	(97)	14	(68)	17	(38)	17
(103)	9	...		(147)	22	(179)	18	(84)	19	(48)	21
(123)	10	...		(146)	27	(147)	19	(33)	21	(25)	22
(193)	12	...		(54)	24	(49)	22	(5)	45	(9)	22
(267)	12	...		(8)	17	...		...		(2)	22

In comparing these results we must remember that Greenwich and Paris have received double weight, and that they will naturally therefore diverge less from the mean than the others. We must also remember that the fainter stars have on the average fewer observations than those of moderate brightness; and no account has been taken of the number of observations for each star at each observatory in making the above comparisons. Nevertheless, the apparent accuracy of the fainter stars falls off very little, except in the case of Helsingfors and San Fernando. The latter series was measured in one orientation only, which may account for much. A more detailed discussion of the Helsingfors series will be given in a later paper, which may help to explain this result.

The general conclusion, that the accuracy of the very faint star places is little inferior to that of the brighter, is as welcome as it is unexpected.

§ 17. *Comparative and Absolute Accuracy of the results from different Observatories.*—Since it has been shown in the last paragraph that magnitude has small influence on accuracy, we may combine the material into date groups without inquiring closely whether the mean magnitude is the same in each group, and may thus discover whether the same accuracy was achieved at different dates, in different parts of the planet's path. And we may further examine whether the rough system of weights adopted fairly represents the relative merits of the results from different observatories.

Table IX.

Average Discordance of Right Ascensions, in Date Groups.

Standard System minus—

Group	Bordeaux.		Catania.		Greenwich.		Helsingfors.		Northfield.	
		s		s		s		s		s
I. Sept. 15–Oct. 23	(117)	·010	(176)	·020	(70)	·007	(76)	·025	(35)	·011
II. Oct. 24–Nov. 24	(163)	13	(290)	23	(104)	7	(154)	25	(57)	18
III. Nov. 25–Dec. 15	(42)	12	(100)	22	(52)	8	(145)	14	(36)	19
IV. Dec. 16–Dec. 31	(73)	10	(61)	18	(71)	10	(81)	16	(54)	12
Mean	(395)	11	(627)	22	(297)	8	(456)	20	(182)	15

	Paris.		Pulkowa.		San Fern.		Toulouse.		Upsala.		Minneapolis.	
		s		s		s		s		s		s
I.	(243)	·009	...		(243)	·019	(216)	·015	(39)	·014	(42)	·019
II.	(390)	11	(14)	·011	(334)	22	(295)	21	(149)	20	(69)	21
III.	(129)	13	(32)	15	(118)	21	(156)	16	(97)	25	(33)	32
IV.	(173)	10	(15)	12	(135)	16	(194)	13	(74)	18	(42)	19
Mean	(935)	11	(61)	13	(830)	20	(861)	17	(359)	20	(186)	22

Table X.

Average Discordance of Declinations, in Date Groups.

Group	Standard System minus— Bordeaux.		Catania.		Greenwich.		Helsingfors.		Northfield.	
I. Sept. 15-Oct. 23	(117)	″·10	(176)	″·23	(70)	″·07	(74)	″·18	(35)	″·11
II. Oct. 24–Nov. 24	(163)	12	(307)	·18	(104)	8	(154)	19	(57)	12
III. Nov. 25–Dec. 15	(42)	11	(100)	·21	(52)	6	(145)	13	(35)	14
IV. Dec. 16–Dec. 31	(73)	10	(60)	·27	(71)	8	(81)	18	(54)	14
Mean	(395)	11	(643)	·21	(297)	8	(454)	17	(181)	13

	Paris.		Pulkowa.		San Fern.		Toulouse.		Upsala.		Minneapolis.	
I.	(243)	″·10	...	″	(243)	″·17	(216)	″·16	(39)	″·19	(42)	″·20
II.	(390)	10	(14)	·06	(338)	20	(294)	18	(149)	16	(69)	18
III.	(129)	15	(32)	11	(118)	21	(155)	19	(97)	18	(32)	20
IV.	(172)	11	(15)	9	(135)	18	(194)	16	(74)	15	(42)	25
Mean	(934)	11	(61)	9	(834)	19	(859)	17	(359)	17	(185)	21

We may draw the following conclusions from these tables:—

The concordance of the observations is fairly uniform along the path of the planet, though there are considerable exceptions in the R.A.'s from Helsingfors, Minneapolis, Toulouse, and Upsala.

The Greenwich and Paris places were fully entitled to the double weight which they received.

The Bordeaux places might also have received double weight.

The Northfield and Pulkowa places might have been given some extra weight. But as they are chiefly well-observed *repère* stars, the effect upon the standard system would have been small in any case.

Judged by their average discordance, the Catania and San Fernando places are barely entitled to the unit weight which they received. Attention has already been called to very large and irregular discordances in the former series (*S.P.P.* No. 5, p. 84). The San Fernando plates have generally only a single exposure, measured in a single orientation, whereas other observatories usually have three or four exposures, measured in at least two orientations.

A second of time is equivalent to ten seconds of arc in declination 48°, which is close to the mean declination of the stars in our standard system. A comparison of the figures in the above table shows that the right ascensions and declinations are very closely of the same average accuracy.

§ 18. *Probable Error of the Standard System.*—To obtain from the above discussed discordances an exact estimate of the probable error of a star place in our standard system would be very laborious, if not impracticable, and the labour is not warranted in the present stage of the work.

We may, however, estimate the probable error roughly, as follows:—

The average discordance of the contributions to which unit weight has been given is well under 0″·20. A majority of these depend upon one or two observations, and have received weight 1. The probable error of a contribution of weight 1 is likely to be not greater than 0″·20, probably less. And a star is not admitted to the standard system unless its weight is at least 4, while the average weight is 7 or 8. The probable error of a standard place in each co-ordinate will therefore be well under 0″·10, but how much under it is impossible to say at present.

§ 19. *The use of the Standard System.*—The standard system provides fairly completely for the reduction of the Crossley Reflector plates, furnishing from 8 to 12 comparison stars along a good part of the planet's path. These star places have been already communicated to Professor Perrine. There are thin places in the catalogue, due to bad weather in Europe while it was fine at Mount Hamilton; these must be filled up if possible. By the kindness of the Astronomer Royal, a considerable part of the material required has been already supplied from special measures made on Greenwich plates, which are not included in the above discussion.

The catalogue also provides for the complete reduction of the Cambridge, Oxford, and Tashkend series, and any others that may be outstanding.

Further, I propose to try the effect of making the equivalent of a re-reduction of the published photographic places, in the following way:—

The programme of Monsieur Loewy included the measurement of all stars in a square of 20′, having the planet at its centre; these were reduced and published separately for each plate. The "*carré de vingt minutes*" generally contains six or eight of our standard system stars, sometimes as many as twenty, and we can compare the individual plate places with those in the standard system. If anything abnormal has happened to the plate, either in making or in reduction, the effect upon the mean of a group of stars scattered over the square would be very nearly the same as upon the planet at its centre. And if we apply to the planet's place a correction derived from the mean difference Standard System minus Stars of the Square, it seems that we ought to get rid of any troubles of a systematic nature in a very simple way.

Suppose, for example, that one telescope has an optical distortion that displaces the centre of the plate with respect to its edges; that another has an optical distortion that skews the plate and makes the scale value in x and y sensibly different, so that there is a large and systematic difference between a six constant and a four constant linear reduction; suppose that a third observatory has used a quite wrong expression in calculating the second order refraction corrections; and that in the work of a fourth the errors in the solution of the plate are undesirably common,—and these suppositions are not so extravagant as they may appear,—the simple

process of reduction to the standard system by means of the stars in the 20′ square will avoid the systematic effects of all these untoward circumstances, and will give a result equivalent to, though not so smooth as, a complete re-reduction.

The standard system will also be indispensable for the reduction of the micrometric comparisons of the planet. In Paris Circulars 8 and 9 Monsieur Loewy published a list of stars used in these comparisons, and desired that they should be measured on the photographs whenever possible. As a result of this, the standard catalogue contains good places of a very large majority of these stars, and provides almost completely for the reduction of most of the micrometer series. The only serious difficulty may be found in reducing the observations made at Washington and Yerkes Observatories, where they used some stars so faint that they are not found on the ordinary photographs. These stars were all marked on charts sent to Professor Perrine for use in selecting his comparison stars, and he has succeeded in finding and measuring a large part of them on the Crossley plates.

We have therefore reason to hope that this standard photographic system of stars may provide, directly or indirectly, for the complete reduction of the Eros observations of 1900 on a homogeneous system.

The extension of the system to the early months of 1901 is making excellent progress.

§ 20. As before, the additional assistance required in this work has been provided by a grant from the Government Grant Fund of the Royal Society.

Acknowledgments are also due to Miss Julia Bell, Girton College, who has continued to perform a great part of the computations; and to S. E. Bowd, who has made the card catalogue and assisted efficiently in all parts of the work.

Cambridge Observatory:
1907 *November* 19.

Note on the Position of the Sun's Axis of Rotation, as deduced from Greenwich Sun-spot Measures 1886–1901. *Papers of the I.U.S.R. Computing Bureau, No. I.* By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The present note is to be regarded rather as a statement of problems than as a solution of them. A discussion of the Greenwich sun-spot measures was begun a few years ago, as a necessary preliminary to computations in connection with the International Union for Solar Research; and it is being continued as the work of the Computing Bureau, of which the writer was asked to take charge. But the resources of this Bureau are at present very slender, and the time of the Director is much occupied with the printing of the Astrographic Catalogue, so that it may be some time before a definitive discussion is completed. Meanwhile the following provisional statement may be of interest; it has been drawn up in response to one or two inquiries for information. Another note will shortly be presented, dealing with the velocity of rotation.

2. The material used is taken from the Greenwich sun-spot ledgers, 1886–1901 inclusive. Later publications, especially the welcome volume of "photoheliographic results, 1874–1885," which has just been issued, will allow of a considerable extension of the discussion; but these were not available when the following computations were commenced. The discussion was confined to spot groups which were seen for a period of ten days at least; and no notice is taken in what follows of the identity of recurring groups. The longitude and latitude on each day were compared with the mean values given in the ledgers; and the differences were tabulated under each 10° of distance from central meridian, so as to show the motions in latitude and longitude. The spots were further grouped according to their latitudes, in groups 0° to 10°, 10° to 15°, 15° to 20°, 20° to 25°, and over 25°, N. and S. latitudes being kept separate.

3. When we have a set of mean residuals for each 10° of longitude, various methods may be proposed for deducing the mean drift from the series. Take, for instance, the following series of mean residuals in latitude for spots in N. latitudes 0° to 10° in the month of October (all years combined).

Dist. from Central Merid.	Mean Resid.		Dist. from Central Merid.	Mean Resid.	
−90° to −80°	+0°·50		+90° to +80°	—	
−80 to −70	+ ·28		+80 to +70	−0·38	
−70 to −60	+ ·73	+0°·40	+70 to +60	− ·32	−0°·38
−60 to −50	+ ·20		+60 to +50	− ·44	
−50 to −40	+ ·27		+50 to +40	− ·33	
−40 to −30	+ ·23	+ ·22	+40 to +30	− ·04	− ·24
−30 to −20	+ ·16		+30 to +20	− ·34	
−20 to −10	+ ·15	+ ·06	+20 to +10	− ·36	− ·24
−10 to 0	− ·04		+10 to 0	− ·12	

Differences of method would turn chiefly upon the relative weights to be assigned to the outer and inner residuals. The outer give better intervals for measuring the motion; but, on the other hand, they are affected by larger errors, owing to foreshortening, and possibly to optical distortion. We shall do no harm by making two separate determinations of drift from outer and inner groups and comparing the results. If we reject the outermost groups (80° to 90°) as too much foreshortened, and take the mean of the next three, we get results for $+65°$ and $-65°$, a difference of 130° in longitude. The next three groups will give us mean values for $+35°$ and $-35°$, a difference of 70°.

4. A drift in longitude may be taken to increase steadily with the time, and hence the differences for $\pm 65°$ should be to those for $\pm 35°$ in the ratio of 13 to 7. Thus, if the above figures had related to longitudes, the mean drift per 10° would have been $(-0°·38 - 0°·40)/13 = -0°·060$ from the outer groups and $(-0°·24 - 0°·22)/7 = -0°·066$ from the inner groups.

5. The same might be true of a drift in latitude if it were a physical drift; but if it is an apparent drift, due to a faulty determination of the Sun's axis, these formulæ will not apply. The error due to a wrongly assumed axis is of the form $k \sin(\theta + 65°) - k \sin(\theta - 65°) = 2k \cos\theta \sin 65°$ in one case, and (similarly) $2k \cos\theta \sin 35°$ in the other. The ratio is thus $\sin 65°/\sin 35° = 1·58$ instead of $65/35 = 1·86$.

6. To take the latitude first: if we have adopted a wrong axis, the effect will be a spurious latitude drift, varying in amount as we go round the Sun during the year. The drift will in fact be, roughly, similar to the Sun's drift in declination, if for a moment we regard the equator as an erroneous determination of the ecliptic: it will be northwards at one time of year and southwards six months later, with no drift midway. Hence, if we arrange the results according to the time of year, they will form a cycle. The results for latitude were accordingly collected in months, each spot being assigned to the calendar month in which it appeared on the central meridian; and the mean drifts between the meridians above specified are given in Tables I. and II., which require no further explanation.

TABLE I.

Latitude drift between $+65°$ *and* $-65°$ *in each month (of the years* 1886–1901).

Latitude.	$-25°$	$-20°$	$-15°$	$-10°$	$0°$	$+10°$	$+15°$	$+20°$	$+25°$	
Jan.	+78	−94	−77	−25	−23	−85	−87	−33	−32	−88
Feb.	+71	−67	−13	−7	−29	−7	+1	+30	−118	—
Mar.	−67	−15	−20	−39	−37	−43	−33	−76	+25	+186
Apr.	−28	—	+17	+14	−4	−19	−3	+22	+13	—
May	+15	+76	−54	+31	+77	−5	0	+2	−29	—
June	—	+5	−54	−17	+74	−20	−17	−2	+33	+120
July	+115	−18	+42	−75	+18	−19	−25	+93	−145	+270
Aug.	−43	−27	−12	−37	+6	+90	−41	−59	+16	−25
Sept.	+115	−29	−53	−13	−28	−33	−43	+13	+50	−12
Oct.	−113	−115	−7	−16	−3	−78	−75	−115	−188	−40
Nov.	−15	−159	−52	+33	−41	−59	−124	−116	−148	
Dec.	−82	−66	−57	−112	−13	−57	−119	−29	+70	+180
Mean	—	—	−28	−22	0	−28	−47	−22	—	—

The unit in the table is $0°{\cdot}01$, and the figures are differences between the mean residual at $+65°$ and that at $-65°$. The row of figures at the top represents solar latitude.

TABLE II.

Latitude drift between $+35°$ *and* $-35°$ *in each month (of the years* 1886–1901).

Latitude.	$-25°$	$-20°$	$-15°$	$-10°$	$0°$	$+10°$	$+15°$	$+20°$	$+25°$	
Jan.	+30	+5	+20	−18	−42	−39	−48	−25	+87	−5
Feb.	+32	−30	−8	−16	−6	−26	+12	+22	−65	+25
Mar.	−35	−3	−9	−26	−19	−23	−20	−28	+30	+54
Apr.	−22	+107	+3	+9	+20	−15	−4	−15	+3	—
May	+28	−21	−50	+5	+22	+34	+25	+45	−42	—
June	−85	−27	−25	−30	+58	+4	−6	+6	+49	+20
July	+25	+15	+38	−6	−5	−29	−18	+50	−101	+165
Aug.	+22	−25	−12	−57	−6	+30	−39	−25	+15	+35
Sept.	+118	−5	−40	−9	+28	−1	−10	−37	+33	+32
Oct.	+50	−30	−55	−5	+7	−46	−38	−19	−193	+10
Nov.	+55	−54	−42	+53	−33	−16	−61	−86	−85	...
Dec.	−98	+10	−25	−57	−48	−67	−32	−29	−43	+125
Mean	—	—	−17	−13	−2	−16	−20	−12	—	—

Multiplying the coefficients A and B by $\frac{1}{2}$ cosec $65°$ for Table I., and $\frac{1}{2}$ cosec $35°$ for Table II., we get the error of the Sun's axis,

which we may now express (as Carrington does) in minutes of arc. The separate determinations are given in Table III.

Excluding, at any rate in the first instance, the outside groups (for latitudes above 20°), we can now analyse the remaining 6 columns in each table harmonically, to find an expression of the form—

$$A \sin \theta + B \cos \theta + C,$$

where $\theta = 0$ for the middle of January.

TABLE III.

Quantities representing the Errors of the Sun's adopted axis.

Latitudes.	−20°	−15°	−10°	0°	+10°	+15°	+20°	Mean.
A from Table I.	+2·′9	+3·′7	+6·′6	+4·′2	+14·′5	+12·′1		+7·′3
A ,, ,, II.	+7·′1	−0·′8	+6·′5	+5·′7	+12·′1	+14·′5		+7·′5
B from Table I.	−5·′8	−1·′0	−11·′6	−12·′0	−9·′5	−9·′7		−8·′3
B ,, ,, II.	+0·′3	+1·′3	−16·′7	−13·′7	−4·′9	−10·′6		−7·′2

Inspection of Table III. suggests the following conclusions:—

(*a*) The determinations from meridians ±65° and ±35° are satisfactorily accordant. Thus the Sun's disc may be treated as free from distortion for the purpose in view.

(*b*) The adopted axis is sensibly in error for the period under discussion.

(*c*) The different zones give sensibly different values for the error, zones −20° to −15° and −15° to −10° being specially discordant. But the results seem really to fall into three groups:

	−20° to −10°	−10° to +10°	+10° to +20°
A	+3·′2	+5·′8	+13·′3
B	−1·′3	−13·′5	−8·′7

The conclusion (*c*) was not altogether unexpected, for an inspection of Carrington's material had indicated something of the kind. The material is given on pp. 240–242 of his volume *Observations of Solar Spots* (published in 1863 by Williams & Norgate); and if it is divided into four groups according to the latitude of the spots, we get approximately for his X and Y—

Latitudes.	X	Y
>+15°	−3·′8	+12·′9
+15° to 0°	−9·′4	+10·′2
0° to −15°	−4·′1	−15·′4
>−15°	−15·′0	+9·′0
Mean	−8·′1	+4·′2

It thus seems doubtful what precise meaning can be attached to the mean of such discordant values. On forming groups corresponding to those used in this paper there are still clear discordances; but the whole question must be reserved for further examination.

Nothing has yet been said of the constant C indicated by the mean values of the columns. It will be seen that it is persistently negative, the mean value from Table I. being $-0°{\cdot}25$, and from Table II. $-0°{\cdot}13$. These values are consistent, and indicate a steady drift of $-0°{\cdot}019$ or $-1'{\cdot}14$ for every 10° of advance across the disc. Carrington found a phenomenon of this kind, and thought its reality so improbable that he set it down to some error in reduction. His value was $1'{\cdot}5$ for 30° of rotation, or 18′ for a whole rotation; and his direction was *towards* the North Pole, *i.e.* in the direction opposite to that here found. It thus becomes important to examine whether this general drift shows any clear indication of change during our period (1886–1901). For this purpose the material must be rearranged in consecutive years, as in Table IV.

TABLE IV.

Drift in Latitude between +65° and −65° from central meridian.

Year.	−25°	−20°	−15°	−10°	0°	+10°	+15°	+20°	+25°
1886	—	—	-4_{9}	-13_{7}	$+8_{14}$	-14_{3}	$+27_{2}$	-57_{2}	—	—
1887	—	—	—	-70_{1}	$+29_{10}$	-4_{4}	-65_{3}	—	—	—
1888	—	—	—	-128_{2}	-102_{6}	$+110_{1}$	-145_{1}	—	—	—
1889	-2_{2}	$+30_{1}$	-60_{1}	—	-66_{5}	$+25_{1}$	—	—	—	—
1890	—	-68_{1}	—	—	-21_{2}	—	—	$+65_{1}$	-33_{1}	$+60_{1}$
1891	$+51_{2}$	$+98_{4}$	-27_{9}	$+35_{1}$	—	—	$+19_{6}$	-12_{17}	-41_{13}	$+42_{5}$
1892	$+2_{10}$	-114_{15}	-30_{13}	-26_{9}	$+57_{3}$	-35_{9}	-55_{26}	$+6_{10}$	$+5_{7}$	$+119_{5}$
1893	$+61_{4}$	-43_{11}	-47_{20}	-24_{26}	$+21_{23}$	-16_{10}	-19_{15}	-4_{17}	-83_{8}	-25_{1}
1894	$+47_{5}$	-25_{1}	-40_{16}	-50_{20}	$+3_{17}$	-11_{22}	-65_{18}	-7_{15}	$+4_{4}$	—
1895	—	$+63_{4}$	-49_{10}	-50_{18}	-31_{14}	-68_{5}	-48_{25}	-37_{14}	-26_{4}	—
1896	—	-42_{5}	-4_{10}	-16_{16}	$+16_{4}$	-95_{5}	-127_{3}	-122_{5}	-15_{1}	—
1897	—	—	-5_{2}	$+33_{6}$	-24_{19}	-77_{14}	-79_{5}	—	—	—
1898	—	—	—	$+3_{9}$	-26_{11}	-109_{6}	-100_{6}	—	—	—
1899	—	—	—	$+20_{7}$	$+40_{5}$	-23_{2}	—	—	—	—
1900	—	—	—	$+60_{1}$	-22_{6}	$+5_{2}$	—	—	—	—
1901	—	$+117_{1}$	—	—	—	$+17_{2}$	—	—	—	—

The unit in the table is $0°{\cdot}01$: the subscript figures represent the number of spots included in the mean.

The mean values of the drift, weighting all spots equally, are as follows:—

TABLE V.

Year.	Mean Drift.	No. Spots.	Year.	Mean Drift.	No. Spots.
1886	$-0°{\cdot}04$	37	1894	$-{\cdot}24$	118
1887	·00	18	1895	$-{\cdot}40$	94
1888	$-{\cdot}90$	10	1896	$-{\cdot}39$	49
1889	$-{\cdot}34$	10	1897	$-{\cdot}39$	44
1890	$-{\cdot}03$	6	1898	$-{\cdot}47$	32
1891	$-{\cdot}02$	57	1899	$+{\cdot}21$	14
1892	$-{\cdot}30$	107	1900	$-{\cdot}07$	9
1893	$-{\cdot}19$	135	1901	$+{\cdot}50$	3

There are some indications of change, but the small number of observations in the terminal years makes it difficult to pronounce with certainty. The reduction of the material recently made available should go far to settle the point. Meanwhile we may sum up the indications by taking means for four years as follows, Carrington's result being rendered comparable and added:—

Mean drift during a whole rotation.

1853–1861 Carrington.		1886–1889	1890–1893	1894–1897	1898–1901
+18′	***	−30′	−32′	−56′	−30′

[NOTE, added December 28.—A provisional reduction of the results for 1874–1885 shows that the drift is distinctly *positive* in those years, being smaller at the ends and large in the middle. An oscillation of about 26 years' period with maximum northerly drift about 1854, 1880, 1906, and maximum southerly about 1867, 1893, would roughly fit the facts observed hitherto.]

On the Mean Distances of the Groombridge Stars.

By A. S. Eddington, B.A. M.Sc.

In a former paper * I have given some calculations as to the mean distance of the stars of Groombridge's catalogue for the purpose of comparing the distances of the two star-drifts, with which that paper dealt. The results obtained were, I believe, sufficient to show that the two drifts must be at, roughly, the same mean distance from us, but there seemed no means of judging how closely the actual numerical results could be relied on. Some further reflection has, however, shown that the method has advantages (as regards freedom from systematic error) which were not at that time noticed; it also incidentally furnishes an additional test of the two-drift hypothesis. I have accordingly developed the results more precisely in this paper.

The principle of the method of determining the mean parallax of a large group of stars is well known. It depends on determining the apparent angular displacement corresponding to a known linear displacement. For example, in Professor Kapteyn's researches the parallactic motion has been mainly used as the known linear displacement; if that is assumed to be the same for all the different groups of stars discussed, the corresponding angular motion is proportional to the mean parallax of the group. In the present case the mean peculiar speed (*i.e.* the mean individual motion, irrespective of direction, which remains after all drift or parallactic motion has been abstracted) is taken for a similar purpose. If its linear amount is assumed to be the same in all parts of the heavens, the angular amount will be proportional to the mean parallax of the stars. We must, of course, be prepared for the possibility that neither the parallactic motion nor the mean peculiar speed is strictly constant in different parts of the sky or for different classes of stars. The determinations of "kV" (or the ratio of the drift motion to the mean peculiar speed) made in the previous paper throw some light on this question; they appear to favour the assumption of constant mean peculiar speed, but the test is at present a rather rough one.

As the proper motions of the stars include not only their peculiar motions but also the drift-motions, the mean peculiar motion cannot be found without mathematical investigation. The theory developed in the previous paper enabled this to be done. There are two stages in the calculation:—(1) The constants of the drifts must be determined from the numbers of stars moving in the different directions (without regard to the magnitudes of their proper motions); and (2) from these constants the theoretical relation between the mean peculiar motion and the whole mean motion can be found.

For example, consider that division of the sky which I have

* *Monthly Notices*, vol. lxvii. p. 34, referred to hereafter as "*Systematic Motions.*" The portion especially dealing with mean distances is pp. 55–57.

called Region B. In fig. 1, A is for this region the theoretical curve of the kind considered in the former paper, in which the radius vector is proportional to the *number* of stars moving in the corresponding direction; B is the theoretical curve in which the radius vector is proportional to the *mean proper motion* of the stars moving in the corresponding direction. The two curves are rather similar; the bi-lobed character of B is readily recognised, although the elongations and minima are much less pronounced than in A.

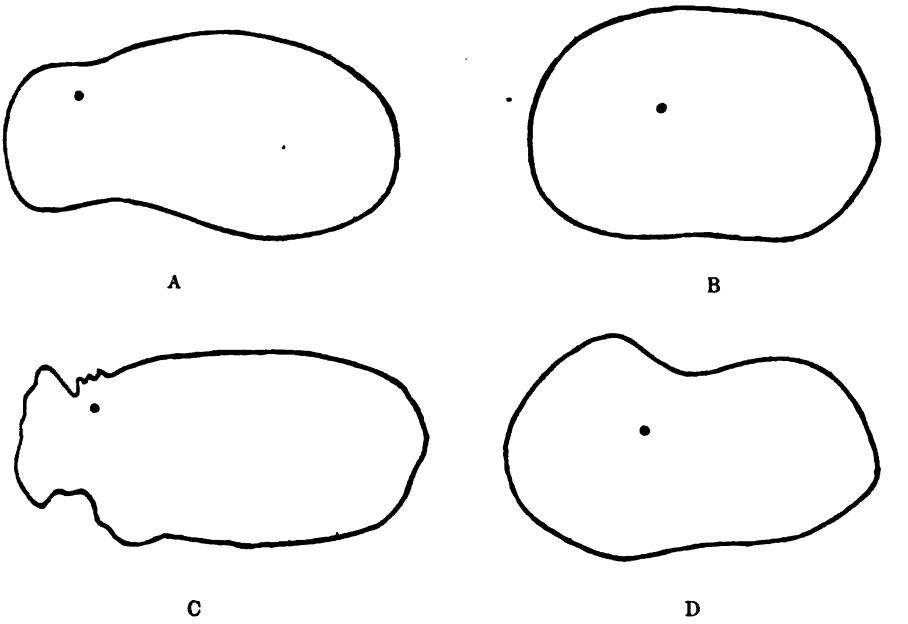


FIG. 1.—Diagrams for Region B.

A and C—theoretical and observed curves, showing total number of proper motions in different directions.

B and D—theoretical and observed curves, showing mean proper motion in different directions.

It should be mentioned that in drawing B the two drifts have been assumed to be at the same mean distance; as the two lobes of the curves correspond respectively to stars of the two drifts, it is clear that if one of the drifts is nearer than the other, this will be shown by the lobe of the curve B, which corresponds to that drift, being exaggerated in size compared with the other. It is in this way that the mean distances of the drifts can be compared. The curves C and D are those derived from observation, and are to be compared respectively with the theoretical curves A and B. Although our main purpose in this investigation is to discuss the distance of the stars of the two drifts, attention may be called to

the clear way in which, in this and in other regions, the mean proper motions support the two-drift hypothesis. The evidence of the observed curves C and D respectively in favour of that hypothesis must be regarded as, at least to a large extent, independent; it could hardly be expected, *a priori*, that the mean proper motions would be greatest in the two directions in which the total number of proper motions is greatest. It may be noticed that the two drifts are slightly more prominent in the observed curve D than in the theoretical curve B; some of the other regions considered show this rather more strongly, and it appears to be a general result. I have not been able to arrive at any satisfactory explanation of it.

For the other regions, reference must be made to Table I. In it will be found, for each of the seven regions covered by the Groombridge catalogue, the mean proper motion of stars moving in each of twelve directions at intervals of 30°. (For forming the mean proper motion in any direction, stars moving in directions within 15° on either side of it were used; thus each direction corresponds to a 30° sector.) Curves similar to D in fig. 1 could readily be constructed from these data, and we should find that they are clearly bilobed, indicating two drifts. Region D constitutes an exception; for, since it contains the apex of Drift II., that drift is not very apparent in it. The same thing may be seen by inspecting the numbers in the table: in each column there are two maxima and two minima well indicated, and a comparison with the Tables II. to VII. in "*Systematic Motions*" shows that they correspond with the maxima and minima in the numbers of stars moving in the different directions.

TABLE I.

Mean Proper Motions (rejecting highest eighth).

Direction θ. Regions A and B	Regions C D E F and G	Region. A	B	C	D	E	F	G
°	°	″	″	″	″	″	″	″
5	15	5·65	4·64	2·75	3·23	4·22	2·99	2·29
35	45	3·99	3·37	4·22	3·54	3·85	4·72	2·11
65	75	3·00	2·40	3·80	4·72	5·24	6·76	2·64
95	105	2·74	2·51	2·61	5·04	4·72	3·49	2·93
125	135	3·11	2·57	3·34	5·11	5·86	3·20	2·87
155	165	3·35	2·90	2·82	3·76	6·42	2·94	2·09
185	195	3·32	2·54	2·74	3·15	6·90	4·47	1·70
215	225	4·28	2·21	1·53	2·10	4·95	5·94	1·94
245	255	3·50	2·14	1·94	2·71	4·33	4·88	2·92
275	285	2·49	1·54	1·61	2·07	2·06	4·39	3·53
305	315	3·80	1·46	1·32	2·00	2·77	3·81	4·40
335	345	4·84	3·60	2·50	3·76	5·49	2·98	2·81

In calculating these means, it is necessary to adopt some rule for rejecting excessive proper motions. The difficulty is to find a limit which will affect the two drifts equally. The rule which

I have adopted, which seems to meet this, has been to reject the fastest eighth moving in each direction. That is to say, if there are n proper motions in one of the twelve 30° sectors, I pick out and reject the $n/8$ highest of these, and take the mean of the remainder. The net result must be nearly the same as if the nearest stars of each drift, to the number of rather more than one-eighth of the whole, were rejected. This means a much more drastic rejection of high proper motions than is usually resorted to; but some such plan seems necessary if the more distant stars are not to be altogether outweighted.

Each of the mean proper motions provides us with an equation involving linearly the unknown mean parallaxes of the two drifts (see "*Systematic Motions*," p. 56, equation (1)). Thus for each region twelve equations were formed, involving these two unknowns; these were properly weighted and solved by least squares. Table II. contains the results. "d_1" and "d_2" denote the distances of the drifts. For the meaning of "h" reference must be made to the mathematical analysis of the former paper; it is connected with the mean peculiar speed of the stars. It is, however, the relative rather than the absolute values of the distance that are of interest, so that the precise unit used is not of great importance; but, accepting Campbell's provisional value (20 k.m. per sec.) for the speed of the solar motion, and my own determinations of the drift-constants, the actual mean parallaxes may be obtained by dividing the numbers in Table II. by 500. (It must, however, be remembered that a great many near stars have been rejected, as explained above, so that the mean parallax has a rather artificial significance.)

TABLE II.

Mean Parallaxes.

(The quantities $\frac{1}{hd_1}$ and $\frac{1}{hd_2}$ tabulated are the mean parallaxes multiplied by a factor whose value is probably about 500.)

Region.	*N.P.D.*	R.A.	$\frac{1}{hd_1}$.	$\frac{1}{hd_2}$.
	° °	h h	" "	" "
A	0–20	0–24	2·96±·07	3·41±·13
B	20–52	22– 2	2·45±·07	2·40±·11
C	,,	2– 6	2·39±·08	2·65±·12
D	,,	6–10	3·35±·12	3·23±·21
E	,,	10–14	3·65±·14	4·78±·29
F	,,	14–18	3·74±·23	4·15±·31
G*	,,	18–22	2·77±·16	2·55±·18

The equality of distance of the two drifts is evidently confirmed. Region E is the only one in which there is any indication of a considerable difference between their distances. The probable errors are of interest, not only as indicating the reliability of the

* The restricted region G (excluding subdivision γ) was used; see *Systematic Motions*, p. 52.

results found, but as a measure of the accordance between theory and observation, for they are derived from the discordance of the calculated and observed mean proper motions in the twelve directions. Region B, which was represented in fig. 1, shows the best agreement between theory and observation; but it may be remarked that, where the agreement is not so good, that is always due to the phenomenon mentioned above—that the mean proper motions show the effect of the two drift-motions more prominently than calculation leads us to expect.

Before we can rely on the values of the probable error given in Table II., it is necessary to consider whether a possible systematic error in the catalogue, from which the proper motions are derived, may not lead to errors of importance not included in the theoretical probable error. The conclusion is rather unexpected; a systematic error in the proper motions does not invalidate these results. The reason is this: a systematic error affects the mean proper motion, and also the total number of proper motions, in a particular direction; owing to the latter error, we arrive at incorrect values of the constants of the drifts; but it is not difficult to see that, if these incorrect values of the drift-constants are used in forming the equations of condition, this will compensate for using incorrect values of the mean proper motions. Looked at in another way, the analysis was designed to extricate the mean peculiar motion from the systematic drift-motions with which it is involved; it at the same time extricates it from any possible systematic error which is hardly distinguishable in its effect from systematic drift or parallactic motion. In order that this elimination of error may take place, it is necessary to use drift-constants derived from the observed proper motions in the particular region considered, not those determined from an examination of the whole sky; this has been attended to.

Assuming that the two drifts are actually at the same mean distance, and accordingly combining their parallaxes given in Table II. with weights inversely proportional to the squares of their probable errors, we have the following table of mean parallaxes (multiplied by a factor) of the Groombridge stars in different parts of the sky.

TABLE III.

Variation of the Mean Parallax with Galactic Latitude.

Region.	Mean Galactic Latitude.	Mean Parallax (multiplied by a factor).
	°	″
B	−12	2·43
C	− 2	2·47
G	+ 8	2·67
A	+27	3·06
D	+32	3·32
F	+46	3·88
E	+66	3·87

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The mean parallax steadily increases with the distance from the galaxy, a result which is in accordance with the generally accepted ideas of the distribution of stars, viz. that the increased number of stars in the low galactic latitudes is due to additional more distant stars being visible, and not to any crowding among the nearer stars. The table apparently indicates a *gradual* change in the mean distance; but, as the regions are rather extensive in area, it is not definitely incompatible with the hypothesis that the differences are caused by a belt of distant stars almost limited to the galaxy.

On Ancient Eclipses. By P. H. Cowell, M.A., F.R.S.

I am much obliged to Mr. Crommelin for repeating my calculations in the case of six solar eclipses, and so setting the question of their accuracy beyond doubt.

The fact that emerges from the discussion of the eclipses is that there is an unexpected rate of change in what may be termed the "nodical year," or period of revolution of the Sun relatively to the Moon's node.

This abnormal rate of change may arise from secular changes in the motion of the Moon's node (other than that arising from the change of the eccentricity of the Earth's orbit round the Sun), or from secular changes in the motion of the Sun, or, of course, from a combination of both hypotheses.

The motion of the node, as found from observation, differs very slightly from the formula given for it by Professor Brown in *M.N.*, lxiv. p. 532. This formula is differentiated and the secular acceleration of the node obtained on the supposition that the eccentricity and inclination of the Moon's orbit are constant. We know no reason why these quantities should vary, and observation shows that if they do vary, the variations do not exceed certain limits. It happens, however, that these limits are sufficiently large to admit of a considerable part of the unexpected change in the nodical year being attributed to the motion of the node (at least as an alternative hypothesis), thus diminishing the part to be ascribed to the Sun.

The formula $+4''(T-18{\cdot}25)^2-1''{\cdot}0$ takes numerical values $+1''{\cdot}0$ in 1755 and 1895, and $-1''{\cdot}0$ in 1825, with lesser values numerically at intermediate dates. A second of arc is not an impossibly large quantity to attribute to systematic errors, or even to unknown long-period terms, and it is clear from Professor Newcomb's table (*Ast. Const.*, p. 22) that residuals of over 1″ cannot be avoided by any formulæ.

Hence it must be understood that a possible change in the eccentricity and inclination of the Moon's orbit is put forward as

an alternative suggestion. There are no modern observations capable of discriminating between various hypotheses, and, conversely, it is unnecessary to discriminate before approaching questions of chronology. All that is necessary in that case is to verify that certain empirical formulæ fit the records in a way that cannot reasonably be considered accidental.

It is a curious reflection that Oppolzer's formulæ should have been put aside merely because they contain impossible mean motions. Because they are based on so much coincidence and because Oppolzer's numerical accuracy is beyond reproach, there must be some solid foundation beneath the superficial blemishes of impossible mean motions. If we discard Oppolzer's position of the lunar perigee as not sensibly affecting the general agreement with the eclipse records, and if, wherever a mean motion needs correction, we apply to Oppolzer's formulæ some multiple of $20T + T^2$, a quantity that vanishes twenty centuries ago, we get to results closely resembling mine; and I attribute the small differences to the fact that Oppolzer nowhere claims to have satisfied but merely to have improved the solar eclipses, and, I believe, he obtained his formulæ from the lunar eclipses.

Let the supposed variation of the principal elliptic term in the Moon's longitude be $+2''\cdot2x$ in a century (i.e., x parts in 10,000); and let the supposed variation of the principal term in the Moon's latitude be $+1''\cdot8y$ in a century (i.e., y parts in 10,000), and let z'' be the (sidereal) secular acceleration of the Sun's longitude, then

$$+6''\cdot2x - 2''\cdot6y + z'' = 3''\cdot6 \pm 0''\cdot5 \quad \ldots\ldots \text{(i)},$$

the right-hand side being determined by the eclipses, and modern observations indicating that x is less than 0·2 and y less than 0·3. In this way the solar eclipses are compatible with a secular acceleration of $2''$ for the Sun.

If the underlying cause be tidal friction, there is nothing improbable in the suggestion that slow changes are caused in the eccentricity and inclination of the Moon's orbit.

The present tables would expose on an average about one-tenth part of the Sun's diameter, and these 200″ are reduced to less than 50″ by a single empirical term in a large number of cases. The suggestion that this is mere chance appears to me untenable. When the observations of the eighteenth century present a difficulty of one second, what is one second against two hundred? The one second may be evidence that a particular hypothesis requires modification, but that is all. I believe equation (i) exhausts every geometrical possibility.

The Perturbations of Halley's Comet in the Past.—First Paper. The period 1301 *to* 1531.

By P. H. Cowell, M.A., F.R.S., and A. C. D. Crommelin, B.A.

We commence this paper by expressing our great indebtedness to Dr. Smart, F.R.A.S.; to Mr. F. R. Cripps, of 22 Hornsey Rise Gardens, N.; and to Mr. Thomas Wright, of 39 Cringle Road, Levenshulme, Manchester; who are really entitled to be considered as joint authors, since they have carried out by far the larger portion of the mechanical quadratures, the results of which are given below. Without their co-operation the completion of the calculations would have been indefinitely delayed; and they are again offering their help in carrying them back to a still more distant date.

The calculation of the perturbations of this comet has already been carried back to 1531 by de Pontécoulant; the identity of the two apparitions before that (those of 1456 and 1378) is universally admitted; before that date Dr. Hind has given (*M.N.*, x. p. 51) a list of conjectured identifications, some fairly certain, like those of 451, 760, 1145, others admittedly vague and uncertain. It occurred to us that new light might be thrown on the question by carrying back the calculation of the perturbations as far as possible, and seeing whether a sufficiently accurate correspondence existed between the conjectured and calculated dates. It is fairly evident that, to bring the labour of computing a large number of revolutions of the comet within reasonable limits, some abbreviations must be introduced, and the following was the plan adopted.

(1) For the purpose of computing the perturbations, the comet in each revolution was assumed to move in an ellipse of constant eccentricity (experience having shown that the perturbations in eccentricity are always small), and with a major axis corresponding to the observed period.

(2) The perihelion and node were supposed to change uniformly from revolution to revolution, the rates of change being deduced from the accepted elements of the comet from 1531 to 1910; subsequently these rates were modified by the use of M. Laugier's discussion (*C.R.*, vol. xxiii.) of the apparitions of 451 and 760, for which fairly definite statements of position are found in the Chinese annals. M. Laugier has been able to represent these observations exactly by elements differing but little from those at the present time. Curiously enough, he does not print his elements, but he gives the longitudes and latitudes calculated from them, and the elements must be very approximately as follows—

(the old system of comet elements is here employed, as more convenient for our purpose):—

T=451 July 3·5		T=760 June 11	
$\varpi = 284°$		$\varpi = 289°$	
$\Omega = 32·5$	Eq^{x}. 451	$\Omega = 36·5$	Eq^{x}. 760
$i = 16$		$i = 17$	
$q = 0·60$		$q = 0·60$	

The following are the collected values of ϖ, Ω, i, reduced to the Equinox of 1910:—

	ϖ	Ω	i
451	304°·8	53°·3	16°
760	305·0	52·5	17
1378	306·9	54·67	17·9
1456	305·26	50·08	17·62
1531	306·47	50·77	17·00
1607	305·41	52·66	17·14
1682	305·09	54·35	17·76
1759	305·27	55·92	17·62
1835	305·55	56·19	17·76
1910	303·64	57·18	17·78
Assumed change per revolution of the comet,	−0·06	+0·15	

The secular variation of i has been neglected altogether in the period now under discussion, as it appears to be too small to deduce with certainty, considering the roughness of the elements for all the apparitions before 1682.

(3) The co-ordinates of the planets referred to the cometary axes are assumed to repeat themselves at uniform intervals, but the interval for x', y' is taken slightly different from that for z', to allow for the adopted motions of the perihelion and the node.

	Period for x', y'. d	Period for z'. d
Venus	224·7005	224·7023
Earth	365·2544	365·2614
Jupiter	4332·47	4332·86
Saturn	10758·6	10760·7
Uranus	30681·3	30700·6
Neptune	60167	60238

In consequence of the change in the period and major axis of the comet's orbit, it is more convenient to use the ordinary

astronomical unit of length and the mean solar day as the unit of time. In our previous papers on the perturbations by Jupiter and Saturn 1835–1910 (*M.N.*, lxvii. pp. 386, 511) every quantity tabulated was of zero dimensions in space and time.

(4) In the case of Jupiter and Saturn, the method of mechanical quadratures is used for values of u (the comet's eccentric anomaly) from 0° to 90°, and from 270° to 360°. For the outer half of the orbit the definite integral method described in the above paper (p. 391) is employed; since the eccentricity of the comet's orbit is treated as constant, all the values on p. 409 can be used unchanged except those relating to the position and motion of the perturbing planet; it may be noted that $\frac{x'}{a}, \frac{y'}{a}\frac{1}{na}\frac{dx'}{dt}, \frac{1}{na}\frac{dy'}{dt}$ vary as $\frac{1}{a}$.

The portion of the perturbations in the outer half of the orbit that we have denoted by V_4 (p. 391) is neglected altogether; this may make an error of a few days in the calculated period, but not sufficient to cause any doubt as to the identity of an observed comet.

The intervals of u for which the calculations are made are $4\frac{1}{2}°$ for Jupiter and Saturn, 9° for Uranus, 18° for Neptune (the calculations for these two planets being made throughout the whole orbit), 1° for the Earth and Venus (the calculations for these two planets being made from the values of u 0° to 30°, 330° to 360°).

(5) We have omitted the discussion of the perturbation of e, Ω, i, having merely found the quantities that come directly into the periodic time.

The following are the assumed co-ordinates of Venus and Earth, referred to the axes of the comet's orbit; they are deduced by a simple transformation from those given by Stratford in the Supplement to the *Nautical Almanac* for 1839, and are in units of the 3rd decimal:—

Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391492	−548	−452	+122	+969	−053	−296
94	569	419	134	966	087	299
96	589	386	146	961	121	302
98	606	352	158	955	154	304
1500	624	317	169	948	187	305
02	638	279	179	941	220	306
04	650	242	189	932	253	307
06	660	204	199	922	286	308
08	668	165	208	910	318	308
10	674	125	216	898	349	308
12	678	085	224	885	380	308
14	679	045	231	871	411	307
2391516	−679	−004	+237	+856	−442	−306

Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391518	−676	+036	+242	+840	−472	−304
20	671	075	247	822	501	302
22	663	115	250	804	530	300
24	654	155	253	785	558	298
26	643	194	255	765	586	295
28	631	233	257	744	613	292
30	615	270	257	723	639	288
32	597	307	256	700	664	284
34	578	343	255	677	689	280
36	557	378	253	653	713	276
38	534	412	250	628	736	271
40	509	444	247	602	758	266
42	483	475	242	576	779	261
44	456	504	237	550	799	255
46	427	532	232	522	819	249
48	397	558	226	493	838	244
50	365	582	218	464	855	236
52	331	604	209	435	871	229
54	297	624	201	405	886	222
56	263	643	192	375	901	214
58	227	660	182	344	914	207
60	191	674	172	312	927	199
62	154	686	161	281	938	191
64	116	696	149	249	948	182
66	078	704	137	217	957	173
68	040	711	124	185	965	164
70	−002	714	111	152	972	155
72	+037	715	098	119	977	146
74	075	714	085	086	981	137
76	113	712	071	052	983	127
78	150	706	057	+019	986	117
80	187	698	042	−014	987	107
82	223	688	028	047	987	097
84	259	676	+015	080	985	087
86	294	662	000	114	982	077
88	329	646	−014	147	978	067
90	362	628	029	179	972	057
92	394	608	044	211	966	046
94	424	586	058	243	958	036
2391596	+453	+562	−072	−275	−949	−025

Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391598	+482	+537	−086	−306	−939	−015
1600	509	510	099	337	928	−004
02	534	481	112	369	916	+007
04	557	451	124	400	903	017
06	578	419	137	430	888	028
08	598	386	149	459	872	038
10	616	352	161	488	855	049
12	632	317	172	515	838	059
14	646	281	183	543	819	070
16	658	244	193	570	799	080
18	668	207	202	596	778	090
20	677	169	211	621	756	100
22	683	130	219	646	734	110
24	687	090	226	670	711	120
26	688	050	233	693	687	130
28	688	+010	239	715	662	140
30	686	−030	244	736	636	149
32	682	069	249	757	609	158
34	675	108	253	777	581	167
36	666	147	256	796	552	176
38	655	186	258	813	523	184
40	643	224	260	829	493	192
42	628	262	260	844	463	200
44	611	299	260	858	432	208
46	593	335	259	871	401	216
48	573	369	257	884	369	223
50	551	403	254	895	337	230
52	527	435	250	905	304	237
54	502	466	246	914	271	243
56	475	495	242	922	237	249
58	446	523	237	929	203	255
60	416	550	231	934	169	260
62	385	574	224	938	135	265
64	353	597	216	941	100	270
66	319	618	208	943	065	275
68	285	638	199	944	−030	279
70	250	655	189	943	+004	283
72	214	670	179	942	039	286
74	177	683	168	940	074	289
2391676	+140	−694	−157	−936	+108	+292

Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391678	+103	−703	−145	−931	+142	+295
80	065	709	133	925	176	297
82	+027	713	120	917	210	298
84	−011	715	107	909	244	299
86	049	715	094	899	278	300
88	087	713	080	888	311	300
90	125	709	066	876	344	300
92	162	702	052	863	376	300
94	199	693	038	849	408	300
96	236	682	023	835	439	299
98	271	669	−009	819	470	298
1700	305	653	+006	802	500	296
02	339	635	020	784	530	294
04	372	616	034	765	559	292
06	403	595	048	745	588	289
08	433	572	062	724	615	286
10	462	547	076	702	642	283
12	489	520	090	680	668	279
14	514	492	103	657	693	275
16	538	462	116	633	717	270
18	−561	−431	+129	608	741	265
20	...	...	...	582	764	260
22	.	...	...	556	786	255
24	...	...	...	529	806	249
26	...	...	...	502	826	243
28	...	...	...	474	845	237
30	...	...	...	445	863	230
32	...	...	...	416	880	223
34	...	...	...	386	896	216
36	..	...	...	356	911	209
38	...	...	...	326	924	201
40	...	...	...	295	936	193
42	...	...	...	264	948	185
44	...	...	...	232	959	177
46	...	...	...	200	968	169
48	...	...	...	168	976	160
50	...	...	...	136	983	151
52	...	...	...	103	989	142
54	...	...	...	071	994	133
2391756	...	...	...	−038	+997	+124

Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391758	...	...	...	−005	+1000	+114
60	...	...	...	+028	1001	104
62	...	...	...	061	1001	094
64	...	...	...	094	1000	084
66	...	...	...	127	998	074
68	...	...	...	159	994	064
70	...	...	...	191	989	054
72	...	...	...	223	983	044
74	...	...	...	255	977	034
76	...	...	...	287	969	023
78	...	...	...	318	960	013
80	...	...	...	348	949	+003
82	...	...	...	379	938	−008
84	...	...	...	409	926	018
86	...	...	...	438	913	029
88	...	...	...	467	899	039
90	...	...	...	496	883	049
92	...	...	...	524	866	059
94	...	...	...	551	848	069
96	...	...	...	577	829	079
98	...	...	...	603	810	089
1800	...	...	...	629	790	099
02	...	...	...	654	769	109
04	...	...	...	678	747	119
06	...	...	...	701	724	128
08	...	...	...	723	700	137
10	...	...	...	745	676	146
12	...	...	...	766	651	155
14	...	...	...	786	625	164
16	...	...	...	804	598	173
18	...	...	...	821	571	181
20	...	...	...	838	543	189
22	...	...	...	854	515	197
24	...	...	...	870	485	205
26	...	...	...	885	455	213
28	...	...	...	898	425	220
30	...	...	...	910	394	227
32	...	...	...	921	363	234
34	...	...	...	931	332	240
2391836	...	...	...	+940	+300	−246

Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391838	...	...	...	+948	+268	−252
40	...	...	...	955	235	258
42	...	...	...	961	202	264
44	...	...	...	966	169	269
46	...	...	...	969	136	274
48	...	...	...	972	102	279
50	...	...	...	973	069	283
52	...	...	...	974	035	287
54	...	...	...	973	+002	290
56	...	...	...	971	−032	293
58	...	...	...	968	066	296
60	...	...	...	964	099	299
2391862	...	...	...	+958	−132	−302

The co-ordinates of the comet itself are found from the table (*M.N.*, lxvii. p. 387) by multiplying $\frac{x}{a}, \frac{y}{a}$ by a, using the special value for each revolution, and the number of days from perihelion to each value of u by multiplying nt by $\frac{\text{period in days}}{6{\cdot}28318}$.

The co-ordinates of Jupiter are found from the table (*M.N.*, lxvii. p. 394) by multiplying $\frac{x'}{a}, \frac{y'}{a}, \frac{z'}{a}$ by 17·7. The following table gives the Julian days corresponding to certain values of g':—

g'	Julian Day.	g'	Julian Day.
°		°	
0	2390645	225	2393353
45	1186	270	3894
90	1728	315	4436
135	2269	360	4977
180	2811		

The values for intermediate degrees may be found with sufficient accuracy by reckoning 12^d per degree from the nearest table value. The co-ordinates of Saturn are found from the similar table, referred to (but not printed in detail), *M.N.*, lxvii. p. 512. As the formulæ on which the table is based are given there, the table is not printed here.

As the perturbations by Uranus and Neptune are always small, approximate co-ordinates suffice; those of Uranus are taken from de Pontécoulant (*Mémoire sur le calcul des perturbations de la comète de Halley*, 1829, pp. 37, 38, 48). They are as follows, in units of the first decimal:—

Julian Day.	x'	y'	z'	Julian Day.	x'	y'	z'
2358000	+187	+26	−55	2369400	−90	−171	+07
300	188	15	57	700	100	165	10
600	189	+04	58	2370000	109	158	14
900	189	−08	59	300	117	151	18
2359200	188	20	60	600	125	143	21
500	187	31	61	900	132	135	25
800	185	42	62	2371200	140	126	28
2360100	182	53	62	500	147	116	31
400	179	64	63	800	153	106	34
700	175	74	63	2372100	158	96	37
2361000	171	84	63	400	162	86	40
300	166	94	62	700	166	75	42
600	160	104	62	2373000	170	64	45
900	154	113	61	300	173	52	47
2362200	147	122	60	600	175	40	49
500	140	131	59	900	175	29	51
800	132	139	58	2374200	176	17	53
2363100	124	147	56	500	176	−05	54
400	116	155	54	800	175	+08	55
700	107	161	53	2375100	174	20	56
2364000	98	167	51	400	172	32	57
300	89	173	49	700	169	44	57
600	79	178	46	2376000	165	56	58
900	69	183	43	300	160	67	58
2365200	59	187	41	600	154	78	57
500	48	190	38	900	148	89	57
800	37	193	35	2377200	141	99	56
2366100	26	195	32	500	133	108	54
400	15	196	28	800	125	118	53
700	+04	196	25	2378100	118	127	52
2367000	−07	196	22	400	109	136	51
300	18	195	18	700	100	144	49
600	29	194	14	2379000	91	151	47
900	40	192	11	300	81	157	45
2368200	50	189	07	600	71	163	42
500	60	186	−04	900	60	169	39
800	70	182	00	2380200	49	173	36
2369100	−80	−177	+03	2380500	−37	+176	+33

Julian Day.	x'	y'	z'	Julian Day.	x'	y'	z'
2380800	−25	+179	+30	2385000	+124	+142	−22
2381100	14	181	27	300	133	135	25
400	−02	183	24	600	141	127	29
700	+09	184	20	900	148	119	32
2382000	21	184	17	2386200	155	110	35
300	32	183	13	500	161	100	38
600	43	181	09	800	167	90	41
900	55	178	05	2387100	172	80	43
2383200	66	175	+01	400	176	70	46
500	77	172	−03	700	180	59	49
800	87	167	07	2388000	183	48	51
2384100	97	162	11	300	185	37	53
400	106	156	14	600	187	26	55
2384700	+115	+149	−18	2388900	+189	+15	−57

The perturbations produced by Neptune are still smaller than those of Uranus, and, the eccentricity of its orbit being small, it has been considered to move in a circle. The following are its co-ordinates at intervals of 6° of longitude, or 1003 days, in units of the first decimal :—

Julian Day.	x'	y'	z'	Julian Day.	x'	y'	z'
2411217	−134	−269	00	2427265	−240	+155	+94
2220	160	254	+10	8268	222	180	92
3223	184	237	20	9271	202	204	90
4226	206	217	29	30274	179	226	86
5229	226	194	38	1277	154	245	82
6232	244	169	47	2280	128	261	76
7235	259	143	56	3283	100	275	70
8238	271	115	63	4286	071	285	63
9241	280	85	70	2435289	42	293	56
20244	286	55	76	6292	− 12	297	47
1247	288	− 24	82	7295	+ 18	298	38
2250	288	+ 07	86	8298	48	295	29
3253	284	38	90	9301	78	290	20
4256	278	68	92	2440304	107	281	+10
5259	268	98	94	1307	134	269	00
2426262	−256	+127	+95	2442310	+160	+254	−10

The co-ordinates recur with reversed signs after 30090 days.

The dates of perihelion passage in 1378, 1456 were already known with accuracy from the observations; for that of 1301 Hind's value was taken as a working hypothesis, to be tested by the result; the Julian days of the various passages are given here for convenience of reference:—

Perihelion Passage.		Julian Day.	Interval.	Corresponding log a.
			d	
1301	Oct. 22·7	2196543·70	28141·07	1·2578
1378	Nov. 8·77	2224684·77	28336·44	1·2598
1456	June 8·21	2253021·21	27471·59	1·2508
1531	Aug. 25·8	2280492·80	27811·07	1·2544
1607	Oct. 26·87	2308303·87	27351·94	1·2496
1682	Sept. 14·81	2335655·81	27936·76	1·2557
1759	Mar. 12·57	2363592·57	28006·37	1·2564
1835	Nov. 15·94	2391598·94	27199·36	1·2480
1910	May 6·3	2418798·30		

The last value is uncertain by several days; the value given was that employed in our Jupiter and Saturn work.

The following are the assumed co-ordinates of Jupiter and Saturn at the times given:—

		Jupiter.		Saturn.	
	u	x'	y'	x'	y'
1301	0°	−4·03	−3·07	−7·07	+5·25
	90	+4·88	+0·28	+5·37	+8·43
	270	−0·48	−4·95	+9·05	+3·63
1378	0	+3·46	+3·97	+3·49	−8·36
	90	−5·02	+0·58	−8·33	−2·49
	270	+1·19	+5·25	−7·44	+4·65
1456	0	−4·58	−2·09	+5·36	+8·44
	90	+4·79	−0·42	+8·51	−3·53
	270	+2·70	−3·92	+6·29	−6·55
1531	0	+0·28	+5·37	−6·60	−5·92

The method pursued has been identical with that used by de Pontécoulant in the memoir quoted above.

$x' - x$, $y' - y$ were found for each value of u; then $(x'-x)^2+(y'-y)^2+z'^2=\rho^2$, where ρ is the distance between the planet and comet.

Then X, Y, the perturbing forces parallel to the axes, are proportional to

$$\frac{x'-x}{\rho^3}-\frac{x'}{r'^3},\quad \frac{y'-y}{\rho^3}-\frac{y'}{r'^3}.$$

dn, the alteration in the mean motion, is equal to X sin u − Y cos $u\sqrt{1-e^2}$ multiplied by a reducing factor, which is not introduced till after the summation; each value of dn is multiplied by the number of days to elapse before the next perihelion passage.

The following quantities are then formed for each value of u:— $xY - yX$, rX, $r(xX + yY)$. In taking the sums of the columns, only half of the term at the top and bottom of each column is taken; the reason is obvious from the geometry of mechanical quadratures.

The factor for reducing $\int dn$ to seconds of arc is $\frac{6\pi}{365{\cdot}256}\, a^{\frac{1}{2}} m' du$.

The same factor multiplies the sum of $dn \times$ No. of days to next perihelion passage, and gives a quantity which we call A.

The factor to multiply

$$\int (xY - yX) \sin u - \sqrt{1-e^2}\int rX \text{ is } \frac{m' a\, du}{e};$$

the result is $\int d\varpi$.

The factor to multiply $\int r(xX + yY)$ is $2\, m' du$: this gives a quantity which we call C.

Then
$$\int d\zeta = A - \int d\varpi \sqrt{1-e^2} - C.$$

And period in days is given by the expression

$$\frac{1296000 - \int d\zeta}{n \text{ at beginning of revolution}}.$$

du is the selected interval of u expressed in seconds of arc; thus for interval—

1° $du = 3600$, for $4\frac{1}{2}$° $du = 16200$, for 9° $du = 32400$, for 18° $du = 64800$
log = 3·5563 log = 4·2095 log = 4·5105 log = 4·8116

The following are the assumed values of log m' for the different planets:—

♀ 4·3893, ⊕ + ☽ 4·4781, ♃ 6·9799, ♄ 6·4558, ♅ 5·6408, ♆ 5·7122; also

$$\sqrt{1-e^2} = {\cdot}2540, \text{ i.e. } e = {\cdot}9672, \quad \log \frac{1}{e} = 0{\cdot}0145,$$

$$\log \frac{6\pi}{365{\cdot}256} = 8{\cdot}7127.$$

The following tables exhibit the calculated quantities:—

Revolution 1456 *to* 1531.

Planet.	Limits of u.	$\int dn$.	A.	$\int d\varpi$.	C.	$\int d\zeta$.
	° °	"	"	"	"	"
Venus	0– 30	+ ·007	+ 200·9	− 0·4	− 1·1	+ 202·1
,,	330–360	+ ·040	+ 1·3	+ 1·9	·0	+ 0·8
Earth	0– 30	− ·006	− 164·2	− 0·8	+ ·7	− 164·7
,,	330–360	+ ·007	+ 0·6	− 0·4	+ ·8	− 0·1
Jupiter	0– 90	+1·070	+29020·	+653·9	− 591·4	+29445·3
,,	90–270	+ ·209	...	−553·4	...	+ 2241·
,,	270–360	− ·943	− 1105·	− 0·6	−1290·8	+ 186·0
Saturn	0– 90	− ·043	− 1096·	+ 49·0	+ 76·9	− 1185·4
,,	90–270	+ ·050	...	−128·9	...	+ 790·2
,,	270–360	− ·018	− 36·6	− 9·4	− 42·6	+ 8·4
Uranus	0–360	·000	+ 0·1	− 4·0	+ 14·8	− 13·7
Neptune	0–360	+ ·002	+ 28·9	+ 6·0	+ 25·4	+ 2·0
Sums	...	+ ·375	...	+ 12·9	...	+31513·

Hence for this revolution we have the equation

$$\frac{1296000'' - 31513''}{n \text{ at } 1456} = 27471{\cdot}59,$$ the observed period in days.

Hence n at $1456 = 46''{\cdot}029$

. . . . . $1531 = 46{\cdot}029 + {}''0{\cdot}375 = 46''{\cdot}404$.

The last result is in good accord with de Pontécoulant, who gives (*C.R.*, lviii. p. 708) $46''{\cdot}410$ for the value of n in 1531; as he has computed the perturbations by Jupiter and Saturn only for the revolutions 1531 to 1607, 1607 to 1682, his value is obviously not to be relied on beyond the first decimal.

Revolution 1378 *to* 1456.

Planet.	Limits of u.	$\int dn$.	A.	$\int d\varpi$.	C.	$\int d\zeta$.
	° °	"	"	"	"	"
Venus	0– 30	+·0147	+ 418·7	− 1·4	− 1·1	+ 420·2
,,	330–360	− ·0019	− 0·9	− 1·5	− ·4	− 0·1
Earth	0– 30	+ ·0148	+ 417·3	− 1·2	− 1·4	+ 419·0
,,	330–360	+ ·0003	− 0·9	− 2·2	− ·3	0·0
Jupiter	0– 90	− ·1320	− 4065·5	− 611·0	− 187·3	− 3722·
,,	90–270	− ·182	...	+ 16·4	...	+ 2991·
,,	270–360	+ ·6725	+ 636·5	− 380·7	+ 841·1	− 108·3
Saturn	0– 90	+ ·0593	+ 1553·2	− 95·3	− 112·1	+ 1689·5
,,	90–270	− ·011	...	+ 72·8	...	+ 1586·
,,	270–360	− ·0240	− 74·5	− 85·6	− 53·4	+ 0·8
Uranus	0–360	− ·0013	+ 17·1	+ 1·3	− 1·6	+ 19·0
Neptune	0–360	− ·0026	+ 11·2	− 10·5	− 153·0	+ 167·2
Sums	...	+ ·4068	...	− 1101·5	...	+ 3452·3

Hence for this revolution we have the equation—

$$\frac{1296000'' - 3452''\cdot3}{n \text{ at } 1378} = 28336\cdot44,$$ the observed period in days.

Hence n at $1378 = 45''\cdot614$

. . . . . $1456 = 45''\cdot614 + ''0\cdot407 = 46''\cdot021$.

There are now two independent determinations of n at 1456, which are satisfactorily near each other, considering the many abbreviations that we have introduced; we are justified in adopting their mean, viz. $46''\cdot025$. The corresponding values for 1378, 1531 are $45''\cdot618$, $46''\cdot400$ respectively; the close accord of the latter value with that of de Pontécoulant is satisfactory.

Revolution 1301 *to* 1378.

Planet.	Limits of v.	$\int dn$.	A.	$\int d\varpi$.	C.	$\int d\zeta$.
	° °	″	″	″	″	″
Venus	0– 30	+ ″0023	...	− 2″0	...	+ 63″4
,,	330–360	+ ·0020	...	− 1·0	...	0·0
Earth	0– 30	+ ·0121	...	− 1·7	...	+ 357·9
,,	330–360	+ ·0074	...	+ 2·4	...	+ 0·1
Jupiter	0– 90	+1″115	+30905″	+611″2	− 721″2	+31471″
,,	90–270	+ ·1983	...	−263·6	...	+ 1303″
,,	270–360	− ·5980	− 1230″	−418·8	−1303·3	+ 180″
Saturn	0– 90	+ ·0295	+ 776·4	− 14·6	− 59·8	+ 839·9
,,	90–270	− ·040	...	−106·4	...	− 606·1
,,	270–360	+ ·0367	+ 80·9	+ 51·6	+ 76·7	− 8·8
Uranus	0–360	− ·0017	− 7·7	+ 1·0	− 6·9	− 1·1
Neptune	0–360	− ·0001	+ 7″	+ 8·6	+ 28·3	− 23″
Sums	...	+ ·763	...	−133·4	...	+33576·3

We now proceed as follows: n at $1301 = 45''\cdot618 - 0''\cdot763 = 44''\cdot855$.

Then theoretical period in days $= \frac{1296000'' - 33576''\cdot3}{44''\cdot855} = 28144^{d}\cdot5$.

This only differs $3^{d}\cdot5$ from the value obtained by using Hind's identification of the comet of 1301, viz. $28141^{d}\cdot07$. The dis-

cordance is so small that we are justified in accepting Hind's result with absolute confidence. The long discussion as to the identity of the comet of 1301 is thus happily ended. Dr. Galle, in his *Cometenbahnen*, p. 155, says of the comet of 1378, "Die erste als sicher zu betrachtende Erscheinung des Halley'schen Cometen." We may now claim to have brought the "sicher" returns one revolution further back, and to have thus obtained a firm starting-point for further investigations, which have, in fact, already been commenced.

It will be remembered that Pingré, using the European observations alone, obtained elements quite unlike those of Halley; subsequently Laugier, using a combination of European and Chinese observations, obtained elements resembling those of Halley, except the position of the node, which was 90° greater. Finally, Hind rejected the European observations entirely, and showed that the Chinese ones could be well represented by the Halley elements. Our result indicates that he was justified in this course; it is rather a curious reversal of the present relations of European and Chinese astronomy. Many of Dr. Hind's older identifications rest on Chinese observations, and it is satisfactory to find that the accuracy of the latter in 1378 and 1301 is fully vindicated.

Having once satisfied ourselves of the identity of the comet of 1301, we may rewrite the last equation in the form—

$$\frac{1296000'' - 33576''\cdot3}{n \text{ at } 1301} = 28141\cdot07, \text{ the observed period in days.}$$

Hence n at $1301 = 44''\cdot861$

. . . . . $1378 = 44''\cdot861 + 0''\cdot763 = 45''\cdot624$.

Taking the mean of this and the previously adopted value for 1378, viz. $45''\cdot618$, we obtain $44''\cdot858$, $45''\cdot620$ for the adjusted values in 1301, 1378. It is advisable to make these successive adjustments in order to diminish cumulative error through quantities that we have neglected.

Observations of Comet d 1907, *from photographs taken with the 30-in. Reflector of the Thompson Equatorial and the Astrographic 13-in. Refractor at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

The following positions of Comet *d* 1907 were obtained from photographs taken with the 30-in. reflector and the astrographic 13-in. refractor, with exposures of from 20 seconds to 3 minutes.

The plates were measured in the astrographic micrometer. Six reference stars were, as a rule, measured with the comet, their positions being derived from the Catalogues of the Astronomische Gesellschaft.

The positions given are not corrected for Parallax.

Log Parallax Correction = log Parallax Factor − log Δ.

Date and G.M.T. 1907.				Apparent R.A.			Apparent Dec.			Log Parallax Factor R.A.	Log Parallax Factor Dec.
				30-in. Reflector.							
d	h	m	s	h	m	s	°	′	″		
June 19	14	3	1	0	16	32·60	+ 1	14	10·8	−9·525	+0·833
July 4	14	15	1	1	14	23·48	5	49	54·7	9·516	0·817
4	14	18	47	1	14	24·23	5	49	57·5	9·512	0·817
16	14	11	38	2	22	8·84	10	28	11·1	9·540	0·805
24	14	14	19	3	22	19·51	13	42	0·7	9·562	0·802
30	14	47	11	4	15	15·97	15	44	19·6	9·564	0·792
31	14	34	8	4	24	26·04	16	0	37·8	9·573	0·799
Aug. 1	14	59	5	4	33	56·40	16	16	0·5	9·565	0·790
6	14	57	39	5	21	17·16	17	8	57·3	9·579	0·800
10	15	12	37	5	58	53·15	17	22	59·5	9·581	0·803
11	15	18	16	6	8	5·99	17	22	42·7	9·581	0·802
12	15	18	36	6	17	10·80	17	20	58·7	9·582	0·805
15	15	32	32	6	43	45·84	17	7	56·9	9·581	0·805
18	15	37	17	7	9	6·29	+16	44	44·8	−9·582	+0·811
				Astrographic 13-in. Refractor.							
20	15	46	44	7	25	20·77	+16	24	19·4	−9·581	+0·811

Royal Observatory, Greenwich:
1907 *December* 12.

Note on Photographs of Phœbe, taken at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

Photographs of Phœbe have been obtained with the 30-in. reflector on 16 nights, viz.—

	m		m
1907 Aug. 10	Exp. 105	1907 Oct. 2	Exp. 90
11	133	4	120
18	143	8	53
19	120	12	90
29	93	30	100
Sept. 2	53	Dec. 3	120
9	107	5	120
10	70	6	90

The photograph on October 30 was at or very near elongation, and will thus be of special value for a determination of the mass of Saturn, for which this satellite is peculiarly useful, its distance at elongation being about 28′—three times that of Iapetus.

Royal Observatory, Greenwich:
1907 *December* 13.

Occultations of Uranus by the Moon in 1908, *visible at British Observatories.* By A. M. W. Downing, D.Sc., F.R.S.

It is hoped that the publication of the following particulars will enable astronomers who are favourably situated to observe some at least of these interesting phenomena, noting any peculiarities of appearance of the planet at the time of occultation that may strike them.

Day. 1908.	Place.	Standard Mean Time of Place.		Angle from North Point.	
		Immersion.	Emersion.	Immersion.	Emersion.
		h m	h m	°	°
April 22	Adelaide	10 53	11 46	106	266
,,	Melbourne	11 25	12 20	106	266
,,	Sydney	11 20	12 18	88	284
,,	Wellington	13 10	14 18	81	288
May 19	Natal...	8 44	9 29	64	308
July 13	Perth, W.A. ...	5 41	6 35	99	275
,,	Adelaide	7 19	8 19	78	296
,,	Melbourne	7 57	8 59	79	293
,,	Sydney	8 3	8 57	58	315
,,	Wellington	10 11	11 19	62	299
Aug. 9	Cape of Good Hope...	10 4	10 55	37	321
Oct. 3	Wellington	12 20	13 7	58	287

1907 *December* 6.

The Transit of Mercury, 1907 *November* 14.

(Communicated by R. T. A. Innes.)

At Johannesburg, at ingress, thin clouds obscured the Sun and made observation difficult; at egress the Sun was cloudless except for a few moments just before external contact.

At Pretoria the Surveyor-General organised a party, but clouds prevented any useful observations.

The observations made here are tabulated below. The individual observers' reports are as follows:—

Mr. R. T. A. Innes, using a 9-in. refractor by Grubb, with a sun-diagonal eyepiece:—

When I looked at the Sun at $15^h\ 46^m\ 4^s$ sid. (clock) T. Mercury was already about $\frac{1}{8}$ on. At the time of internal contact rather thick clouds were passing, but the planet was never invisible. Internal contact took place, as nearly as I could estimate, at $15^h\ 47^m\ 48^s$, but it was possibly a few seconds later. Heavy clouds passed for a few seconds, making confused definition. At $15^h\ 48^m\ 1^s$ the sky cleared, but contact was passed and the planet clear of the limb by $\frac{1}{8}$ part or so. Before egress the planet was carefully scrutinised; it was a mere circular black dot, without any shading; definition was perfect, the Sun's limb being without a trace of serration. Second internal contact was at $19^h\ 8^m\ 56^s$ and was good; last contact at $19^h\ 11^m\ 36^s$ was fair, as some thin clouds were passing.

Mr. W. M. Worssell, M.B.A.A., took part in the observations with his 3-in. refractor and a direct-vision eyepiece:—

Ingress: definition was very bad, a decided line was seen at $15^h\ 47^m\ 59^s$ sid. T. Egress: suspected black drop at $19^h\ 8^m\ 48^s$, certain black drop at $19^h\ 8^m\ 51^s$; contact passed at $19^h\ 9^m\ 0^s$, external contact at $19^h\ 11^m\ 27^s$. Definition medium.

Mr. J. Innes, 3-in. refractor, direct-vision:—

Dark glass split during observations, observations uncertain. Internal contact at egress $3^h\ 46^m\ 41{\cdot}5^s$ stand. clock T., egress $3^h\ 49^m\ 1{\cdot}5^s$. One-quarter weight has been given to these observations.

Mr. H. E. Wood, M.Sc. $2\frac{1}{8}$-in. refractor (Rheinfelder & Hertel), direct-vision:—

Good definition, image small; internal contact at egress good, $19^h\ 8^m\ 56^s$ sid. T., last contact at $19^h\ 11^m\ 25^s$ more uncertain.

For the timing arrangements, electrical sounders, connected with the sidereal clock, were placed near the observers. The clock had

been rated by star observations with a transit instrument for five weeks preceding the 14th November, and had a steady rate of 0·28 seconds per day losing. Its correction at 16 hours S. T. on November 13 was +0·56 seconds. Cloudy weather prevented the observation of time stars on the 14th and 15th, but from the clock rate the corrections at the times of observations of the transit on the 14th are +0·84 seconds at ingress, +0·88 seconds at egress. The mean time clock was rated by comparisons with the sidereal clock, and its corrections are −1·3 seconds at ingress, −1·2 seconds at egress. When the corrections are applied to the above times, reduction to Greenwich mean time made, and the following reductions to the centre of the Earth applied, ingress internal contact +3·4ˢ, egress internal contact +30·9ˢ, external contact +31·2ˢ, we have:—

Observer.	Telescope.	Phase.	Geocentric Greenwich Mean Time. h m s	
R. T. A. Innes ...	9-in.	II.	22 26 4·8	
W. M. Worssell ...	3-in.	II.	22 26 15·8	
R. T. A. Innes ...	9-in.	III.	1 47 7·5	
*W. M. Worssell ...	3-in.	III.	1 47 7·0	
J. Innes	3-in.	III.	1 47 11·1	½ weight
H. E. Wood ...	2½-in.	III.	1 47 7·5	
R. T. A. Innes ...	9-in.	IV.	1 49 47·3	
W. M. Worssell ...	3-in.	IV.	1 49 38·4	
J. Innes	3-in.	IV.	1 49 31·5	½ weight
H. E. Wood ...	2½-in.	IV.	1 49 36·4	

Results of Johannesburg observations:—

	Obs.	Nov. 1907. d	h	m	s	N.A. s	O−C. s
Internal contact at ingress	2	13	22	26	10·3	18	−7·7
,, ,, ,, egress	4	14	1	47	7·6	17	−9·4
External ,, ,, ,,	4	14	1	49	40·0	58	−18·0

The direct-vision observers using the 3-in. telescope had six sun-glasses either blistered or cracked during their observations.

* Mean of "certain black drop" and "contact passed."

Johannesburg Observatory:
1907 *November* 15.

Observations of the Transit of Mercury, 1907 November 14
By E. T. Whitelow.

Made at Birkdale, Lancashire.

Long. W. 3° 1′ 37″.
Lat. N. 53° 26′ 48″.

Neither ingress nor egress was observed, owing to clouds.

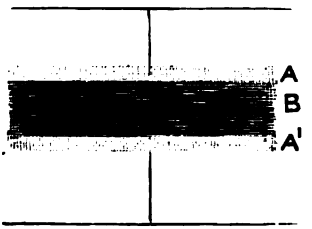
A. *Direct Observations* were made with 7½-in. refractor, stopped down to 4-in. A Thorp's polarising eyepiece: power 150. At 12h. 15m. G.M.T., Mercury showed a clean cut, round disc, with no indication of halo or bright spots. The duration of observation was half a minute or so of bright sunshine, and intermittent observations of a few seconds at a time when the Sun was slightly obscured by cloud, perhaps 2 minutes in all.

At 2h. 20m. there was a sudden burst of bright sunshine in a patch of clear blue sky. I put the edge of the Sun just outside the field of the eyepiece, uncrossed the prisms of the eyepiece so as to get the maximum light, and found Mercury in the field. It had much the appearance of Venus when in inferior conjunction, viz. a very thin curved line of light extending about 120° round the planet. There was no trace of this on the side away from the Sun, nor could I see *any* trace of the dark portion of Mercury contrasted against the sky.

B. *Spectroscopic Observation.* 12h. 48m. G.M.T.

Instrument 3½-in. refractor, Brashear grating, 15,500 lines per inch; 1st order. Power 120 and 180.

The planet was observed on the Sun's disc only, with the slit parallel to the direction of diurnal rotation. Along the centre of



the spectrum was seen a shaded band, consisting of a dark central band B, bordered by slightly lighter margins A, A^1. Each was about ¼ the width of the central darker band when Mercury was bisected by the slit. To make sure these were not due to dust lines, I pulled the clock slow-motion cord to and fro, and found these margins travelled with Mercury along the slit. I also slightly increased the opening of the slit, to clear it of any possible effect from dust lines. When I moved the telescope up or down in

declination, the central dark band B became relatively narrower than the margins A, A^1. Fraunhofer lines in A, A^1 exactly corresponded with those on the Sun's surface outside these margins. They were, however, very faint, or hazy, and difficult to focus, although lines on the Sun's surface were perfectly defined. I could not say they were wide, and the difficulty of focussing seemed to be due to lack of sufficient contrast as seen on the margins A, A^1.

The total duration of the Spectroscopic observations was about $1\frac{1}{2}$ minutes of brilliant sunshine and about 2 minutes of haze.

NOTE—*November* 18, 1907.—The foregoing observations having been submitted to Father Cortie, he suggested that the failure to see a lighter ring when Mercury was *on* the Sun's disc while making the observation A might be due to the polarised field of the Thorp eyepiece being then too dark to permit it to be seen.—E. T. W.

Observation du Passage de Mercure sur le Soleil le 14 Nov. 1907.
Par Robert Jonckheere, Observatoire de Roubaix.

La situation de l'observatoire est approximativement à 50° 40′ de latitude et à 3^m 19^s de longitude Est du méridien de Paris.

L'instrument utilisé pour les observations est une lunette équatoriale de 220 mm. diaphragmée à 150 mm. La distance focale est de $2^m{\cdot}50$. Le pied est faible et les trépidations rendent difficiles les mesures micrométriques. Le micromètre est bifilaire. Toutes les mesures ont été prises à l'aide d'un grossissement de 150, mais pour les observations visuelles des grossissements divers, variant de 70 à 600, ont été employés.

L'atmosphere pendant toute la durée du passage de la planète avait le zénith complètement dégagé de nuages mais l'horizon était brumeux. Le soleil étant à une hauteur de 30° en moyenne durant l'observation il a été par instants masqué de nuages, et c'est la raison pour laquelle le premier contact intérieur seul a pu être observé.

L'état atmosphèrique n'a pas permis de chercher à constater la présence de la planète avant son entré sur le disque du soleil.

Le 1^{er} contact extérieur n'a pu être observé et la planète n'a été aperçue que lorsque la moitié de son disque découpait le bord solaire.

Aucune déformation des cornes n'a été constatée bien que l'image se présentait à ce moment avec une grande netteté.

Aucun ligament ni pont n'a été remarqué au 1^{er} contact intérieur.

La planète était beaucoup plus noire que les taches solaires qui, en comparaison, paraissaient bleues. La corde tracée par Mercure était trop boréale pour occulter aucune de ces taches.

Aucun point obscur pouvant être considéré comme satellite n'a été observé.

Determination du Diamètre et de l'Aplatissement.

Afin de réduire le plus possible les erreurs personnelles, les mesures micromètriques ont été prises en placant l'œil horizontalement puis verticalement. Le micromètre n'a pas été fixé à un angle déterminé, il a été constamment maintenu à l'une des deux positions verticale ou horizontale.

Mesure du Diamètre Horizontal.

L'œil placé horizontalement.

	Vis 1.		Vis 2.		Mesures Différentielles.	
De 11 à 12 heures	8″·3	51″·0	8″·8	50″·4	16″·5	24″·3
	8·3	50·9	9·2	51·1	59·6	50·8
	8·2	50·5	8·0	51·2	3·9	11·0
Moyennes	8·27	9·20	8·67	9·10	7·90	

Moyenne générale 8″·63.

L'œil placé verticalement.

De 12½ à 1 heures	8″·2	50″·6	8″·7	51″·3	42″·3	31″·7
	8·0	50·4	8·8	50·8	32·4	42·2
	8·3	51·0	8·4	51·9	7·4	15·2
Moyennes	8·17	9·33	8·63	8·67	9·40	

Moyenne générale 8″·84.

Moyenne pour les deux séries de mesures 8″·73.

Mesure du Diamètre Vertical.

L'œil placé horizontalement.

	Vis 1.		Vis 2.		Mesures Différentielles.	
De 12h à 12½h	9″·8	49″·0	9″·6	51″·0	33″·5	23″·5
	9·3	50·2	9·9	51·0	31·4	21·0
	9·8	50·5	9·4	51·6	34·7	44·6
Moyennes	9·63	10·10	9·63	8·80	10·10	

Moyenne générale 9″·65.

L'œil placé verticalement.

De 1h à 1h 25m	9″·2	50″·0	9″·0	50″·6	11″·9	21″·6
	9·0	50·8	9·3	51·0	55·1	3·5
	9·5	50 2	9·1	50·7	30·7	40·0
Moyennes	9·23	9·67	9·13	9·23	9·13	

Moyenne générale 9″·28.

Moyenne pour les deux series 9″·46.

Pour l'ensemble de toutes les mesures 9″·10.

L'existence des anneaux n'a pas été constatée, mais un léger défaut de mise au point donne une image faisant croire à une apparence d'atmosphère et ressemblant entièrement aux dessins de Schroeter de 1799. La nuance de ce halo est alors identique à celle de la pénombre des taches solaires.

J'ai encore remarqué que les facules et granulations du soleil semblaient plus distinctes aux environs de la planéte : probablement l'effet du contraste produit par le disque noir.

Aucune apparence lumineuse n'a été aperçue. Le disque s'est toujours présenté uniformément noir.

Les derniers contacts allaient se produire lorsque le disque solaire disparut derrière les nuages : une éclaircie se produisit et il fut possible d'observer la planète alors qu'elle échancrait le bord solaire de la moitié de son diamètre.

P.S.—5 minutes et 6 secondes après le premier contact intérieur, un petit point noir a passé trés rapidement en face du soleil, de l'Ouest à l'Est a environ 20° de déclinaison boréale : vraisemblablement une étoile filante.

NOTES.—Le phénomène dans sa marche régulière n'a donné lieu à aucune observation autorisant l'hypothèse de l'existence d'une atmosphère, même trés faible.

L'examen des mesures micrométriques montre une différence entre les diamètres vertical et horizontal de la planète.

La moyenne pour le diamètre horizontal est de 8″·73 et pour le diamètre vertical 9″·46. Cette différence de 0″·73 est surprenante. Il ne peut étre question d'une erreur personnelle ou instrumentale.

La valeur verticale 9″·46 est celle qui approche le plus du diamètre généralement adopté et cela se conçoit car les phases de Mercure ne permettent habituellement que les mesures de ce diamètre.

Il me semble également intéressant de faire remarquer que pour la première série des mesures du diamètre horizontal, la moyenne des résultats est plus faible que pour la seconde : alors que pour les mesures du diamètre vertical, c'est l'inverse qui se produit. Les mesures effectuées avec l'œil dans la position horizontale ayant été prises en premier lieu, le mouvement diurne peut expliquer cette différence étant donné le déplacement horaire de la planète dans l'intervalle des premières aux dernières mesures.

Dans le cours de mes observations visuelles, j'avais remarqué une légère déformation du disque dont l'axe était à 20° à gauche de la verticale, mais j'avais prié mon aide de ne pas noter cette particularité qui me paraissait invraisemblable. Cependant, après avoir pris la moyenne des mesures, je m'aperçus que ce n'était pas une illusion puisque les chiffres confirmaient l'apparence que j'avais constatée.

D'après ces observations l'on serait donc conduit à devoir accepter que la planète Mercure a une déformation trés sensible, et, bien qu'elle paraisse trop grande pour ne pas avoir été aperçue jusqu'ici, on peut admettre qu'elle ait échappé aux observateurs,

car ils ont bien rarement l'occasion de prendre les mesures du disque entier de cette planète.

Cette déformation pourrait servir à justifier l'hypothèse d'un mouvement de rotation dont l'axe coinciderait avec l'orbite de la planète, mais il serait peut être plus simple d'admettre qu'elle n'est pas le résultat d'un mouvement de rotation rapide, mais de l'attraction du soleil. Il faudrait alors reconnaitre avec M. Schiaperelli que cette planète n'effectue qu'une seule rotation durant sa révolution autour du soleil.

Roubaix :
le 19 *Novembre* 1907.

LUNAR NOMENCLATURE.

1. At a meeting of the International Association of Academies in Vienna in May 1907, a Committee was appointed to consider and report upon Lunar Nomenclature, with a view to securing consistency and uniformity. The Committee consisted originally of Loewy (Chairman), Franz, Newcomb, Saunder, Turner, Weiss. The names of W. H. Pickering and Puiseux have since been added, and it was hoped that an eminent German astronomer might have also joined the Committee, but he felt unable to do so. A heavy blow has fallen on the Committee in the sudden death of its Chairman, M. Loewy.

By request of the Chairman, the undersigned undertook the duties of Secretary to the Committee; and he has accordingly received several suggestions from individual members. From these it becomes clear that it will be necessary to take a preliminary decision of the first importance:—viz. shall an essentially new system of nomenclature be propounded, or do astronomers prefer to retain simply the traditional system, with necessary corrections and possible modifications?

To propose a new system requires some courage and may only increase confusion. On the other hand, no more favourable moment than the present is likely to occur for considering such a proposal; for if the Committee succeeds in correcting and improving the present system, it will naturally be rendered more tenacious of life. Finally, the proposal for a new system need not mean the abolition of the old. In the case of minor planets we have already a double system of nomenclature—names and numbers; and there may be advantages in having a dual system also for the Moon.

But such a new departure calls for very careful consideration; and it would be of great help to the Committee to know whether it were likely to meet with approval. The circumstances suggest that if the Royal Astronomical Society will publish in the *Monthly Notices* the sketch-proposals themselves, so that brief comments upon them may be invited, the Committee might be greatly

assisted in deciding whether to put forward some such scheme definitely or not.

Accordingly, the present document is submitted to them for publication, containing the proposals of Dr. Franz, and the counter-proposals of M. Puiseux (to which our late Chairman had given a general approval) and of Professor W. H. Pickering. The Committee will welcome any comments made upon them by astronomers interested; and would specially appreciate replies to the following questions:—

(1) Do you approve the proposal of some mechanical or numerical system of nomenclature *in place of* the old?

(2) Do you approve the proposal of some such system *in addition to* the old?

(3) If such a system is to be adopted, which of the three proposed below seems, on the whole, preferable?

(4) Are your comments and replies intended solely for the Committee, or may they be published if thought desirable?

H. H. Turner,
Secretary to the Committee on Lunar Nomenclature.

University Observatory, Oxford:
1907 *November* 20.

Dr. Franz's Proposals.

Breslau, le 13 Août 1907.

Monsieur,—Invité à présenter des propositions pour la nomenclature systématique de la lune, je pense qu'il n'est pas trop tard pour réformer entièrement les dénominations. Car la mesure d'un grand nombre d'objets n'a été faite que depuis 1900. M. Saunder a publié les siennes pour une grande partie sans nom, et les mesures de Breslau, qui sont aussi de plus de mille, ne sont pas encore publiées et portent dans les manuscrits des dénominations provisoires. La commission ne peut se passer de publier une carte de la lune avec une nomenclature systématique, et alors dans les années et siècles à venir on aura rarement besoin de recourir aux descriptions de Mädler, qui ne se vendent plus chez les libraires, ou à celles de Neison, qui en sont un résumé.

Je m'occupe aujourd'hui seulement des cratères principaux comme Mösting, et je laisse de côté les cratères secondaires comme Mösting *A*. Les noms des astronomes, que Riccioli a introduits, n'ont aucune relation avec les cratères, et, à de très rares exceptions, jamais un astronome n'a vu le cratère qui porte son nom.

Pour avoir une dénomination systématique il faut:

(1) que les noms accusent directement la position du cratère.

(2) que les noms soient courts, afin que leurs inscriptions sur les cartes ne couvrent et ne cachent pas trop de détails.

(3) que les noms soient sonores et faciles à prononcer.

Voici, ce que je propose :

On désigne les *longitudes* (dans l'image inverse, que la lunette montre, de gauche à droite, comme on écrit) de 10 à 10 degrés par les *consonnes* : longitude plus de

				par	b	ch	d	f	g	k
+10°	+0°	−0°	−10°	−20°	−30°	−40°	−50°	−60°	−70°*	
l	m	n	p	r	s	t	w	x	z	

On désigne les *latitudes* (de haut en bas, comme on écrit) par des voyelles. Latitude plus de

−70°	−60°	−50°	−40°	par	Aa	Ae	Ai	Ao
−30°	−20°	−10°	−0°		Ea	Ee	Ei	Eo
+0°	+10°	+20°	+30°		Ia	Ie	Ii	Io
+40°	+50°	+60°	+70°		Oa	Oe	Oi	Oo

On met la consonne entre les deux voyelles.

Si dans un carré il y a plusieurs cratères à dénominer, on les ordonne par ordre de longitude et on ajoute les consonnes l, m, n, p, r, s, ... z, b . . . k, dont les premières, les plus souvent employées, sont appelées demi-voyelles par les linguistes et se prononcent facilement. Si jamais l'interpolation d'un nom nouveau devenait nécessaire, la commission pourrait facilement donner une règle d'interpolation (par exemple l'addition d'une voyelle) et je n'ai pas besoin de m'occuper maintenant de cette question spéciale.

Pour montrer cet usage, j'ai joint une liste de tous* les cratères dénominés par Neison, quoique je croie, que la commission jugera un moindre nombre nécessaire, de même qu'on préfère n'avoir pas trop de constellations dans le ciel.

J'ai aussi joint un tableau des carrés.

On n'objectera pas, que les noms nouveaux sont difficiles à se rappeler.

Il y aura partout des associations d'idées, qui aident la mémoire, et qui ne sont pas plus étrangères à l'objet que les anciens noms d'astronomes.

Par exemple Apol (Tycho) rappelle Apollon, traînées divergentes, etc.

Au sujet des cratères secondaires comme Mösting *A* ou Enom *A* je vous enverrai une troisième lettre.

* J'ai omis h, parce que les Français ne prononcent pas cette consonne.
J'ai omis q, parceque cette consonne n'a pas de son propre.
J'ai omis v, parcequ'on prononce cette consonne de même que f ou w.
J'ai placé ch au lieu de c, parceque c se prononce comme k ou comme z.
Mais je fais toutes les concessions désirées ; par exemple on peut mettre v au lieu de w (ce serait plus court) et le prononcer comme w (comme en France).
La voyelle e doit être prononcée en France comme é. Les Français peuvent écrire é, s'ils veulent.
Je pense qu'on n'aura jamais besoin d'épeler les lettres.
Je ne crains pas, que la commission trouve une difficulté en ce que les Anglais prononcent les voyelles moins sonores que les Italiens et les autres nations. On n'a pas besoin d'une désignation pour les longitudes et les latitudes qui surpassent 80°. L'aire entre 70° et 90° est plus petite que celle entre 60° et 70° dans la projection orthographique.

Agréez, Monsieur et cher Collègue, l'assurance de ma haute considération,

JULIUS H. G. FRANZ.

Bogualawsky	= afa	Pontécoulant	= achi	Fabricius	= afom
Schömberger	= aka	Biela	= adi	Lockyer	= ago
Simpelius	= ala	Hagecius	= afi	Nicolai	= ako
Malapert	= ana	Rosenberger	= afil	Barocius	= alo
Short	= anal	Vlacq	= agi	Maurolycus	= alol
Newton	= apa	Nearch	= agil	Clairant	= alom
Cabeus	= ara	Hommel	= agim	Cuvier	= amo
Klaproth	= aral	Pitiscus	= agin	Faraday	= amol
Casatus	= asa	Tannerus	= aki	Stöfler	= amom
Le Gentil	= aza	Bacon	= ali	Licetus	= amon
Boussingault	= ade	Kinau	= alil	Nasireddin	= ano
Mutus	= age	Jacobi	= alim	Saussure	= anol
Manzinus	= ake	Lilius	= ami	Pictet	= anom
Kinau	= ale	Zach	= amil	Street	= apo
Pentland	= alel	Deluc	= ani	Tycho	= apol
Curtius	= ame	Maginus	= anil	Heinsius	= apom
Cysatus	= ane	Clavius	= api	Wilhelm I.	= apon
Moretus	= anel	Röst	= asi	Longomontanus	= aro
Gruemberger	= ape	Bayer	= asil	Hainzel	= aso
Blancanus	= are	Weigel	= asim	Drebbel	= ato
Scheiner	= arel	Schiller	= asin	Schickhard	= awo
Wilson	= ase	Segner	= ati	Wargentin	= axo
Bettinus	= asel	Phocylides	= awi	Inghirami	= azo
Kircher	= asem	Pingré	= azi	Marinus	= eba
Zuchius	= asen	Oken	= abo	Adams	= echa
Bailly	= axe	Vega	= acho	Furnerius	= eda
Hausen	= aze	Steinheil	= afo	Fraunhofer	= edal
Hanno	= abi	Janssen	= afol	Stevinus	= edam

* Only a part of these are printed.

M. PUISEUX'S PROPOSALS.

Sur la proposition de M. Franz, concernant l'établissement d'une nouvelle nomenclature lunaire.

Il ne faut pas se dissimuler qu'une nomenclature entièrement nouvelle sera difficilement acceptée. Il demeurera nécessaire d'acquérir un certain degré de familiarité avec l'ancienne pour profiter des travaux descriptifs faits jusqu'à ce jour, et qui ne

pourront être remplacés ou traduits dans le nouveau système qu'après un temps assez long.

Si l'on se résout à établir un système entièrement nouveau, il doit présenter l'élasticité nécessaire pour s'étendre par degrés à des objets plus petits et moins bien délimités que ceux qui auront reçu des noms en premier lieu. Il est inévitable que les études futures amèneront à considérer de tels objets.

On ne peut pas adopter un système pour les formations qualifiées principales, un autre pour les formations qualifiées secondaires, à moins que la première catégorie ne soit extrêmement restreinte, et composée seulement d'objets familiers à tous les sélénographes.

Pour les mers, les golfes, les grands massifs de montagnes, il est à peu près inévitable que les dénominations actuelles, d'origine géographique, se maintiendront en usage concurremment avec les noms nouveaux que l'on peut souhaiter de voir adopter. Ces objets sont peu nombreux, leur nom évoque aisément le souvenir de leur position sur la Lune, ils n'impliquent nulle prétention à la ressemblance ou à l'équité, enfin ils correspondent à des habitudes déjà enracinées.

Pour les grands cirques, il faut s'attendre à voir survivre, quoi que l'on fasse, Platon, Aristote, Eratosthène, Archimède, Ptolémée, Tycho, Copernic, Képler. Tous les objets sont bien connus, très faciles à retrouver. Les noms qui leur sont donnés rappellent des illustrations incontestables, et assez anciennes pour n'éveiller aucune jalousie. Aristarque, Eudoxe, Pline, Posidonius, Euclide, Hésiode, Manilius, Proclus, se recommandent par les mêmes raisons, si ce n'est qu'ils rappellent des personnages moins célèbres ou de moindres contributions à l'Astronomie.

Les noms de savants modernes et ceux de savants anciens donnés à des objets peu apparents seront beaucoup plus facilement abandonnés, et présentent des inconvénients généralement ressentis. Leur répartition n'est pas conforme à l'équité. Une répartition nouvelle ne le serait pas non plus, ou cesserait, après peu d'années, de paraître telle. Le nombre des cirques décorés de noms d'astronomes est déjà une charge pour la mémoire, tout en restant très au dessous des exigences de la topographie. Beaucoup de cirques portent deux noms différents dans les ouvrages les plus répandus, ceux de Schmidt et de Neison. Assez souvent le nom du même astronome a été appliqué à deux cirques différents. Un changement total, en ce qui concerne les noms des cirques, a donc des chances d'être adopté, s'il est rationnel dans son principe et facile dans son application.

La proposition du Dr. Franz revient en somme à affecter une combinaison de trois lettres, facile à prononcer, à chaque rectangle de dix degrés de côté, tant en longitude qu'en latitude. Une quatrième lettre est ajoutée s'il est nécessaire.

On remarquera que la désignation du rectangle peut être faite au moyen de deux lettres seulement. Adoptant les mêmes divisions que M. Franz, on aura 16 fuseaux en longitude, 16 bandes en latitude. On leur affectera les 16 premières lettres de l'alphabet

latin, respectivement majuscules et minuscules. On aura ainsi le tableau I, dont le rétablissement n'exigera, pour ainsi dire, aucun effort de mémoire.

La désignation de ces couples de lettres sera moins dénaturée par les prononciations diverses que celle des mots de trois lettres. Elle éveillera moins souvent des associations d'idées bizarres ou triviales, et il sera plus aisé de la rétablir sans avoir le tableau sous les yeux.

Il s'agit maintenant de subdiviser les rectangles. Le Dr. Franz se sert à cet effet de bandes méridiennes, de largeur indéterminée. A chacune de ces bandes est affectée une consonne, dans un ordre qui n'est pas l'ordre alphabétique. D'où nécessité de demander un effort nouveau à la mémoire ou de recourir au tableau.

Il me semble préférable de faire suivre le couple de lettres d'un couple de chiffres. Ces deux chiffres indiqueraient le nombre de degrés comptés de l'origine du carré, respectivement en longitude et en latitude, jusqu'au centre de la formation. Ainsi le nom de Proclus pourra être remplacé par le signe Dg, 66, celui d'Hyginus par le signe Hh, 68.

Dans le voisinage des bords, on pouvrait être amené à introduire, au lieu du couple final de chiffres, deux couples de deux chiffres. Pour éviter cette complication, il vaudra mieux adopter uniformément 10° comme largeur de tous les fuseaux de longitude et de toutes les bandes de latitude. Cela conduit à introduire seulement deux lettres de plus, comme le montre le Tableau II. Le signe de Proclus devient alors Eh, 66, celui d'Hyginus Ii, 68.

Il sera superflu d'inscrire sur les cartes le couple de lettres à côté de chacune des formations comprises dans un même rectangle de 10°. Il suffira que cette indication figure une fois vers le centre du rectangle, là où se trouvera une place libre. L'inscription des chiffres sur les formations sera remplacée avec avantage par une petite chiffraison de 1 à 9, le long des parallèles et des méridiens.

Les signes proposés jusqu'ici correspondent aux points d'intersection d'un réseau divisant la surface de la Lune en rectangles de 1° de côté. La plupart des objets qui méritent des noms, mesurant un degré carré ou davantage en surface, se trouveront ainsi suffisamment désignés. On pourrait encore convenir de souligner le couple de chiffres, inférieurement s'il s'agit d'un cirque, supérieurement s'il s'agit d'une montagne isolée. On éviterait ainsi d'avoir à spécifier si l'on a entendu nommer une saillie ou une dépression.

Le cas où plusieurs points à désigner se présenteraient dans un cercle de 0°,5 de rayon, ayant pour centre un sommet du réseau sera exceptionnel. On peut cependant le prévoir. Le cercle en question pouvra être divisé en 12 secteurs de 30°, désignés chacun par une des douze premières lettres de l'alphabet grec, $\alpha, \beta, \ldots . \nu$. L'ordre alphabétique suivra celui des secteurs, du Nord vers l'Est, d'après la convention adoptée pour les angles de position. On fera, dans ce cas, suivre le symbole précédemment adopté de la lettre grecque convenable. Ainsi Ii, 68 γ sera un petit cratère situé dans

un rayon de 1° environ autour d'Hyginus, l'origine de l'angle de position étant non pas le centre d'Hyginus, mais le sommet voisin du réseau.

Pour les fissures la position du centre est trop sujette à discussion. On pourrait se contenter des numéros du catalogue de Schmidt, précédés de la seule lettre S. Ainsi S 31 sera la grande fissure d'Ariadæus. Je recommanderais, sans l'imposer, de faire suivre ce symbole de l'angle de position moyen, exprimé en degrés et mis entre parenthèses. Mais je serais d'avis de ne reporter sur les cartes que les fissures qui peuvent être tracées sans incertitudes au moyen des photographies.

En résumé je propose :

1° de ne rien changer á l'usage actuel pour les mers, les golfes, les massifs de montagnes, et pour seize grands cirques dont j'ai donné la liste ; cette liste pouvant être, si on le désire, un peu étendue.

2° de diviser l'hémisphère visible par des méridiens et des parallèles en rectangles de 1° de côté, et de représenter chaque sommet de ce rectangle par un symbole (deux lettres et deux chiffres), facile à retrouver à tout instant sans l'aide d'aucun document.

3° d'affecter aux cirques, aux montagnes isolées, aux taches, le symbole du sommet du réseau qui tombe le plus près de leur centre de gravité apparent.

4° d'indiquer par un trait, quand cela semblera utile, s'il s'agit d'une formation à centre saillant ou à centre déprimé.

5° de garder la nomenclature de Schmidt pour les fissures.

On peut facilement pousser la division de la surface plus loin par l'emploi de douze lettres grecques, ou l'étendre en longitude jusqu'aux méridiens +100° et −100° par l'emploi des lettres accentuées A′ et R′. Il est peu probable que l'on ait jamais à recourir à ces expédients, qui d'ailleurs ne demanderaient nulle dérogation aux conventions précédentes et imposeraient peu d'effort à la mémoire.

Les tableaux suivants (I et II) donnent la signification des symboles littéraux, suivant que l'on adopte le mode de division du Dr. Franz ou le mien. Le tableau III donne, dans la seconde hypothèse, les symboles qui répondent aux points du premier ordre. Les coordonnées adoptées sont celles de Neison. Il y aurait lieu, si ma proposition est adoptée, de revoir ce tableau en ayant égard aux déterminations plus récentes.

P. Puiseux.

1907 *November* 16.

TABLEAU I.

Longitude.		Latitude.	
+90°	A	+90°	a
80		80	
70		70	b
60	B	60	c
50	C	50	d
40	D	40	e
30	E	30	f
20	F	20	g
+10	G	+10	h
0	H	0	i
−10	I	−10	j
20	J	20	k
30	K	30	l
40	L	40	m
50	M	50	n
60	N	60	o
70	O	70	
80	P	80	p
−90		−90	

TABLEAU II.

Longitude.		Latitude.	
+90°	A	+90°	a
80	B	80	b
70	C	70	c
60	D	60	d
20	E	50	e
40	F	40	f
30	G	30	g
20	H	20	h
+10	I	+10	i
0	J	0	j
−10	K	−10	k
20	L	20	l
30	M	30	m
40	N	40	n
50	O	50	o
60	P	60	p
70	Q	70	q
80	R	80	r
−90		−90	

TABLEAU III.

Agrippa	Hi,	04	Burg	Ge,	85	Gassendi	Nk,	07
Albategnius	Ik,	41	Byrgius, A.	Pl,	44	Goclenius	Ek,	40
Alphonse, A.	Jk,	33	Campanus	Ll,	78	Grimaldi, A.	Qj,	15
Archytas	Id,	48	Capella	Mj,	58	Gueriké, C.	Kk,	22
Archimède, A.	Jg,	78	Carlini	Lm,	43	Hainzel, A.	Ln,	93
Aristarque	Ng,	73	Cassini, A.	In,	40	Halley	Ij,	68
Aristillus	If,	14	Censorinus	Fj,	20	Hansen, A.	Bh,	43
Aristote, C.	Gd,	47	Céphée, A.	Ee,	61	Harding	Qe,	13
Bessarion, E.	Mh,	75	Clavius, C.	Ko,	57	Harpalus	Nd,	42
Bessel,	Hg,	72	Cléomède, A.	Dg,	48	Hell	Jm,	82
Billy,	Ok,	04	Conon	Ig,	21	Héraclide	Me,	41
Biot,	Dl,	02	Copernic	Li,	09	Hercule	Fe,	86
Bode	Ji,	37	Crüger	Pk,	77	Herschel	Jj,	26
Bode, A.	Ji,	19	Cyrille	Gk,	34	Hésiode, B.	Kl,	77
Bode, B.	Ji,	39	Delambre	Hj,	72	Hipparque, C.	Ij,	87
Bohnenberger, A.	Fk,	97	Flamsteed	Nj,	45	Hipparque, L.	Ij,	97
Brayley	Mg,	61	Furnerius, A.	Cm,	83	Horrocks	Ij,	64
Bouillaud	Ll,	20	Gambart, A.	Ki,	91	Hortensius	Li,	86

TABLEAU III.—*continued.*

Képler	Mi,	88	Moretus	Jq,	70	Römer	Fg,	65
Lahire	Kg,	57	Mösting	Jj,	61	Rosse	Fk,	48
Lalande	Jj,	94	Mösting, A.	Jj,	53	Sacrobosco	Hl,	64
Landsberg	Lj,	60	Murchison, A.	Ij,	14	Scheiner, A.	Lp,	70
Landsberg, A.	Mi,	10	Mutus	Gp,	93	Schubert, A.	Bi,	73
Langrenus	Cj,	18	Olbers	Qi,	88	Seleucus	Pg,	61
La Peyrouse, A.	Bj,	49	Parry, A.	Kj,	69	Struve, B.	Ce,	53
Laplace, A.	Le,	73	Petavius	Dl,	95	Taruntius	Ei,	66
Le Monnier, A.	Gg,	96	Phocylides, E.	Oo,	65	Thalès	Ec,	92
Lichtenberg	Pf,	71	Picard	Dh,	44	Thebit, A.	Tl,	61
Lindenau	Gm,	42	Piccolomini	Fl,	29	Théophile	Gk,	61
Linné	Hg,	28	Picon	Je,	95	Timocharis,	Kg,	37
Mädler	Gk,	91	Pitiscus	Fo,	oo	Tycho	Kn,	23
Maginus	Jo,	70	Pline	Gh,	35	Ukert	Ii,	18
Manilius	Ih,	94	Posidonius, A.	Gf,	92	Vega, A.	Cn,	95
Marius	Ok,	02	Proclus	Eh,	76	Viète, A.	Om,	73
Maskelyne	Mi,	03	Ptolémée, A.	Jj,	19	Vitello	Mm,	70
Maurolycus,	Hn,	43	Pythagore	Pc,	23	Vitruve	Fh,	18
Mayer	Lh,	96	Pytheas	Lg,	10	Werner	Il,	38
Menelaus	Hh,	66	Ramsden	Mm,	22	Wichmann	Mj,	28
Messier	Ej,	72	Reinhold	Li,	33	Wollaston	Nf,	70
Milichius	Mh,	oo						

PROFESSOR W. H. PICKERING'S PROPOSALS.

It does not seem to the writer practicable to abolish the system of names at present applied to the chief lunar craters. A supplementary system of nomenclature for designating the minor points, however, seems to him very desirable. He would therefore make the following suggestions to the Committee on Nomenclature.

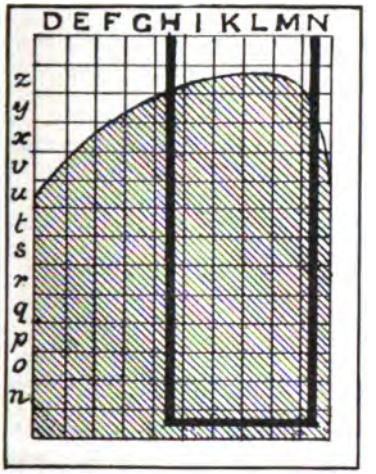
A series of photographs should be taken, or, if practicable, negatives already taken should be used. These should be enlarged through a reticule to a uniform scale of 400 mm. to the Moon's diameter. The positives should be on plates measuring 20 × 25 cm. The enlargements should be made through a reticule of squares, each square measuring 20 mm. on a side. These positives would form a set of standards which should be safely stored From them negatives should be printed which could be furnished to engravers, photographers, or the public. The reticule should be oriented parallel to the Moon's axis.

Since it is desirable to get all the detail possible, and since the maximum detail is shown only near the terminator, it is proposed

to use photographs taken at eight different ages of the Moon, one set representing the northern hemisphere and the other the southern. This will give us sixteen different photographs. Each of these should be selected in as favourable a libration as possible.

Besides the lunar detail which shows near the terminator, there is an entirely different kind of detail, consisting of bright and dark surfaces, and of bright points which show only when remote from the terminator. Six plates should be taken to show this detail, three for each hemisphere. These would be taken when the terminator is distant, but not necessarily at full moon. This would give us a class of detail which has not been so carefully studied as that at the terminator, and for this reason is of all the more importance.

We thus have in all twenty-two standard plates. While it would be desirable to have an atlas engraved from them which



could be issued to the public, this would not be necessary, since a photographic edition of the whole twenty-two would not be very expensive. It should be possible for those interested to buy photographs of a single region.

The positive standards should be lettered in capitals running from the western to the eastern limb, and in small letters running from north to south. Twenty-four letters should be used, j and w being omitted. Around the selected region of each standard a heavy black line should be drawn, as shown in the figure, and the regions outside of this should not be used. The selected regions should overlap slightly on the different plates, which the use of twenty-four letters will permit. Otherwise only twenty letters would be needed. Beginning at the western limb, two capitals should be assigned to the first plate, three to the second, and three to the third. Four capitals will be assigned to the fourth plate, as

shown in the sketch, and the same arrangement will apply to the other quadrants.

Any area of the Moon's surface can now be indicated by two letters. Thus Kq will be found on only a single sheet of the atlas, and its exact area will be there clearly shown. The six sheets taken away from the terminator, under high illumination, could be indicated by the use of Greek letters.

The sides of the squares may be divided to tenths by the eye, and the position of any point within the square indicated thus, "Kq 63," indicating that the point measures ·6 way from the side between J and K, and ·3 way from the bottom of the square. Should still greater accuracy of designation be required, a millimetre scale could be used, and the point designated thus, "Kq 65, 37." The squares measure approximately 100″ on a side, so that this system should give us all the accuracy needed.

A double catalogue would go with the atlas, one part giving the old nomenclature of every point hitherto named in terms of the new, and the other giving the new in terms of the old. It is not intended that this nomenclature should supersede the older one; the idea is simply to furnish one which shall be purely supplementary, and absolutely free from ambiguity. It could be made readily accessible to all, so that for purely scientific work anyone could use it who chose to do so; and since the older nomenclature would also be given in the catalogue, no one need feel compelled to use the supplementary system who did not like it.

WILLIAM H. PICKERING.

1907 *November* 8.

MR. SAUNDER'S PROPOSALS.

I have already expressed my views as to the best method of dealing with the problem presented by the present condition of Lunar Nomenclature in *R.A.S. Monthly Notices*, lxvi. p. 41, and in a note printed in view of the submission of the question to the Vienna Congress. It might, however, be convenient if I summarise them here.

I am not in favour of any radical change. So much work of permanent value has been published in terms of the old nomenclature that it can never be entirely superseded, and to create another, to be used concurrently with it, would, in my opinion, only increase the confusion.

The present nomenclature, however, does require careful revision in order to ensure that—

(1) No two formations should have the same name.

(2) No formation should have more than one name.

In assigning names for future use I would suggest the following general rules—

(1) That Mädler's principal names be retained.

(2) That where Mädler has not given principal names, as little alteration as possible be made in names now generally accepted. These rules should not prevent the Committee from rejecting a name altogether where none seems required, or from introducing a new one where one would be advantageous.

(3) For minor formations only two alphabets should be used; the capital Roman to denote craters, depressions, and dark areas; the small Greek to denote peaks and bright spots.

(4) Such rills as are in Schmidt's catalogue might be denoted by their catalogue numbers, others by a short reference to their position.

I have always thought that the best method of publishing the decisions of the Committee would be to embody them in an accurate map, which must be specially prepared. But the practical difficulties appeared so great that I suggested the use of photographs, very much as Professor W. H. Pickering now does, though I had not thought out the arrangement in so much detail.

Dr. Franz has, however, in a recent letter, suggested the construction of an atlas giving accurate outlines and positions of all the more prominent features, but not presenting the minute detail. The different parts of the surface would be represented in a series of maps, and he has offered to be responsible for the limb regions if I would undertake the central parts. Dr. Franz has made a special study of the limb, and has accumulated a large number of unpublished measures. His maps would be constructed under favourable librations of 10°. I have a large number of measures of the central regions besides those already published; and as Mr. W. H. Wesley has kindly placed his great skill as a draughtsman at the disposal of the Committee for the delineation of these parts, it seems to me that the construction of such a map is now quite feasible. The maps of the central regions would be under mean libration.

I believe that with the data now available a map might be constructed in which the positions of all objects shown would be accurate to within less than 1″ of arc (geocentric), corresponding to less than one mile on the surface of the Moon for the central part, whilst the measured points are so thickly distributed that an object not represented in the map, but whose position could be located by cross bearings from points that are represented, could have its co-ordinates read off to very nearly this degree of accuracy.

With such a map it would be possible to refer to unnamed formations by means of their selenographical co-ordinates, and I believe this to be the best solution of the problem that has yet been suggested.

S. A. Saunder.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Henry Boase Austin, J.P., Government Buildings, Bloemfontein, Orange River Colony, South Africa;
W. Geoffrey Duffield, Physical Laboratory, Manchester;
Wm. Earnshaw Etzel, B.S., Litt.L., M.S.M.F., Professor of Sciences, St Bernard's, Rochester, N.Y., U.S.A.;
John M. Field, 1 Hart Street, Edinburgh;
James D. Maddrill, Ph.D., International Latitude Observatory, Ukiah, California, U.S.A.;
William Henry Rees, B.Sc., County School, Pontypridd, and 2 Craigwen Place, Pontypridd, South Wales;
Thomas James Forrester Smith, Newstead, Wavertree, Liverpool; and
Captain James Weir, F.R.G.S., Examiner of Masters and Mates in Navigation and Seamanship to the Board of Trade, 5 Clive Terrace, Penarth, South Wales,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Frederic Hermann Albert Alfred Buss, 2 Lansdowne Terrace, Grosvenor Square, Ashton-on-Mersey, near Manchester (proposed by A. Fowler);
Arthur du Pré Denning, M.Sc., Ph.D., Lecturer in Physics, Birmingham University, 18 Lightwoods Hill, Birmingham (proposed by William Briggs);

Herbert Shaw, Royal College of Science, South Kensington, S.W. (proposed by A. Fowler); and

James Henry Worthington, Student in the University of Oxford, Bindon, Wellington, Somerset (proposed by H. H. Turner).

Sixty presents were announced as having been received since the last meeting, including, amongst others:—

E. T. Whittaker, The Theory of Optical Instruments, presented by the author; framed photograph of 14th century clock restored by J. J. Hall, presented by Mr. Hall; Calendrier Perpétuel, presented by M. Bosson.

Seven transparencies of Sun-spots, Comet, Planets, and Nebulæ, from negatives taken at the Royal Observatory, Greenwich, presented by the Astronomer Royal.

J. C. Kapteyn, On the number of stars of determined magnitude and determined galactic latitude (Pub. Astron. Laboratory, Groningen, No. 18), presented by Professor Kapteyn.

On the Lunar Inequalities due to Planetary Action.

By Ernest W. Brown, D.Sc., F.R.S.

1. I have lately finished the computation of the terms in the Moon's motion due to the actions—direct and indirect—of the planets, and it is of interest to compare the results with those of previous investigators. There are two extensive lists in print: one given by Radau * in 1892, and the other by Newcomb † a few months ago. Both of these give only the terms in longitude, and I shall therefore limit the results set forth here for discussion to these terms, reserving the complete list for a memoir, containing also my methods and the details, which I hope to present to the Society at a later date. I have also omitted here the secular terms and those which depend solely on the lunar arguments, since the results for them are not in doubt, and since they will ultimately be combined with terms arising from the figure of the Earth. Further, I only consider quantities of the first order relative to the planetary masses, while Newcomb has included the mutual perturbations in his values for the direct action. But the latter do not affect any of the terms except one or two of long period, mentioned in No. 10 below.

The great majority (about 400) of the terms are of short period,

* "Recherches concernant les Inégalités planétaires du mouvement de la Lune," *Paris Obs. Ann.* (*Mém.*), vol. xxi.

† "Investigation of Inequalities in the motion of the Moon produced by the action of the planets," *Carnegie Inst. Publ.*, 72.

and are most conveniently added to the true longitude. The terms of very long period are best added to the mean longitude, and they are so set down here; but the changes in the other elements for these long period terms are not given, since they can be exhibited as simple ratios to the mean longitude portions. Radau adds these long period terms to the true longitude, and I have therefore dropped the corresponding short period terms which he gives.

In his final results, Newcomb drops coefficients less than $0''{\cdot}003$. I have done the same in transcribing from my manuscripts, except occasionally where a smaller coefficient comes in the middle of a long series, or where Newcomb or Radau give a coefficient greater than $0''{\cdot}002$. All my coefficients have been computed to $0''{\cdot}001$ except those of a few long period terms in δw_1; these exceptions are noted by a comma placed before the zero, which is set down as the last figure in order not to break the continuity of the representation.

2. The notation for the arguments of the tables is as follows:—

l, $2D$, F, the mean anomaly, the argument of the "variation" and the principal argument of the latitude of the Moon.

V*, the Moon's true longitude.

w_1, the Moon's mean longitude.

Q, V, T, M, J, S, the mean longitudes of Mercury, Venus, the Earth, Mars, Jupiter, and Saturn.

$h = D - F + T$, the mean longitude of the Moon's node.

The results are arranged in six columns. The first pair, headed B, contains the coefficient and angle as computed by myself; the second pair, headed N, those of Newcomb; the third pair, headed R, those of Radau. The coefficients are given in units of $0''{\cdot}001$. The angle has, except in δw_1, been divided into four parts for convenience: ϕ contains the lunar arguments l, D, F, h; α, the constant part of the angle so taken that the coefficients are all positive; j, j' the multiples of T − P and T (or j, j'' the multiples of P − T and P) so that j' (or j'') represents the order of the term with reference to the eccentricities and inclination of the Earth and planets. The values of j' (or j'') and ϕ, once given, are understood to be the same for every following term until new values are set down. The differences between the results of Newcomb and myself are marked with the letters E_1, E_2, E_3, U whenever they exceed $0''{\cdot}020$, and they are considered in No. 8 below.

* An italic capital to distinguish from the mean longitude of Venus.

$$\delta V = +0''.001\ C \sin\{\phi + j(T - V) + j''T + \alpha\}, \text{ Venus.}$$

$\phi=0$		B		N		R		
j	j'	C	α	C	α	C	α	
			°		°		°	
1	0	822	0·0	882	0·1	860	0	E_1
2		307	179·8	401	179·3	283	180	E_1
3		42	359·3	37	1·6	65	0	
4		46	0					
5		33	0					
6		24	0					
7		17	0					
8		12	0					
9		8	0					
10		6	0					
11		4	0					
21		5	0					
−2	1	10	254					
−1		16	84	14	85			
1		42	82	55	81·5			
2		348	272·9	354	276·2	348	275	
3		176	271·7	197	272·2	181	272	E_1
4		2	82					
5		4	271					
6		6	272					
7		4	272					
−1	2	3	27					
1		5	25					
2		3	33	5	34			
3		92	199·0	82	208·0	80	198	
4		26	204					
5		9	17					
6		4	207					
5	3	26	114	23	112			

2D		B		N		R		
j	*f*	C	α	C	α	C	α	
			°		°		°	
−11	0	3	0					
10		5	0					
− 9		6	0					
− 8		8	0					
− 7		8	0					
− 6		11	0					
− 5		11	0					
− 4		8	0					
− 3		34	180·0	56	359	32	180	E_1
− 2		36	0·0	24	0·0	48	0	
− 1		23	180	8	180	30	180	
1		99	0·0	96	0·0	105	0	
2		136	179·5	143	178·8	148	180	
3		13	178	35	180	23	180	E_1
4		4	180					
18		3	0					
1	1	3	232					
2		40	271·0	102	271·8	46	270	E_2
3		37	271·5	92	272·4	44	270	E_2
4		5	89					
20		10	273					
−11	−1	3	78					
−10		4	78					
− 9		4	78					
− 8		4	78					
− 7		5	78					
− 6		5	78					
− 5		11	82					
− 4		7	78					
− 3		2	288	5	268·2	12	270	
− 2		19	271	49	267·8	28	270	E_2
1		9	98					
2		13	281					
3		3	281					

2D		B		N		R		
j	j'	C	α	C	α	C	α	
			°		°		°	
3	2	11	199	26	199·7			
4		3	202					
5		4	20					
−6	−2	6	162					
−3		15	342	35	339·8			
−2		3	7					
15		4	151					
l								
−7	0	4	180					
−6		5	180					
−5		6	180					
−4		6	180					
−3		8	180	6	180	10	180	
−2		61	0·0	71	0·6			
−1		129	180·0	149	180·0	66	180	E_1
1		152	0·0	166	0·0	80	0	
2		48	180·0	93	180			E_1
3		127	180·0	8	0	138	180	E_1
4		11	180					
21		30	0			56	0	
−2	1	4	258					
−1		1	79	4	80			
1		8	75	3	80			
2		46	271	82	273·1			E_1
3		40	272	41	271·8			
4		5	272					
5		4	92					
23		6	272			12	273	
−3	−1	32	268	37	267·0			
−2		46	264	80	266·5			E_1
−1		9	104	3	100			
1		3	102	4	280			
2		7	282					
3		7	280					

l		B		N		R	
j	j'	C	α	C	α	C	α
			°		°		°
3	2	14	210	23	201		
4		4	205				
5		3	19				
6		16	198				
−5	−2	3	161				
−4		4	336	22	336		
−3		15	331				
5	3	4	115	5	115		
−5	−3	4	65	5	65		
$2D-l$							
−10	0	3	180				
−9		6	180				
−8		8	180				
−7		13	180				
−6		22	180				
−5		38	180				
−4		83	180·0				
−3		658	180·0	646	179·8	681	180
−2		137	0·0	142	0·0	192	0
−1		13	180	25	180	42	180
1		133	0·0	146	0·0	143	0
2		157	179·6	174	180·0	172	180
3		14	178	14	180	18	180
4		3	180				
18		11	0			12	0
−2	1	3	259				
−1		3	270				
2		65	271·4	71	269·5	70	270
3		49	271·9	53	273·7	50	270
4		5	90				
20		126	273·0			140	272

2D−l		B		N		E	
j	*j'*	C	α	C	α	C	α
− 8	− 1	3	269				
− 7		5	269				
− 6		8	269				
− 5		24	269	25	268		
− 4		30	89				
− 3		0	...			13	270
− 2		38	268	41	264·0	45	270
1		9	101				
2		13	281				
17		3	253				
3	2	19	200	21	202	22	198
4		4	202				
5		4	19				
− 7	− 2	3	340				
− 6		79	162·6	83	161·3	82	163
− 5		4	165				
− 3		24	340	26	337	27	342
− 2		3	6				
15		25	151				
5	3	4	114				
12	− 4	33	303				
2D+l							
− 5		3	0				
− 3		4	0				
− 2		4	0				
1		11	0	6	180		
2		15	180	9	0		
3		3	180				
2	1	5	271				
3		4	271				
− 2	− 1	3	269				

$2l$ j	j'	B C	B α	N C	N α	E C	E α
− 2	0	5	0				
− 1		10	180				
1		10	0				
2		6	180				
3		9	180				
2	1	3	273				
− 2	− 1	3	267				
$2l-2D$							
− 2	0	7	0	9	0		
− 1		5	180	7	180		
2		3	180	8	180		
3		73	180·0	72	180·0	75	180
4		3	0				
4	1	4	92				
− 20	− 1	7	267				
− 3		3	268				
− 2		3	268	3	261		
6	2	62	17·4	62	17·5	63	17
$4D-l$							
1	0	6	180				
2		7	0				
$3l-2D$							
3	0	3	180	3	180		
6	2	5	18	3	26		
$2l-4D$							
3	0	8	0				
$-4D$							
3	0	7	0				
$-D$							
3	1	5	273			31	91·4
$l-D$							
3	1	11	273			48	91·4
$2l+2D$							
2	0	0		3	0		

h		B		N		R	
j	f	C	α	C	α	C	α
			°		°		°
− 5	−2	3	255				
− 4		9	255				
− 3		16	75			16	256
− 2		5	75				
$h+2F$							
− 2	−3	3	75				

$$\delta w_1 = +0''{\cdot}001\ \mathrm{C} \sin(\phi + \alpha).$$

	B		N		R		
ϕ	C	α	C	α	C	α	
		°		°		°	
13T − 8V	237	313·9	247	316·1	250	318·7	
l + 16T − 18V	1449,0	151·0	1477,0	150·1	1442,0	150	E_2
l + 3T − 10V	35,0	33					
l + 29T − 26V	108	112·0			110	121	
2F − 2D + 6T − 5V	54	270·0			60	269	
3l − 2D + 24(T − V)	10	0			12	0	
D + l − F + 17T − 18V	8	75					
F + 24T − 23V	3	285			8	0	
D + 12T − 15V	13	262			16	261	
D + 25T − 23V	13	190			19	194·5	
3D − 2F + 18T − 18V	2	272			8	0	
2D − 2F − l − 22T + 23V	< 10				4	284	
D − 2l − 20T + 21V	0				3	288	

$$\delta w_1 = +0''{\cdot}001\ \mathrm{C} \sin(\phi + \alpha), \text{ Mercury.}$$

Q − 4T	3	239°		
2D − l − 3Q + T	75	105	57	150
2D − l − 4Q + 5T	3	113	6	105
l − 2F + 4Q − 3T	3	113	7	105

$\delta V = +0''\cdot001\ C \sin\{\phi + j(M - T) + j'M + \alpha\}$, Mars.

$\phi=0$		B		N		R		
j	j'	C	α	C	α	C	α	
			°		°		°	
1	0	11	180	11	174	14	180	
2		195	180·2	224	180	228	180	E_1
3		14	357	14	0	16	0	
4		5	349					
−3	1	6	260					
0		0		30	82			U
1		327	224·4	372	229·2	422	229	E_1
2		38	212·4	42	213·6	53	213	
3		48	212·5	47	212·9	55	212	
4		10	331					
2	2	93	244·8	95	249·8	112	247	
3		20	245	30	120			U
4		14	244					
5		6	62					
3	3	16	277					
4		13	276					
5		6	275					
6		3	94					
2D								
−2	0	5	0	6	0	16	0	
1		4	180					
2		44	181	50	180·0	59	180	
3		5	0					
1	1	23	224	52	225·0	30	224	E_2
2		6	212					
3		8	214	28	214			E_2
4		3	37					

2D		B		N		R		
j	j'	C	α	C	α	C	α	
			°		°		°	
− 5	− 1	3	149					
− 3		1	327	5	333			
− 2		3	328					
− 1		23	317	58	316·8	34	315	E_2
3		3	280					
2	2	5	244					
3		4	244					
4		4	246					
− 2	− 2	8	297					
l								
− 3	0	3	180	3	180			
− 2		38	0	43	0·0			
− 1		4	0	4	0			
1		5	180	4	180			
2		43	180	48	180			
3		3	0	3	0			
4		3	180					
0	1	0		3	80			
1		73	223·3	55	219·9	35	230	
2		10	212	9	215			
3		13	213	10	215			
5		8	198					
− 3	− 1	9	330	10	324			
− 2		8	327	9	324			
− 1		74	306·3	55	318·9	36	310	
0		0		3	99			
2	2	17	245	17	244			
3		5	245					
4		3	244					
6		6	63					
− 4	− 2	3	296					
− 3		5	295					
− 2		18	295	17	296			
26		0				12	306	

2D−l		B		N		R	
j	*f*	C	α	C	α	C	α
			°		°		°
−5	0	3	180				
−4		20	182			19	180
−3		5	0	6	0		
−2		13	0	17	0	34	0
−1		3	0	4	0	7	0
1		8	180	5	180	9	180
2		61	181	66	180·0	73	180
3		5	353	3	0		
1	1	31	220	41	226·4	47	223
2		11	212	9	215		
3		14	214	13	215		
4		3	27				
−6	−1	3	149				
−5		43	162			42	148
−4		3	329				
−3		3	327	5	333		
−2		6	328	6	324		
−1		35	320	43	316·9	51	317
3		4	280				
2	2	11	244	9	244		
3		6	244				
4		5	245				
−6	−2	33	298				
−3		3	296				
−2		14	297	11	294		
2D+l							
2	0	6	180	4	0		
1	1	3	82				
−1	−1	3	93				
2l							
−2	0	3	0				
2		3	180				
1	1	3	232				
−1	−1	3	308				

$2l-2D$		B		N		R	
j	j''	C	α	C	α	C	α
			°		°		°
−2	0	4	0	4	0		
5	1	17	209			17	212
6	2	18	244				

$\delta w_1 = +0''{\cdot}001\ \mathrm{C} \sin(\phi + \alpha)$, Mars.

	B		N		R	
ϕ	C	α	C	α	C	α
		°				°
8M − 4T	3	310				
9M − 5T	8	305				
11M − 6T	6	335				
13M − 7T	6	19				
15M − 8T	26	43				
17M − 9T	4	63				
$4D - 3l + 25M - 23T$	4,0	67				
$h + 2M - T$	17	165			19	164
$D - l - 4M + 3T$	<10				3	153

$\delta V = +0''{\cdot}001\ \mathrm{C} \sin\{\phi + j(\mathrm{J} - \mathrm{T}) + j''\mathrm{J} + \alpha\}$, Jupiter.

$\phi=0$		B		N		R		
j	j''	C	α	C	α	C	α	
1	0	643	178°8	741	178°8	646	180°	E_1
2		187	359·6	242	359·8	196	0	E_1
3		10	7	9	6	12	0	
−3	1	6	257	6	80			
−2		18	274	16	275			
0		87	289·9	183	283·0	139	279	U
1		165	241·5	193	242·7	172	242	E_1
2		52	352·0	42	77·7	49	352	U
3		4	355					
1	2	10	250					
0		5	324					
1		25	238	24	83			U
2		6	344.					
1	3	3	230					

2D		B		N		R		
j	j''	C	α	C	α	C	α	
			°		°		°	
−3	0	4	0	18	180			E_2
−2		70	180·0	18	0	54	180	E_1
−1		33	1·1	16	0	45	0	
1		167	178·5	168	177·9	175	180	
2		85	359·2	92	359·3	88	0	
3		7	13	74	2·3			U
0	1	27	349	36	350·5	32	350	
1		35	236·5	89	237·4	39	238	E_2
2		15	352	42	354·5			E_2
−2	−1	30	8	7	9			E_1
−1		6	313	15	311			
0		33	184	40	181·0	38	189	
2		9	273	38	272·0			E_2
3		6	102	19	105			
1	2	5	236	10	240			
2		3	345					
0	−2	3	200					
1		6	110					
l								
−3	0	1	170	4	180			
−2		36	180	45	180	12	180	
−1		144	1·0	140	1·6	36	0	
1		158	179·0	163	178·6	48	180	
2		190	180·0	64	0·0	165	180	E_1
3		5	21	3	0·0			
−2	1	6	274	6	270			
0		62	282·3	38	7·5			E_1
1		39	242	36	268·4			
2		96	352·5	10	351	65	353	E_1

l		B		N		E		
j	j'	C	α	C	α	C	α	
−2	−1	7	188°	10	189°			
−1		35	298	32	278·6			
0		63	257·2	38	172·5			U
2		6	273	6	270			
3		8	286					
0	2	7	326					
1		5	238	3	206			
2		4	343					
−1	−2	5	302	3	334			
0		7	214					
$2D-l$								
−4	0	4	180					
−3		22	182	18	0			U
−2		1137	180·3	1140	180·2	881	180	
−1		51	0·9	64	0·0	101	0	
1		211	178·4	230	179·0	220	180	
2		89	359·2	98	0·7	89	0	
3		6	14	30	1·9			U
−3	1	5	261					
−2		13	310	18	294			
0		56	5·5	60	353·4	60	351	
1		46	237·0	48	238·7	49	236	
2		20	352	21	353			
−3	−1	6	187					
−2		436	7·5	445	7·5	316	7	
−1		18	296	19	311	25	304	
0		60	174·2	62	182·2	63	184	
2		16	273	18	276			
3		7	102					
1	2	6	237	4	237			
2		3	344					
−2	−2	5	19			9	16	
1		3	291					

$2D+l$ j	j'	B C	B α	N C	N α	R C	R α	
			°		°		°	
−2	0	3	180					
−1		5	1					
1		21	178	11	0			U
2		7	359	6	180			
0	1	4	353					
1		5	237					
2		3	352					
0	−1	4	182					
2		3	273					
$2l$								
−2	0	3	180					
−1		11	2					
1		12	178					
2		10	180					
0	1	5	293					
1		3	239					
−1	−1	3	301					
0		5	247					
$2l-2D$								
−2	0	5	180	4	180			
−1		11	2	11	0			
1		3	0					
2		240	179·9	256	180·0	206	180	
0	1	0		4	350			
2	1	284	172·5	258	172·3	194	173	U
−1	−1	2	302	3	298			
0	−1	2	186	4	189			
2	2	3	163			2	164	
$4D-l$								
−2	0	7	180					
1		9	358					
2		5	179					
−2	−1	3	7					
$3l-2D$								
2	0	7	180	10	180			
2	1	5	172	15	257			

$2l-4D$		B		N		R		
j	j''	C	α	C	α	C	α	
			°		°		°	
2	0	9	0					
2	1	7	173					
h								
0	1	4	81			4	75	
0	−2	0				11	277	

$\delta V = +0''{\cdot}001\ C \sin\{\phi + j(S-T) + j''S + \alpha\}$, Saturn.

$\phi=0$		B		N		R		
j	j''	C	α	C	α	C	α	
			°		°		°	
1	0	42	179·6	40	180·0	38	180	
2		8	0	8	0			
0	1	21	273	51	278·9			U
1		13	257	11	255			
0	2	3	297					
$2D$								
1	0	10	180					
2		5	0					
0	1	4	270					
1		3	257					
0	−1	4	255					
l								
−1	0	6	0	8	0			
1		10	180	8	180			
2		3	180					
0	1	12	263	4	275			
1		3	257					
−1	−1	3	283					
0		12	277	4	265			
$2D-l$								
−2	0	19	180					
1		14	180					
2		4	0					
0	1	6	271					
1		3	257					
−2	−1	5	271					
0		6	267					

3. In order that the discussion of the various differences may be made clear, it is necessary to state certain facts in the theory of these inequalities which have considerable effect on the final results, and on the methods of obtaining them.

The disturbing function R is of the form—

$$\begin{aligned}R = \Sigma\{&A_0 \cos Q + A_1 \cos (l+Q) + A_2 \cos (2D - l + Q) \\ &+ A_3 \cos (2D + l + Q) + A_4 \cos (2D + Q) \\ &+ A_5 \cos (2D - 2l + Q) + \ldots\ldots\},\end{aligned}$$

where Q here and throughout this discussion is an angle depending only on the solar and planetary arguments, and where the summation sign refers to the different arguments Q.

Let w_1 be the Moon's mean longitude, and c any one of the other five lunar elements. When the value of R has been substituted in the equations of variations and the latter solved we obtain—

$$\delta w_1 = \Sigma\{B_0 \sin Q + B_1 \sin (l+Q) + \ldots\ldots\},$$

$$\delta c = \Sigma\left\{C_0 \begin{matrix}\sin\\ \cos\end{matrix} Q + C_1 \begin{matrix}\sin\\ \cos\end{matrix} (l+Q) + \ldots\ldots\right\},$$

the term in δc being a sine or cosine according as c is or is not an angular element.

The values of the variations of the elements have to be substituted in—

$$\delta V = \delta w_1 + \left(\frac{dV}{dw_1} - 1\right)\delta w_1 + \frac{dV}{dc}\delta c\,.$$

Now V consists of a non-periodic term w_1 and periodic terms, say,

$$V = w_1 + v_1 \sin l + v_2 \sin (2D - l) + v_3 \sin (2D + l) + \ldots\ldots,$$

and from this we obtain easily the values of the deviatives of V. The terms which arise from the substitution of the variations of the elements in δV may therefore be divided into two classes. The first of these, which I have called elsewhere* the *primary* terms, consists of those terms in δV which arise from the variation of the non-periodic term; the *secondary* terms are those arising from the substitution in the remaining, that is, the *periodic* terms of V. Thus the values of δw_1 constitute the *primary* terms, and all the other portions of δV the *secondary* terms.

Now the primary terms have the same arguments as the terms in R, that is, the arguments

$$Q, l+Q, 2D-l+Q, \ldots\ldots,$$

while the secondary terms have the arguments

$$\begin{aligned}&Q \pm (l), Q \pm (2D - l), Q \pm (2D + l), \ldots\ldots \\ &l + Q \pm (l), l + Q \pm (2D - l), l + Q \pm (2D + l), \ldots\ldots, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}$$

* *The Inequalities in the motion of the moon due to the direct action of the planets*, Pitt Press, Cambridge, 1907.

the arguments arising from the derivatives of V being enclosed in brackets. It is therefore obvious that the arguments Q, $l+Q$, ... which are present in the primary terms will also appear amongst the secondary terms; they are due to the separation of a product of a sine and a cosine into the sum or difference of two sines.

4. The first result to be noted is the fact that *the sum of the secondaries with arguments* $l+Q-(l)$, $2D-l+Q-(2D-l)$, ... that is, those having the argument Q, *is very small compared with the coefficient of the primary with argument* Q.* In many cases these secondaries are of the same order of magnitude as the primary of the same argument, the chief of them in general being those arising from the two arguments just written down. This theorem is a consequence of the method of the variation of arbitrary constants. If we had adopted the straightforward method of integrating the original equations for the Moon's motion with the additional value of R due to planetary action, the terms independent of l in the coordinates arising from the terms in R which contain l, and which therefore have the factor e (the lunar eccentricity), would have possessed the factor e^2, while the principal terms due to $R=A_0\cos Q$ would not have this factor; the former terms must therefore be quite small.

This fact constitutes a useful test of the general accuracy of a large portion of the work, and it was satisfied in all cases as closely as could be expected. It is illustrated in the following table for a few terms of the indirect action of Venus in longitude. The first column gives the arguments of the primaries (that is, of the terms in R), and in line with them are the resulting secondaries for the values of ϕ which stand at the head of each column. The last line but one gives the sums of these secondaries, and the last line the coefficients of the primaries. The sums of these two lines constitute the complete coefficients for the indirect action. The constant angle to be added to each argument is omitted, as it is (in these and in most cases) very nearly the same for each of the numbers in a given column.

$\delta V = 0''{\cdot}001\,C\sin(\phi+\alpha)$, $\phi = iT - i'V$, Venus. Values of C.

Arg. of primary.	T-V	T-2V	2T-2V	3T-2V	3T-3V	4T-3V	5T-3V
$\phi+l$	− 47	− 7	− 92	+ 18	−13	+ 21	+ 5
$\phi-l$	− 8	...	+ 21	+ 7	+ 4	− 3	+ 7
$\phi+2D-l$	+ 42	+ 6	+ 70	− 20	+ 9	− 18	− 5
$\phi-2D+l$	+ 18	...	+ 3	− 8	...	− 3	− 7
$\phi+2l-2D$	+ 1	...	...	....	+ 1	...	...
$\phi-2l+2D$	+ 2	...	+ 2				
Sums	+ 8	− 1	+ 4	− 3	+ 1	− 3	0
ϕ	−352	−14	−507	+275	−51	+155	+129

* This result was not noticed until the computations had been practically completed, when the approximate vanishing of the sums of the secondaries in all cases pointed to a general theorem.

5. At first sight it would appear that the arguments of the two terms formed, for example, from sums of products like $\sin(l+Q)\cos(2D-l)$, that is, $l+Q\pm(2D-l)$, would have coefficients in δV of the same order of magnitude. It is not so whenever the arguments in R and V contain l, unless the argument in R is of very long period (greater than a score or so of years). This fact might have been predicted from considerations of the same nature as those in the previous paragraph. For example, an argument $2l+Q+(l) = Q+3l$ would not be expected from a direct method of treatment to have a coefficient nearly so large as that of $2l+Q-(l) = Q+l$. The latter is frequently sensible, while there are no terms with the former argument.

This arises, again, from the peculiarities of the method of the variation of arbitrary constants. In the volume referred to above I have shown directly from the method that *if a term of* R *contains* il *in its argument* ($i = 1, 2, 3, \ldots$) *and* V *a term* $i'l$ ($i' = 1, 2, 3, \ldots$), *then the secondary arising from the sum of these arguments is always very small compared with that arising from the difference, unless the period of the primary is very long.* This result was proved by inserting the terms due to δl, δe, as given by the equations of variations, in δV. It enabled me to abbreviate the computations by some twenty or more per cent., since it rendered the separate computation of δe unnecessary except in a few easily recognised cases.

6. One other fact should be noted. Although the coefficient of the evection (arg. $2D-l$) is of order $m(1/13)$ compared with that of the principal elliptic term (arg. l), the numerical factors which it contains are so large that in certain functions of the lunar co-ordinates it becomes equal to or of more importance than the principal elliptic term. It is therefore never safe to neglect the former terms unless we know that the latter are quite insensible. This shows most strongly in the arguments $l+\phi$ in R for the indirect action. The terms which multiply $\delta V'$ (the portion of the Earth's longitude due to the action of the planets) are chiefly those due to the evection, while those which multiply $\delta r'$ (the planetary portion of the Earth's radius vector) are due to the evection and the principal elliptic term. Yet the former portions in several cases are larger than the latter.

7. *The differences* B − R. Each of Radau's coefficients depending on the action of Venus is to be diminished by 1/51 of its amount on account of the difference between the value for the mass of Venus adopted by him and that adopted by Newcomb and myself. Each coefficient due to Mercury is to be diminished by 7/60 of its amount for my adopted value of the mass of that planet.

The equations of variations used by Radau appear to require a correction which would increase each coefficient in the ratio 96 : 95, as I have shown in the volume previously referred to. This is confirmed by Newcomb's results, for he obtains values for the principal terms the same as those which I found in the volume quoted.

In the primary terms with arguments independent of the lunar angles, Radau omits the portions due to the term of R which depends on $r^2 \cos 2(V - T)$. These portions constitute about one-eighth of the principal portions for the indirect action, and his coefficients for this portion should therefore be diminished in the ratio 7 : 8. The corresponding correction for the direct action is smaller, and its amount depends on the particular term under consideration.

In the short period terms of the disturbing function for the indirect action of the form $l + \phi$ he omits the portions multiplied by δV. These, as stated in No. 6 above, constitute a considerable fraction of the whole. The terms *proportionately* most affected are those with this argument, but the *differences* for the terms of argument $l + \phi - (l)$ are much larger, for the latter (secondary) terms are about three times the former (primary) terms.

The discovery of the causes of divergence between the results of Radau and myself was rendered difficult owing to the fact that, although his secondary inequalities for the Venus terms of argument ϕ are given separately, and are in several cases quite large when the theorem of No. 4 above shows that they ought to be quite small, yet his final coefficients very nearly agreed with mine. As a matter of fact, I have found, from an examination of the separate portions of my results, that these corrections very nearly balance one another in the case of Venus, though they do not do so with the other planets, nor do they in the case of Venus with arguments other than ϕ.

I have compared most of Radau's individual results with mine, and find a close agreement for nearly all those portions which he has computed. For the direct action this agreement was particularly useful, since my coefficients are obtained from formulæ and methods radically different from his, though reducible to them by algebraical processes.

Two other differences should be noticed, namely, those in the terms with arguments $-D + 4T - 3V$ and $l - D + 4T - 3V$. These arise from the fact that the indirect action is a little greater and of the opposite sign to the direct action; Radau only takes the latter into account.

8. *The differences* B − N. These have been marked with the letters E_1, E_2, E_3 (explained), and U (unexplained), whenever they exceed 0″·020.

Those marked E_1 are due to the fact that Newcomb substitutes the variations of the elements in the value of V limited to the three terms

$$V = w_1 + 2e \sin l + \tfrac{5}{4} e^2 \sin 2l,$$

instead of taking the complete value; he adds that "in nearly or quite all cases we may drop terms of the second order in e."

This third term of V in reality only produces a few secondaries with coefficients greater than 0″·020, and these arise in the form $2D - 2l + \phi + (2l) = 2D + \phi$; but there are many greater than

0″·003, the adopted limit below which coefficients were dropped. The latter arise also in the forms $l+\phi-(2l)$ and $2D-l+\phi+(2l)$.

A much more extended and larger set of differences arises, however, from the neglect of the evection in the expression for V. The majority of the differences E_1 due to this cause will be found in the combinations—

$$\phi \pm (2D-l),$$
$$l+\phi+(2D-l)=2D+\phi,$$
$$2D-l+\phi-(2D-l)=\phi,$$
$$2l-2D+\phi+(2D-l)=l+\phi,$$

the last being of importance only when $2l-2D+\phi$ is of long period of the order of ten years, and the first quite rarely. Hence the final coefficients of terms with arguments ϕ, $2D+\phi$ will be chiefly affected, and a very few in those with arguments $l+\phi$, from this cause. All those differences which may be nearly or altogether explained by the neglect of this and other terms in V are marked E_1. I assume that the terms not present in Newcomb's list have not been computed.

In order to examine the differences E_2 a closer inspection is necessary. On referring to Newcomb's tables for the variations of the elements (p. 154 of his work), I find that under the argument 2D he has only the terms containing $(V-T)$ and $2(V-T)$ for Venus. There are, however, several other arguments with coefficients of the same order of magnitude as these, and the same fact appears to a smaller extent with the other planets. All differences explained by this cause have been marked E_2.

The small difference E_3 in the great long period term due to Venus scarcely needs explanation. Newcomb has included the portion (less than 0″·10) due to the mutual perturbations of Venus and the Earth, and he states that his method indicates a possible error of the order 0″·10. I exclude the former part, but the maximum error by my method should be less than 0″·05.

Of the differences U there are two in Mars, eight in Jupiter, and one in Saturn. The most important is that in Jupiter with argument J; I have compared the several portions of any value for the primary with that of Radau, and the results (after the corrections noted in No. 7 above have been made) agree for the indirect action, but I have been unable to obtain his result for the direct action from the algebraical formulæ which he gives. The causes for the remaining differences I have not been able to trace, and must leave the values of those coefficients an open question.

9. In working out the terms produced by planetary action, the valuable memoir of M. Radau has been available for comparison of results at almost every stage, and has materially assisted in the prevention of errors made during the course of a piece of work of great complexity, though not of great difficulty when once the theory had been put into final shape. The work of Professor Newcomb only appeared when I had finished the greater part of

the computations, and would in any case not have been available for detailed comparison without much labour, owing to the complete difference between his method and mine. He combines the direct and indirect actions at the earliest opportunity. I have kept them separate until each was fully completed. The comparison between his results and mine revealed one error in my work which affected the primaries due to the terms with argument ϕ in R by about 5 per cent., and a few of the small secondaries from these terms about twenty per cent.; an error in the equations of variations had almost no effect. These errors have, of course, been corrected in the results given above.

10. Like Professor Newcomb, I have also made an examination of the inequalities of the second order relative to the planetary masses, and have so far found nothing that could sensibly affect the motion of the Moon. I have found an additional portion to the term with argument $l + 3T - 10V$ of the order of $0''\cdot2$, but the period of this term is so long that it would scarcely affect the observations within the degree of accuracy at present obtainable. The motion of the node of Venus also affects the term with argument $l + 16T - 18V$ by a quantity of the same order of magnitude. The terms of the second order in the Sun's motion, as given in Newcomb's Tables of the Sun,

$$\delta V' = -0''\cdot265\cos(4M_1 - 7T_1 + 3V_1) - 0''\cdot021\sin(4M_1 - 7T_1 + 3V_1) + 3\cdot76\cos(3J_1 - 8M_1 + 4T_1) + 5\cdot18\sin(3J_1 - 8M_1 + 4T_1)$$

(where the suffix denotes the mean anomalies of the planets instead of the mean motions), or

$$+0''\cdot266\sin(1^\circ\cdot190t + 31^\circ\cdot8) \quad \text{(period 300 years)}$$
$$+6''\cdot40\sin(0^\circ\cdot202t + 231^\circ\cdot2) \quad \text{(period 1780 years)},$$

will produce inequalities with the same arguments due to indirect action having the approximate coefficients

$$-0''\cdot04 \quad \text{and} \quad -0''\cdot9$$

respectively. These results are sufficiently accurate for tabular purposes, but I shall give a more complete computation, with an examination of the whole effect of the terms of the second order.

New Haven, Conn. U.S.A.:
1907 *November* 26.

Postscript.—Since this paper was sent in, I have computed the terms due to the motion of the ecliptic and have discovered a few new inequalities containing the arguments of the planets. The most important is one with a coefficient $0''\cdot21$ due to Jupiter, and having a period of 280 years.

1908 *January* 10.

Note on the Single Equation which comprises the Theory of the Fundamental Instruments of the Observatory. By Sir Robert Ball, LL.D., F.R.S.

We may conceive a generalised astronomical instrument of which the essential parts are as follows:—

There is a fundamental axis, which we shall distinguish as axis I. It is capable of rotation in fixed bearings, and to it is attached an index which points to a reading R on a fixed graduated circle A. Axis I passes through the centre of A, and is normal to the plane of A.

Axis II is capable of rotation in bearings fixed on axis I. A second graduated circle B is attached to axis II which passes through the centre of B, and is normal to its plane. The reading of B is R′, as shown by an index rigidly attached to I. It may be observed that an index parallel to the intersections of the planes of A and B will serve for reading both circles, and the geometry of the question is simplified by employing this index.

It is necessary to distinguish between the two poles on the celestial sphere which are defined by the plane of a graduated circle. From one of these poles the graduation would appear to increase clockwise. From the other pole the graduation would appear to increase anti-clockwise. It is the latter pole which we shall here employ. The angle between I and II is the angle ($\not>$ 180°) between the poles of A and B. We shall express it by $90^\circ - q$.

The telescope is rigidly attached to axis II, and when the optical axis of the telescope is directed to a star, the arc ($\not>$ 180°) from that star to the pole of B is also a constant of the instrument. We shall denote it by $90^\circ + r$.

The semiplane through axis II and that half of the telescope which contains the objective, cuts B at the graduation we shall term Δ.

Let R_1, R_1' and R_2, R_2' be the readings of the instrument when directed successively to stars S_1 and S_2, with celestial co-ordinates α_1, δ_1 and α_2, δ_2. These co-ordinates may be altitude and azimuth, or right ascension and declination, or latitude and longitude, or any system in which the fundamental circles are rectangular. Then the equation we desire is obtained by equating two different

expressions for the cosine of the arc between S_1 and S_2, and it is as follows:—

$$\begin{aligned}
&\sin\delta_1\sin\delta_2+\cos\delta_1\cos\delta_2\cos(\alpha_1-\alpha_2)\\
&=+\sin^2 q\sin^2 r\\
&\quad+\cos^2 q\sin^2 r\cos(R_1-R_2)\\
&\quad+\cos^2 q\cos^2 r\sin(R_1'-\Delta)\sin(R_2'-\Delta)\\
&\quad+\cos^2 r\cos(R_1-R_2)\cos(R_1'-\Delta)\cos(R_2'-\Delta) \qquad \text{(i)}\\
&\quad+\sin^2 q\cos^2 r\cos(R_1-R_2)\sin(R_1'-\Delta)\sin(R_2'-\Delta)\\
&\quad+\cos^2 r\sin q\sin(R_1-R_2)\sin(R_1'-R_2')\\
&\quad+\cos q\sin r\cos r\sin(R_1-R_2)\{\cos(R_2'-\Delta)-\cos(R_1'-\Delta)\}\\
&\quad+\sin q\cos q\sin r\cos r\{\cos(R_1-R_2)-1\}\{\sin(R_1'-\Delta)\\
&\qquad\qquad+\sin(R_2'-\Delta)\}
\end{aligned}$$

By assigning suitable values to q and r, this formula can be made to apply to the following astronomical instruments:—the altazimuth, the meridian circle, the prime vertical instrument, the equatorial, and the almucantar. For the meridian circle q and r should be each as near zero as possible, and for the almucantar q is the latitude and r quite arbitrary. The following general proof will show that the complete theory of each of the instruments named must be included in this one formula.

From any such instrument we demand no more than that the two readings R and R′ obtained by directing the instrument to any particular star shall enable us to calculate the co-ordinates α, δ of that star free from all instrumental errors.

Let S_1, S_2, S_3 be three standard stars of which the co-ordinates are known, and let each of these stars be observed with the generalised instrument with results R_1, R_1'; R_2, R_2'; R_3, R_3' respectively. Substituting for each of the three pairs (S_1 S_2), (S_2 S_3), (S_3 S_1) in the typical formula (i), we obtain three independent equations. From these equations, q, r, and Δ can be found. Nor will there be any indefiniteness in the solution, for in each case we may regard these quantities as approximately known, so that to obtain the accurate values of q, r, and Δ we shall have to solve only linear equations. We may thus regard (i) as an equation connecting α_1 δ_1, α_2 δ_2, R_1, R_1', R_2, R_2', and known quantities.

Let S be the star whose co-ordinates α, δ are sought. We write the equation (i) for the pair (S S_1), and substitute their numerical values for α_1, δ_1, R_1, R_1'. We thus have an equation connecting the co-ordinates α, δ of *any* star with its corresponding R, R′ and known numerical quantities. When we substitute for R and R′ the values observed for S, the formula reduces to a numerical relation between the α and δ of the particular star S. From the pair (S S_2) we find in like manner another quite independent numerical equation involving α, δ. As, however, the equations are not generally sufficient to determine α, δ without indefiniteness, we obtain a third equation from (S S_3). This

equation is not independent of the others, but if we make $x = \sin\delta$, $y = \cos\delta\cos\alpha$, $z = \cos\delta\sin\alpha$, we shall obtain three linear equations in x, y, z which can be solved, and thus α and δ are found without any ambiguity whatever.

All the ordinary formulæ used in connection with the different instruments named can be deduced as particular cases of the general equation (i).

In general, there are no real values of R and R' when the instrument is directed to the pole of circle A. In such a case R would have to be set on one of the imaginary circular points at infinity.

2nd January 1908.

The Perturbations of Halley's Comet in the Past. Second Paper. The Apparition of 1222. By P. H. Cowell, M.A., F.R.S., and A. C. D. Crommelin, B.A.

In the first paper of this series we identified the comet of October 1301 with Halley's, and found the value 44″·858 for the mean daily motion at that epoch We have now completed (with the aid of Mr. F. R. Cripps) the calculation of the perturbations by Jupiter and Saturn for the preceding revolution. As a first approximation, Hind's date (mid-July 1223) was assumed for the preceding perihelion passage, and on this assumption the results were as follows :—

Planet.	Limit of u.	dn.	$d\varpi$.	$d\zeta$.
Jupiter	0°– 90°	−″·2261	−357″	−6550″
,,	90–270	−·0823	− 35	+6001
,,	270–360	+·5624	−183	− 353
Saturn	0– 90	−·0619	+ 12	−1784
,,	90–270	+·0636	− 39	+1602
,,	270–360	−·2411	+ 15	+ 41
Sum	...	+·0146	−587	−1043

Hence mean motion in $1223 = 44''\cdot858 - 0''\cdot015 = 44''\cdot843$

and calculated period $= \dfrac{1296000'' + 1043''}{44''\cdot843} = 28924$ days.

This indicated 1222 August 15 as the date of the preceding perihelion passage, or 11 months earlier than Hind's date. This is too large a discordance to be possible, so Hind's identification of

the comet of July 1223 with Halley's is erroneous. There was, however, a much more remarkable comet which appeared at the exact epoch indicated by the calculation; and examination shows that the greater part of the statements made concerning it by contemporary writers are quite consistent with its being Halley's, so that the identity is placed beyond reasonable doubt.

The error of the first assumption is so great that it is necessary to recompute the perturbations; this has as yet only been done approximately, the resulting date being 1 day earlier, or August 14; the small difference between this and the preceding result is an illustration of the general proposition that the date of the more remote perihelion passage need only be very roughly known in order to obtain the periodic time correctly.

Pingré's description of this comet is as follows; "En Automne, c'est à-dire aux mois d'Août et de Septembre, on vit une étoile de première grandeur, fort rouge, et accompagnée d'une grande queue qu'elle étendait vers le haut du ciel, en forme d'un cone extrêmement aigu: elle paraissait fort près de la Terre: on l'observa (d'abord) vers le lieu où le soleil se couche au mois de Décembre. Le 15 Août, jour de la première apparition de cette comète (peut-être à Milan) la Lune fut comme morte; elle n'avait plus d'éclat, et elle joignit la comète. On vit ensuite cette comète à l'occident, et même vers le nord, avant la fin du mois d'Août. En Chine on l'observa le 10 Septembre, entre la constellation Kang (les pieds de la Vierge, = ι, κ, λ, θ Virginis), Arcturus et la chevelure de Bérénice: elle disparut le 8 Octobre. Le Père de Mailla dit que les Chinois virent une comète à l'ouest en 1222, à la première lune: c'est sans doute une erreur du copiste, il faut lire à la huitième lune."

The words in parentheses are not part of the original documents, and Pingré's interpolation "d'abord" appears to be erroneous; it is the place where the comet was last seen, not first seen. When the comet extended its tail towards the zenith it must have been nearly vertically above the Sun.

The following is the description in Williams' *Chinese Observations of Comets*:—"In the reign of Ning Tsung, the 15th year of the epoch Kea Ting, the 8th moon, day Kea Woo (= 1222 September 15), a comet appeared in Yew She Te (η, τ, υ Boötis). Its luminous envelope was 30 cubits long. Its body was small, like the planet Jupiter. It was seen for two months. It passed through Te (α, β, γ, ι Libræ), Fang (β, δ, π Scorpii), and Sing (Antares, etc.), and then disappeared." (Williams, in this and other places, erroneously gives ν Boötis instead of υ as one of the components of Yew She Te.)

Most of the above statements are satisfied by the following orbit, in which the longitude of perihelion is that actually derived from the perturbations, and the date of perihelion passage is 8 days later than that indicated above; a perihelion distance somewhat greater than the present value has been used. Hind found that this was also indicated in the return of 1066.

Elements in old System of Elements.

$T = 1222.$ August 22.
$\varpi = 296°{\cdot}5$
$\Omega = 42$ } Equinox of 1222.
$i = 16{\cdot}5$
$q = 0{\cdot}67$
Motion retrograde.

We have had to assume that the date when the comet was first seen in China (in the region η, τ, υ Boötis) should be one lunar month earlier than that given, or August 17 instead of September 15. Our reasons are: (1) it is distinctly stated that the comet was seen for two months in China, the final date being October 8, the position then being in Scorpio; (2) we cannot make this a month later, for in November Scorpio was invisible; (3) further, we are distinctly told that the comet was a splendid object in Europe in mid-August, so that it is most improbable that the Chinese should have missed it till a month later; (4) further, its geocentric motion was at that time so rapid that had it been in Boötes in mid-September it would have been a morning-star on August 15, which it apparently was not.

Another difficulty that we have to surmount is the phrase "La Lune joignit la comète." It was doubtless this phrase that led Pingré to the conclusion that the motion was from south to north, which is in opposition to the Chinese account. Considering the vagueness of all European cometary observations at that epoch, we may look on the phrase as sufficiently satisfied by the fact that the Moon (at her first quarter) and comet were symmetrically placed above the western horizon an hour after sunset, though separated by some 70°. There is one more point to notice. The comet is said to have been near Antares when last seen (October 8). Our chart shows that it was then in Libra, but the head was too near the Sun to be seen; the tail would point to Antares, and, considering its length, it may well have reached that star. It will be seen that we have given the European observations most weight for dates, the Chinese most weight for the track among the stars. This conclusion is in agreement with that reached in the case of other comets, including that of 1301. We have thus shown that the observations are satisfied with no further alteration than they would in any case require to render them self-consistent, and the exact coincidence of date leaves scarcely any room for doubt as to the identity. Pingré was prevented from making this identification by not knowing the wide range through which the period of Halley can alter, and doubtless his inference that the comet moved at first from south to north put Hind off the track.

It is of interest to point out that the revolution 1222–1301 is the longest on record, being 79 years 2 months. It just exceeds that from 1066 to 1145, which was hitherto looked on as the longest. The round now being accomplished is the shortest on

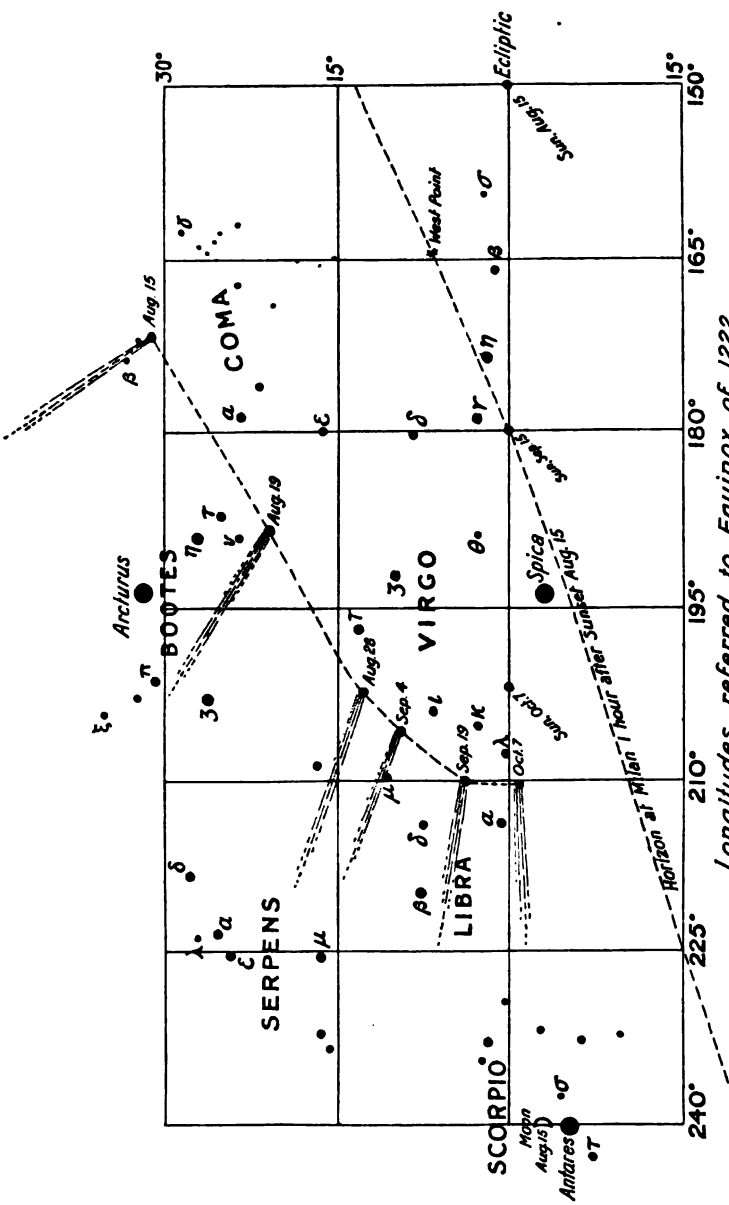


CHART SHOWING THE PATH OF HALLEY'S COMET. 1222.

Longitudes referred to Equinox of 1222.

record, lasting only 74 years 5½ months, a difference of nearly 5 years.

Though the computations for the revolutions 1066–1145, 1145–1222, are still incomplete, enough has been done to make it extremely probable that Hind's identification is correct in each case, though his elements of the apparition of 1066 clearly need modification. They differ more widely from the present elements than perturbations will account for. However, the positions are so vaguely recorded that large alterations in them are possible.

[*Note added January* 28.—Since writing this paper we have done further work on the revolution 1145–1222, the result of which tends to the conclusion that the date of perihelion passage in 1222 was in September rather than August. We are therefore somewhat doubtful whether the date September 15 for the comet being in Yew She Te may not be right after all; if so, the European dates require alteration. We shall return to this point in a subsequent paper. It does not weaken the conclusion as to the identity of the comet of 1222 with Halley's.]

Tables have now been constructed giving the values of the definite integrals for Jupiter and Saturn for the outer half of the orbit. To use them, we have first to find the number of days from perihelion to the extremities of the minor axis, which is done by multiplying the period in days by the number whose log is 8·9825. Hence we find Julian days of passing the points $u = 90^\circ$, $u = 270^\circ$, and then find g' for Jupiter and Saturn by the formulæ—

$$g'_{\mathcal{U}} = \frac{(\text{Julian Day} - 2390645)\ 360^\circ}{4332{\cdot}85},$$

$$g'_{\mathfrak{h}} = \frac{(\text{Julian Day} - 2388249)\ 360^\circ}{10760{\cdot}7}.$$

Then the tables give the values of $\int dn$, $\int d\varpi$, $\int d\zeta$, corresponding to the values of g' at first and third quadrant. The algebraical sum of the quantities has to be taken and added to that deduced from the mechanical quadratures.

The numbers from the tables should be modified as follows:—

Multiply $\int dn$ by the number whose log is $\frac{5}{2}\,[1{\cdot}2585 - \log a]$, the other columns by the number whose log is $[1{\cdot}2585 - \log a]$, where a is given its value for the revolution under consideration. This does not apply to the constant part of $\int d\varpi$, $\int d\zeta$, which is independent of a.

Table of the definite integrals $\int dn$, $\int d\varpi$, $\int d\zeta$, *for Jupiter and Saturn between the values* 90° *and* 270° *of u, the comet's eccentric anomaly.*

	Jupiter, $u = 90°$.			Jupiter, $u = 270°$.			Saturn, $u = 90°$.			Saturn, $u = 270°$.		
g'	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$
°	″	″	″	″	″	″	″	″	″	″	″	″
0	+·3320	−306	+5434	+·0932	+ 23	−125	+·0808	+38	+1143	−·0284	+77	−18
10	+·3476	−253	+5955	+·0986	+ 94	−121	+·0743	+51	+ 959	−·0382	+77	−12
20	+·3595	−191	+6237	+·0959	+161	−113	+·0670	+63	+ 742	−·0476	+75	− 7
30	+·3616	−124	+6306	+·0852	+222	−104	+·0588	+72	+ 501	−·0567	+70	0
40	+·3559	− 53	+6148	+·0673	+275	− 85	+·0498	+78	+ 246	−·0650	+62	+ 6
50	+·3426	+ 19	+5775	+·0426	+316	− 67	+·0406	+81	− 14	−·0723	+52	+12
60	+·3233	+ 90	+5224	+·0133	+347	− 45	+·0319	+82	− 271	−·0783	+40	+17
70	+·2976	+156	+4492	−·0210	+367	− 24	+·0238	+80	− 510	−·0828	+27	+22
80	+·2681	+216	+3649	−·0573	+374	− 1	+·0160	+75	− 729	−·0856	+13	+26
90	+·2351	+268	+2704	−·0953	+368	+ 20	+·0092	+68	− 924	−·0868	0	+30
100	+·2001	+310	+1700	−·1331	+352	+ 41	+·0033	+60	−1092	−·0867	−13	+32
110	+·1638	+343	+ 656	−·1704	+326	+ 59	−·0014	+50	−1224	−·0850	−25	+34
120	+·1271	+367	− 398	−·2057	+291	+ 77	−·0048	+39	−1321	−·0820	−37	+34
130	+·0919	+380	−1406	−·2379	+250	+ 98	−·0070	+27	−1382	−·0779	−48	+34
140	+·0581	+382	−2386	−·2667	+201	+105	−·0080	+14	−1409	−·0726	−57	+33
150	+·0267	+374	−3290	−·2913	+148	+116	−·0079	+ 1	−1404	−·0666	−65	+31
160	−·0016	+356	−4106	−·3116	+ 91	+122	−·0066	−11	−1365	−·0600	−71	+28
170	−·0261	+329	−4815	−·3267	+ 32	+128	−·0040	−23	−1280	−·0529	−75	+25
180	−·0462	+294	−5396	−·3364	− 26	+129	−·0003	−35	−1179	−·0451	−78	+21
190	−·0619	+252	−5852	−·3409	− 84	+126	+·0044	−46	−1042	−·0374	−78	+16
200	−·0724	+204	−6159	−·3400	−141	+121	+·0100	−56	− 879	−·0298	−77	+11
210	−·0774	+149	−6308	−·3334	−195	+113	+·0165	−64	− 692	−·0224	−73	+ 5
220	−·0769	+ 89	−6299	−·3213	−244	+103	+·0236	−71	− 485	−·0152	−68	0
230	−·0710	+ 27	−6155	−·3046	−287	+ 88	+·0312	−76	− 267	−·0085	−61	− 5
240	−·0583	− 36	−5776	−·2817	−321	+ 70	+·0394	−80	− 29	−·0026	−53	−11
250	−·0401	− 99	−5258	−·2545	−349	+ 51	+·0478	−81	+ 207	+·0023	−43	−17
260	−·0164	−161	−4582	−·2234	−366	+ 31	+·0558	−80	+ 442	+·0064	−31	−22
270	+·0120	−220	−3765	−·1890	−375	+ 9	+·0638	−76	+ 668	+·0094	−19	−26
280	+·0453	−273	−2817	−·1517	−371	− 13	+·0710	−70	+ 877	+·0110	− 5	−29
290	+·0819	−319	−1766	−·1131	−356	− 36	+·0773	−61	+1052	+·0110	+ 9	−32
300	+·1210	−355	− 646	−·0740	−329	− 57	+·0827	−51	+1211	+·0097	+23	−33
310	+·1617	−380	+ 523	−·0359	−291	− 78	+·0865	−39	+1322	+·0067	+37	−33
320	+·2023	−392	+1691	+·0003	−242	− 94	+·0891	−25	+1391	+·0023	+49	−33
330	+·2411	−390	+2740	+·0323	−184	−109	+·0897	−10	+1406	−·0036	+59	−31
340	+·2766	−376	+3833	+·0594	−118	−119	+·0885	+ 6	+1370	−·0109	+68	−28
350	+·3074	−347	+4722	+·0800	− 49	−124	+·0855	+23	+1260	−·0196	+74	−24

Constant part of $\int d\varpi$ (to be added to tabular quantities) Jupiter − 112″, Saturn − 24″, Total − 136″.

Constant part of $\int d\zeta$ (to be added to tabular quantities) Jupiter + 2700″, Saturn + 765″, Total + 3465″.

As an example of the use of these tables we may deduce the values already obtained for the revolution 1835–1910 (*M.N.*, vol. lxvii. pp. 410, 411, 519, 520). For Jupiter the values of g' at $u=90°$, $u=270°$, are 296°·44, 322°·24, for Saturn 199°·52, 214°·74.

Hence we obtain—

		$\int dn$	$\int d\varpi$	$\int d\zeta$
4	90°	+″·1068	−343″	−1056″
,,	270°	+·0074	−229	− 97
2	90°	+·0097	− 55	− 887
,,	270°	−·0190	− 71	+ 3
Sum	4	+·1142	−572	−1153
,,	2	−·0093	−126	− 884

Now log a for this revolution = 1·2480, whence we have to multiply $\int dn$ by number whose log is ·0262, $\int d\varpi$, $\int d\zeta$ by number whose log is ·0105.

Performing the multiplication and adding the constants we obtain—

		$\int dn$	$\int d\varpi$	$\int d\zeta$
	4	+″·1213	· 698″	+1519″
	2	−·0099	−153	− 141
Results previously obtained	4	+·1219	−698	+1532
	2	−·0101	−153	− 142

A more complete investigation of the average motion of the comet's perihelion has been made, using the results of all the revolutions for which the perturbations have now been computed. It appears that the perihelion has a direct motion in longitude at nearly the same rate as the node, so that the periods of the various planets for co-ordinates x' y' may be taken the same as those given for z', p. 112. The value of ϖ for 1910 should be 305°·64, not 303°·64 as printed on p. 112.

Photographs of Comet d 1907 (*Daniel*). By Dr. Max Wolf. (Plate 6.)

Comet Daniel has been photographed at the Observatory, Königstuhl, Heidelberg, on the following nights, with the following instruments:—

Date.	1¼-in.	6-in. (*a*)	6-in. (*b*)	16-in. (*a*)	16-in. (*b*)	28-in.	Total Plates.
1907							
July 21	...	...	...	1	1	...	2
22	1	1	1	1	1	...	5
Aug. 2	...	...	...	...	...	2	2
4	1	...	...	1	1	4	7
7	1	...	...	1	1	2	5
8	...	...	...	1	1	3	5
9	...	...	...	1	1	4	6
11	...	...	...	1	1	2	4
14	1	...	...	...	...	2	3
27	...	...	...	1	1	...	2
10 *n.*							41

Six of these plates were taken by Mr. Scheifele, 12 by Dr. Kopff, and 23 by myself; 9 are stereos.

Besides these photographs I have made six drawings with the 28-in. reflector, viz. on August 4, 7, 8, 9, 11, and 14.

The most striking result appears to me the difference between the visual and the photograph appearances. All the drawings show a minimum of luminous radiation in or around the axis of the tail in the region turned away from the Sun. All the photographs show the brightest tails near the axis, exactly in the parts where visually there was a minimum of brightness. This very curious difference, which was remarked several years before in other comets photographed here, seems to be placed beyond doubt by the photographs taken with the reflector.

Besides this, it is very interesting, and perhaps in opposition with accepted theories, that the short arms of the tails furthest outside the axis are nearly all curved.

The following eight specimens are presented to the Society:—

						h	m				m
1.	Plate	D144,	Reflector,	1907	Aug.	2;	14	44·6	Königstuhl	M.T., exposure	10
2.	,,	D147,	,,	,,	,,	4;	14	35·1	,,	,, ,,	15
3.	,,	D152,	,,	,,	,,	7;	14	52·1	,,	,, ,,	15
4.	,,	D155,	,,	,,	,,	8;	14	42·2	,,	,, ,,	25
5.	,,	D161,	,,	,,	,,	9;	14	59·6	,,	,, ,,	12
6.	,,	D167,	,,	,,	,,	11;	15	5·2	,,	,, ,,	15
7.	,,	D173,	,,	,,	,,	14;	15	8·2	,,	,, ,,	12
8.	,,	A4850,	1¼-in. lens,	,,	,,	14;	15	0·2	,,	,, ,,	20

1907 Aug 14 d. 15 h. 0·2 m.

1¼-in. Lens.

1907 Aug. 14 d. 15 h. 8·2 m.

28-in. Reflector.

The lantern slides are a little enlarged from the original negatives. On the slides from the reflector-plates 1 degree equals 86 mm., and on the slide of the 1¼-in. lens 1 degree equals 4 mm.*

It is interesting to compare photographs 7 and 8, which were taken at the same time, but with very different optical means—28-in. and 1¼-in.

The tail on No. 8 is somewhat over 12 degrees long, while the small field of the reflector gives the head and its neighbourhood only. The bright star near the head of the comet on No. 8 is γ Geminorum.

* In the reproduction on Plate 6 one degree on the reflector plate = 79 mm., and one degree on the 1¼-in. lens plate = 3·7 mm.

Astrophysical Observatory,
Heidelberg (Königstuhl),
1907 *December* 10.

On an improved method of illuminating the field in a Transit Instrument, and its effect on the discordance in reversed positions of the instrument. By Sir W. H. M. Christie, K.C.B., F.R.S., and H. A. H. Christie, B.A. (Plates 7, 8.)

It is known that with a reversible transit instrument observations made in the two positions of the instrument generally show discordant results for clock error for stars reduced with the same collimation error for the two positions. This discordance has been attributed to what is called lateral flexure, and a correction depending on cos Z.D. has sometimes been applied, but at Greenwich we never felt satisfied that there was any real ground for this assumption. We have suspected sometimes that it was due to looseness in the mounting of the object-glass, and in the Paris-Greenwich longitude determination of 1888 this was found to be the case with both the portable transits. But though this was remedied by an improvement in the mounting of the object-glass, this puzzling discordance still remained in subsequent longitude determinations. Whilst there is in each series of observations a general tendency in one direction, there are considerable variations in the discordance between the mean clock errors for each night in the two positions of the instrument, as will be seen from Tables I., II., III., IV., V., VII., and VIII.

Mr. Hollis, on thinking over the matter in connection with the observations for longitude of Killorglin in 1898 and of Paris in 1902, had come to the conclusion that to obtain the correct clock-error from the mean of results in reversed positions it was necessary to keep the focus of the eyepiece absolutely fixed, and that to secure this it was expedient to keep the diagonal eye-tube which was used for the observations, and was liable to sag, rigidly attached to the tube of the instrument. In the Paris longitude

observations in 1902 special attention was paid to this, the eyepiece being adjusted to focus at the beginning of the evening, and not altered afterwards. The discordance, however, was not altogether got rid of, and on some nights it exceeded $0^s.2$ or $0^s.3$. There was the further difficulty that, as will presently be explained, the observer's eye is liable to change of focus when the bright star comes into the field.

In 1906 and 1907 Mr. Harold Christie was working with two of the portable transits (B and C) previously used in the Paris longitude determinations of 1888 and 1902, with a view to other longitude determinations, and the large discordances in his results for the two positions again called attention to the question. He noticed that if the eyepiece was adjusted for distinct vision of the wires in an illuminated field a fresh adjustment was required when the star appeared, the eyepiece having to be pulled out further from the object-glass owing to change in the focus of the eye, the wires being, by an unconscious mental process, naturally referred to a finite distance (say 2 or 3 feet), and the star, when it came in, to an infinite distance. This change of focus still took place when the other eye was covered up. He further noticed (on March 1, 1907) that while adjusting the eyepiece for focus on a slow-moving polar star the star seemed to cross the wire from one side to the other as the eyepiece was moved in or out, the apparent movement of the wire being as much as 20″. A similar effect had been previously noticed on the meridian mark (a bright point) seen in an illuminated field. It was at first thought that this might be due to the wires not being exactly in the focus of the object-glass, but readjustment of the focus failed to get rid of the effect. On trying the same experiment on Polaris by daylight next day it was found that no apparent movement of the wires could be got by movement of the eyepiece in or out. This indicated that the effect was due to *illumination of the field.* It was further noted that the movement was less apparent with a strong illumination than with a reduced light. It may here be explained that the illumination of the field is given by a gilt annular reflector in the transit axis, the inclination of which can be varied from a minimum of 45° to a greater angle with the axis so as to reduce the light.

Afterwards observations of stars were made with the eyepiece inside the focus, at the best focus, and outside focus. The results were that the discordance W—E was positive and large ($+1^s.5$) with the eyepiece far in, and negative (about $-0^s.6$ or $-0^s.7$) with the eyepiece far out.

It is to be observed that the mean of the results W and E is sensibly unaffected by the discordance, as will be seen from the last column.

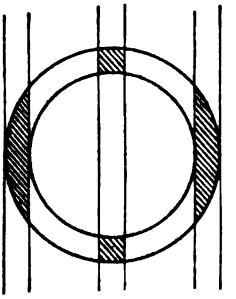
Transit C.—Annular Illumination Experiments with Eyepiece focus.
By Mr. H. Christie.

Date.	Position of Eyepiece.	No. of Stars. W. E.	W—E.	Mean. Clock slow.
1907.			s	s
March 19	Good focus	2, 2	+0·15	+59·48
	Inside focus	1, 2	+1·45	59·66
	Outside focus	2, 2	−0·73	59·45
March 20	Good focus	3, 2	+0·58	59·92
	Inside focus	2, 2	+1·40	59·91
	Outside focus	3, 3	−0·55	59·96

N.B.—The "good focus" position was the ordinary one for star observations; the eyepiece was displaced from this position until observation was only just possible.

These experiments seemed to clearly establish the connection between the discordance and eyepiece focus, and they further suggested that the annular reflector was in fault.

It is to be noted that the wires at night are not seen by their own light, but as dark shadows cutting off the light of the illuminated field, and thus the illuminating surface, which casts the shadow of the wire, has an important influence on its appearance with the annular reflector. When the wire is out of focus the



shadow of each point of the wire is an annulus, and the superposition of these will give two dark lines parallel to the wire with a comparatively slight haze between, the two sides of the annulus being more effective in forming the shadow images than the top and bottom, as will be seen from the accompanying figure, which shows the portions of the annulus at the sides and at the top and bottom respectively, which are effective in forming the corresponding shadow images of a vertical wire. Thus two fairly clear and separated images are formed one on each side of the true position of the wire.* Now if one side of the annulus is more distinctly

* On examination of the wires by daylight illumination from the object-glass, it was found that when the eyepiece was out of focus, instead of two images a single image was seen, with bands on each side.

lighted than the other, and the whole illumination of the field is not very bright, the eye will lose one of the two images of the wire, and only see a single image, which will appear fairly clear, and will be displaced by an amount depending on the diameter of the out-of-focus annular ring.

On investigation this proved to be the case. When the annular reflector was at an angle of 45° to the axis (giving a maximum of illumination) it was fairly evenly illuminated, but when the inclination was increased, in order to reduce the light to a convenient amount for star-observing, one side of the gilt reflector became considerably brighter than the other. This was the actual position always used in practice, as the full illumination was too strong for observing stars. Also, it was found that the side of the reflector which gave most light was the one which would, by the above theory, cause the clock error micr. W to be greater than the clock error micr. E, as the star-observations showed to be the case. In order further to test this, Mr. H. Christie made observations on April 5, 6, and 10 with the reflector kept at 45° to the axis (a rheostat being used to reduce the light from the electric lamp), and obtained better results.

It had long been felt that illumination by means of an annular reflector in the transit axis was not satisfactory, as the image formed at the eye-ring from the illumination of the field was an annulus outside the circle formed by a star, and there was risk of the eye failing to receive the whole of the rays on both sides coming from the reflector.

Illumination by rays coming centrally within the cone of rays from the object-glass would be preferable in this respect, but there are objections to the way in which this has usually been carried out. In 1870, Sir G. B. Airy successfully applied central illumination to the Water Telescope which he planned, mounting a piece of looking-glass at an angle of 45° in front of the object-glass to receive the light from a gas flame, without any condensing lens. In this way the light was well diffused over the field, the gas flame as well as the mirror being well out of focus at the plane of the wires.

But in other forms which have been used since, a condensing lens with a small electric lamp in its focus has been introduced, which, in combination with a small reflecting prism or other specular reflector fixed in front of the object-glass, forms a magnified image of the source of light in the plane of the wires, exaggerating its defects, and making it difficult to get tolerable uniformity of illumination of the field and sharp definition of the wires.

In planning the New Altazimuth, a sketch arrangement was proposed in 1896 January for central illumination by light reflected from the axis lamp inside the telescope tube to a matt surface gilt reflector attached centrally to the inside surface of the object-glass; but, owing to practical difficulties, this plan was not carried out, and the ordinary annular reflector in the axis was at first

used. In 1897 September central illuminations by the usual method of small reflecting prism ($\frac{3}{8}$-inch square) cemented on the outside of the object-glass and collimating lens with a small electric lamp in its focus was arranged for, and was subsequently brought into use. There was difficulty, however, with the illumination, owing to the filaments of the lamp forming an enlarged image in the focus, and the wires were badly defined, owing, as afterwards appeared, to diffraction effects from the small aperture of the illuminating pencil. Further, with such a small pencil, dust on the field lens of the eyepiece would be sensibly in focus with the wires.

Attention being again called to the question, it seemed that too much importance had been given to simply getting enough light without sufficient consideration of the optical conditions required in the arrangement for central illumination of the field. These conditions are:—

(1) That a uniform illumination shall be secured over the field, without such defects as are due to the source of light coming to a focus near the plane of the wires.

(2) That the illuminating pencil should have a sufficient aperture to minimise diffraction effects on the wires, giving rise to shadow-bands.

The condition (1) can be secured by substituting a matt surface, such as opal glass (finely ground) or plaster of Paris, for the specular reflector, the condensing lens with electric lamp in its focus being retained. The parallel rays which fall on the matt surface from each point of the source of light in the focus of the condensing lens are scattered uniformly in directions slightly inclined to the axis of the telescope, and after passing through the object-glass, converge respectively to each point of the field, so that each point is illuminated by a pencil uniformly distributed over the whole of the opal reflecting surface.

As regards condition (2), it is to be borne in mind that in transit observations sharp definition of the wires is as important as sharp definition of stars; and that, as the wires are seen as shadows thrown by the illuminated field, the aperture of the illuminating pencil determines the sharpness of the wires. With the small pencils which have commonly been used even on large instruments, there are necessarily large diffraction effects which are serious with the relatively very high powers employed. At the same time it is to be noted that comparatively little loss of light, and practically no loss of definition for a star, is entailed by cutting out a circle of, say, one-fourth of the aperture at the centre of the object-glass.*

After some preliminary experiments with central illumination for the portable transit, the following arrangement was adopted on 1907 May 8 to secure the two conditions stated above. An elliptical finely ground opal glass was used as the reflector, and mounted in a brass tube which was attached to the outer surface of the object-glass by shellac. The illuminating source was a small

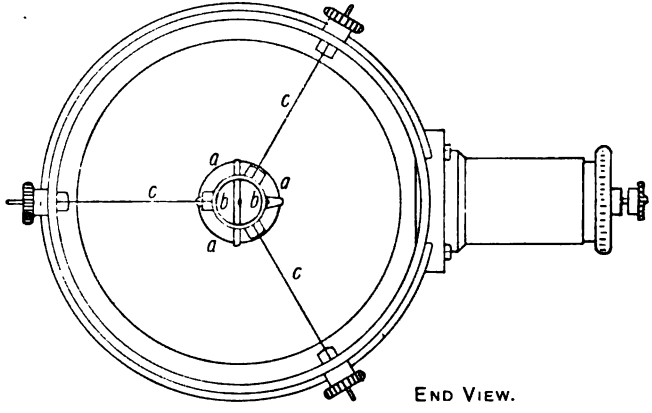
* The loss of light would in this case only be one-sixteenth.

electric lamp in the focus of a condensing lens, throwing a parallel beam of light on the opal reflector. The brightness of the electric lamp was controlled by a rheostat, and a very satisfactory illumination of the field, with the wires sharply defined, was obtained. The diameter of the pencil of rays from the reflector was $\frac{3}{4}$-inch, which was adopted as suitable for the 3-inch object-glass. The results given for Transit C, in Table IX.—central illumination —were all obtained with this arrangement. It was found, however, that the shellac was not a secure fastening for such an instrument as a portable transit, as any jar was apt to dislodge the arrangement, so a more secure form of support was adopted later on, and applied first to transit B in July 1907. This consisted of an opal reflector as before, fixed to the dew cap by three pieces of watch-spring, and is shown in Plate 7. This arrangement was found satisfactory, and so was attached to the other instruments (transits B and D) as well. No trouble has been caused by diffraction effects. The observations obtained are, on the whole, satisfactory, although two nights (September 10 and 11) with transit B give discordances W—E of over $0^s{\cdot}1$. There was probably some other cause at work on these nights, as the collimation determined from the nadir gave a sudden jump of about $2''$ in the *opposite* direction to the error shown by the stars (Tables VI., IX., and X.).

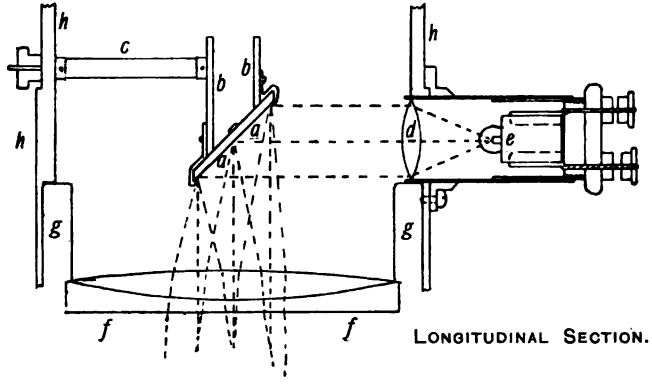
As the application of the improved central illumination to the small transits proved so successful, it was applied to the New Altazimuth on 1907 June 5, the mode of attaching the reflector to the object-glass being slightly modified from the original plan with shellac cement. In order to secure a firmer attachment, a plain glass plate, worked slightly concave on the outside to fit the curvature of the outer surface of the object-glass, was burnished into the end of the tube carrying the opal reflector, and cemented on to the object-glass with Canada balsam. In this way the obstruction of light due to the attachment is reduced to a minimum, viz. the thickness of the brass tube. The general arrangement is shown in Plate 8. The aperture of the illuminating pencil adopted for the 8-inch object-glass was $1\frac{1}{2}$ inches, which was found to give greatly improved definition of the wires, which had been very unsatisfactory with the $\frac{5}{8}$-inch pencil formerly in use. At the same time the troublesome markings previously seen in the illuminated field were got rid of, and a satisfactory uniform illumination substituted.

It is proposed to apply a similar method of illumination to the Greenwich transit-circle as soon as the necessary electrical connections can be made.

The tables show the difference between the mean clock error determined with the instrument micrometer West and micrometer East respectively. They are corrected for clock rate and reduced with a collimation error determined from nadir observations only. The weights given depend on the number of reversals of the instrument, a weight of 1 corresponding to a single reversal, while a weight of 2 corresponds to two or more.



END VIEW.

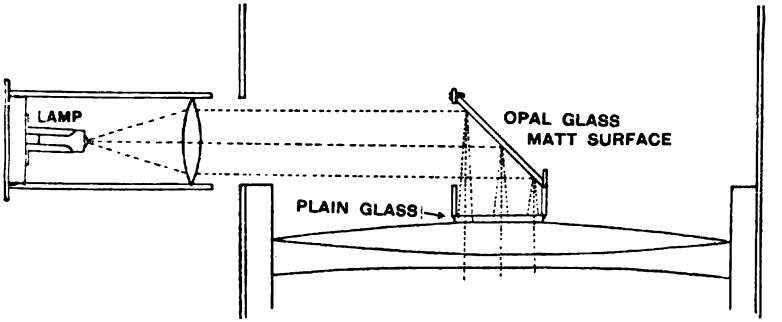


LONGITUDINAL SECTION.

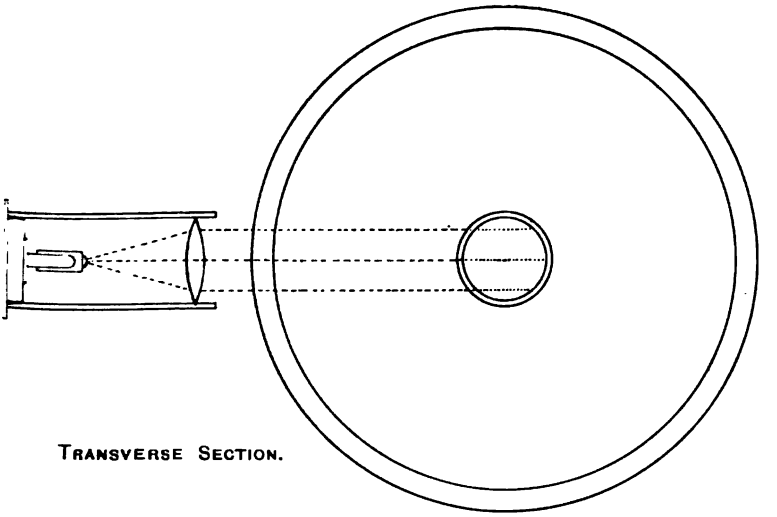
PORTABLE TRANSIT—ILLUMINATION OF FIELD.

a, Matt surface reflector (Opal glass finely ground).
b, Brass tube to carry reflector.
c, Supports for tube, made of watch-spring.
d, Condensing lens.
e, Small electric lamp.
f, Object-glass of telescope.
g, Object-glass cell.
h, Dew cap.

ALTAZIMUTH—ILLUMINATION OF FIELD.



LONGITUDINAL SECTION.



TRANSVERSE SECTION.

TABLE III.—*continued.*

Date. 1902.	W—E. Means.	No. of Stars. W.	E.	Wt.
Paris.				
Apr. 28	+0·088	13,	20	2
May 1	+0·059	20,	17	2
3	+0·024	18,	17	2
Mean	+0·057			
Greenwich.	s			
Sept. 21	+0·015	3,	2	1
22	+0·099	18,	19	2
23	−0·004	8,	5	1
24	+0·158	19,	18	2
25	+0·087	8,	10	1
26	+0·127	17,	16	2
Mean	+0·096			

Date. 1902.	W—E. Means.	No. of Stars. W.	E.	Wt.
Paris.				
Oct. 6	+0·068	10,	7	1
7	+0·127	12,	16	2
8	+0·029	6,	10	1
9	−0·006	9,	9	1
10	+0·047	17,	17	2
13	+0·113	12,	7	1
14	+0·080	20,	20	2
16	+0·076	15,	17	2
17	+0·073	21,	19	2
21	+0·051	19,	15	2
22	+0·024	16,	15	2
Mean	+0·064			
Greenwich.				
Oct. 26	+0·149	9,	2	1
27	+0·212	11,	11	1
29	+0·048	18,	19	2
30	+0·005	21,	10	1
Nov. 1	+0·107	9,	8	1
2	+0·040	9,	8	1
3	+0·114	6,	9	1
4	+0·101	24,	23	2
Mean	+0·093			

TABLE IV.

Transit B.—Observations by Mr. Storey and Mr. Witchell.

Annular Illumination.

Date. 1906.	W—E. Means.	No. of Stars. W.	E.	Obsr.
	s			
Jan. 22	+0·07	6,	6	J. S.
23	+0·06	4,	2	W.
24	+0·13	7,	6	J. S.
26	+0·09	7,	6	J. S.
29	+0·15	4,	4	W.

Date. 1906.	W—E. Means.	No. of Stars. W.	E.	Obsr.
	s			
Jan. 30	+0·15	4,	4	J. S.
Feb. 3	+0·03	4,	4	J. S.
April 4	+0·04	4,	4	J. S.
9	+0·17	4,	3	W.
19	+0·12	5,	5	W.
Mean	+0·101			

On some other nights individual stars are very discordant; in one case there is a range of one second of time.

TABLE V.

Transit B.—Observer, Mr. H. Christie.

Annular Illumination.

Date. 1906.	W—E. Means. s	No. of Stars. W.	E.	Wt.
July 16	+0·081	2,	6	1
18	+0·369	4,	5	2
20	+0·868	5,	7	1
23	+0·480	3,	6	1
24	+0·584	7,	7	1
25	+0·972	15,	7	2
31	+0·368	8,	6	1
Aug. 7	+0·703	7,	4	1
Sept. 20	+0·093	3,	7	1
25	+0·744	5,	7	2
28	+0·521	4,	2	1
Oct. 9	+1·057	3,	9	1
11	+0·409	2,	2	1
22	+0·725	2,	3	1
23	+0·088	2,	3	1
27	+0·455	4,	3	1
Mean	+0·558			

TABLE VI.

Central Illumination.

Date. 1907.	W—E. Means. s	No. of Stars. W.	E.	Wt.
July 30	+0·053	3,	4	1
Sept. 7	+0·091	7,	8	2
9	+0·089	8,	8	2
10*	+0·109	8,	7	2
11*	+0·131	7,	7	2
15	−0·030	4,	4	1
19	+0·038	5,	6	2
24	−0·023	9,	12	2
25	−0·052	7,	7	2
26	−0·034	15,	15	2
Oct. 4	+0·004	8,	8	2
Mean	+0·036			

Observer, Captain Monro, R.N.

Sept. 16	+0·042	4,	3	1
17	−0·052	6,	6	2

Note.—The observations in Table VI. (Central Illumination) were all reduced with the same collimation error throughout; there appears to have been a slow change of collimation, which was also shown by the nadir observations. No correction has been applied for inequality of the pivots. The mean of a number of determinations made by the striding level in 1902, when the pivots were last reground, was 0″·00.

* On September 10 and 11 the nadir readings for collimation are very discordant, differing by about 1″ from the mean. The clock stars appear to be affected also, but with the opposite sign.

TABLE VII.

Transit C.—Observer, Mr. Hollis.

Annular Illumination.

Date. 1902.	W—E. Means.	No. of Stars. W.	No. of Stars. E.	Wt.
Greenwich.	s			
Mar. 17	+0·128	15	12	2
19	+0·106	17	17	2
20	+0·056	10	12	1
21	+0·086	10	7	1
22	+0·066	14	11	1
23	+0·111	8	8	1
25	+0·260	16	18	2
27	+0·416	13	10	1
28	+0·070	14	5	2
Mean	+0·143			
Paris.				
Apr. 7	+0·303	10	8	1
8	+0·316	9	2	1
9	+0·067	10	15	1
11	+0·095	9	11	1
13	+0·267	16	18	2
17	+0·217	17	16	2
18	+0·221	17	16	2
20	+0·162	18	11	2
21	+0·166	16	16	2
23	+0·260	9	15	1
24	+0·137	17	19	2
Mean	+0·199			
Greenwich.				
Apr. 27	+0·052	9	14	1
28	+0·060	18	17	2
May 1	+0·064	16	12	2
2	+0·070	10	14	1
3	+0·007	17	17	2
Mean	+0·048			

Date. 1902.	W—E. Means.	No. of Stars. W.	No. of Stars. E.	Wt.
Paris.	s			
Sept. 21	+0·204	15	11	2
22	+0·120	16	16	2
24	+0·121	17	14	2
26	+0·118	16	18	2
Mean	+0·141			
Greenwich.				
Sept. 29	+0·133	14	14	2
Oct. 2	+0·032	16	16	2
8	+0·079	20	19	2
9	+0·077	10	7	1
10	−0·019	19	20	2
12	+0·057	12	8	1
14	+0·005	19	20	2
15	+0·087	14	10	1
16	+0·081	20	18	2
17	+0·033	16	17	2
18	+0·138	17	14	2
21	+0·041	20	20	2
22	+0·199	15	18	2
Mean	+0·072			
Paris.				
Oct. 25	−0·003	14	13	2
27	+0·058	10	8	1
28	+0·061	7	13	1
29	+0·158	16	17	2
31	+0·172	16	17	2
Nov. 2	+0·157	17	11	2
3	+0·187	20	17	2
4	+0·234	20	19	2
Mean	+0·138			

TABLE VIII.

Transit C.—Observer, Mr. H. Christie.

Annular Illumination.

Date. 1906.	W—E. Means.	No. of Stars. W.	E.	Wt.
	s			
Nov. 5	−0·015	7,	7	1
11	+0·652	2,	7	1
19	−0·281	6,	4	1
22	+0·118	14,	8	2
Dec. 9	+0·253	3,	3	1
10	+0·316	4,	5	1
13	+0·507	11,	10	2
14	+0·410	3,	3	1
1907.				
Jan. 21	+0·386	7,	9	1
26	+0·436	7,	7	2
29	+0·321	5,	3	1
Feb. 1	+0·105	8,	6	1
22	+0·323	17,	13	2
28	+0·406	15,	17	2
Mar. 1	+0·146	9,	8	2
10	+0·010	2,	2	1
11	+0·435	4,	8	2
22	+0·185	4,	6	2
26	+0·198	3,	5	1
Mean	+0·277			

TABLE IX.

Central Illumination.

Date. 1907.	W—E. Means.	No. of Stars. W.	E.	Wt.
	s			
May 8	−0·029	16,	15	2
10	+0·041	1,	6	1
27	−0·015	7,	5	2
June 2	−0·023	12,	11	2
10*	−0·001	11,	5	2
17	−0·045	5,	6	2
19	+0·046	15,	14	2
July 16	+0·047	7,	7	2
17	+0·055	2,	2	1
Mean	+0·003			

* Observer, Mr. Hollis.

Notes.—No correction for inequality of the pivots has been applied. The following determinations of the apparent inequality have been made with the striding level, showing that it is very small.

Date.		Apparent Inequality of Pivots.	Discordance W—E due to Pivot Error.
		s	s
1902	March–May	+0·05	+0·016
1902	Sept. –Nov.	−0·09	−0·029
1906	Nov. –Dec.	−0·09	−0·022
1907	Jan. –March	−0·12	−0·029
1907	April –June	−0·07	−0·017

The pivots have not been re-ground since 1901.

TABLE X.

Transit D.—Central Illumination.

Observer, Captain Monro, R.N.

Date. 1907.	W—E. Means.	No. of Stars. W.	No. of Stars. E.	Wt.
	s			
Sept. 18	−0·137	2,	3	1
19	+0·085	8,	10	2
23	+0·030	10,	8	2
24	+0·009	12,	10	2
30	+0·008	5,	8	2
Oct. 2	−0·014	8,	6	2
4	+0·150	9,	9	2
7	−0·052	8,	6	2
8	+0·021	8,	8	2
Mean	+0·020			

Observer, Lieut. Gibson, R.N.

Date. 1907.	W—E. Means.	No. of Stars. W.	No. of Stars. E.	Wt.
	s			
Sept. 27	−0·149	3,	3	1
Oct. 2	+0·019	7,	7	2
4	+0·096	8,	8	2
7	−0·089	6,	4	1
8	−0·020	5,	8	2
Mean	−0·006			

On the Orbit of the Binary Star β 80.

By T. J. J. See, A.M., Ph.D. (Berol.).

This interesting and rapidly revolving system was discovered by Burnham with his celebrated 6-inch telescope at Chicago in 1874. It was measured by Dembowski the following year, and has since been followed regularly by the most active observers of close pairs. At first the motion was very slight, because the companion was near apastron, with a long radius vector and revolving slowly. But in the last ten years the motion has become very rapid both in angle and distance. The apparent orbit quite well represents the observations, as shown by the accompanying diagram. The material used is the complete measures given by Burnham in his General Catalogue of Double Stars, just published by the Carnegie Institution. The place of β 80 for the epoch of 1880 is $\alpha = 23^h\ 12^m\ 45^s$; $\delta = +\ 4^\circ\ 45'$; mags. 8·2, 9·1.

The elements are as follows:—

P = 63·5 years	$\Omega = 107^\circ{\cdot}8$
T = 1905·30	$i = 17^\circ{\cdot}6$
$e = 0{\cdot}726$	$\lambda = 10^\circ{\cdot}7$
$a = 0''{\cdot}626$	

Apparent orbit:—

Length of major axis	= 1″·25
Length of minor axis	= 0″·79
Angle of major axis	= 117°·4
Angle of periastron	= 117°·8
Distance of star from centre	= 0″·452

No previous orbit for this star has been worked out. Some astronomers have expressed the opinion that the period could not be deduced until the companion returned to apparent apastron. But it will be seen that the observations fix the apastron with singular accuracy.

Comparison of computed with observed places for β 80.

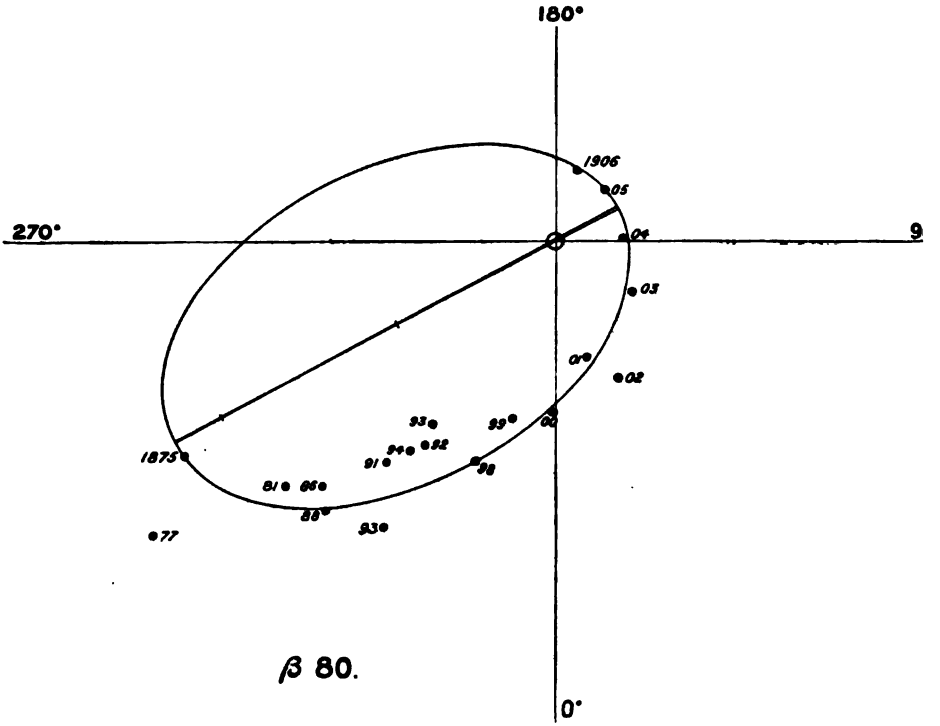
t	θ_c	θ_o	$\theta_o-\theta_c$	ρ_c	ρ_o	$\rho_o-\rho_c$	Observer, Nights.
	°	°	°	″	″	″	
1875·80	300·4	300·9	−0·5	1·07	1·07	0·00	Δ, 5.
1877·79	306·1	302·4	+3·7	1·24	1·07	+0·17	β, 2.
1881·69	312·2	308·6	+3·6	0·91	1·02	−0·11	β, 3.
1886·94	316·1	316·7	−0·6	0·84	0·92	−0·08	LM, 1.
1888·75	319·5	319·8	−0·3	0·88	0·87	+0·01	β, 4; Lv, 3.
1891·57	322·5	325·5	−3·0	0·69	0·78	−0·09	β, 3.
1892·97	327·2	329·0	−1·8	0·60	0·74	−0·14	Sp, 3.
1893·60	328·5	330·5	−2·0	0·83	0·71	+0·12	Lv, 1; W, 3.
1894·84	325·1	334·4	−9·3	0·63	0·66	−0·03	Lewis, 1.
1898·77	340·3	336·0	+4·3	0·59	0·64	−0·05	β, 1; Bow, 1; A, 4.
1899·38	346·0	338·7	+7·3	0·45	0·61	−0·16	Bry, 4.
1900·65	358·6	351·6	+7·0	0·42	0·47	−0·05	Doo, 2; A, 2.
1901·83	13·8	12·4	+1·4	0·30	0·33	−0·03	Bry, 4; Doo, 2.
1902·75	24·3	29·8	−5·5	0·37	0·26	+0·11	Bry, 1; Lewis, 1.
1903·76	55·3	53·4	+1·9	0·22	0·21	+0·01	A, 2; Bry, 2.
1904·73	91·6	90·2	+1·4	0·16	0·18	−0·02	A, 2.
1905·65	136·4	134·8	+1·6	0·17	0·17	0·00	A, 2.
1906·52	163·8	172·4	−8·6	0·18	0·19	−0·01	A, 2.

Ephemeris.

t	θ_c	ρ_c	t	θ_c	ρ
	°	″		°	″
1908·7	209·0	0·28	1914·7	256·4	0·60
1909·7	222·5	0·33	1915·7	260·2	0·65
1910·7	233·3	0·39	1916·7	263·0	0·69
1911·7	241·3	0·45	1917·7	265·7	0·72
1912·7	247·2	0·50	1918·7	268·2	0·76
1913·7	252·4	0·56			

Dembowski measured the position here laid down on five nights in 1875. It happens that his observations both in angle and in distance are perfectly represented by this ellipse which satisfies the law of areas. Thus both ends and a full side of

the apparent ellipse have already been described, and the observations of Aitken in 1905 and 1906, beyond periastron, show that the other side of the ellipse is almost identical with that already described. The elements are thus fixed with unusual accuracy. It seems likely that the period is not uncertain by more than one or two years, and that the eccentricity cannot depart from the value here given by more than ± 0·03.



This orbit illustrates the value of good observations advantageously placed. An ephemeris for the next ten years is added for the convenience of observers. This system deserves careful attention for some time, because the motion will be rapid both in angle and distance, and observations will be valuable in verifying the orbit.

As Burnham's General Catalogue gives all the measures used, it suffices to give a calculated comparison with the annual means as plotted in the diagram, which embraces all the best material.

U.S. Naval Observatory,
Mare Island, California,
1907 *December* 4.

On the Orbit of β 513 = 48 *Cassiopeiæ.*

By T. J. J. See, A.M., Ph.D. (Berol.).

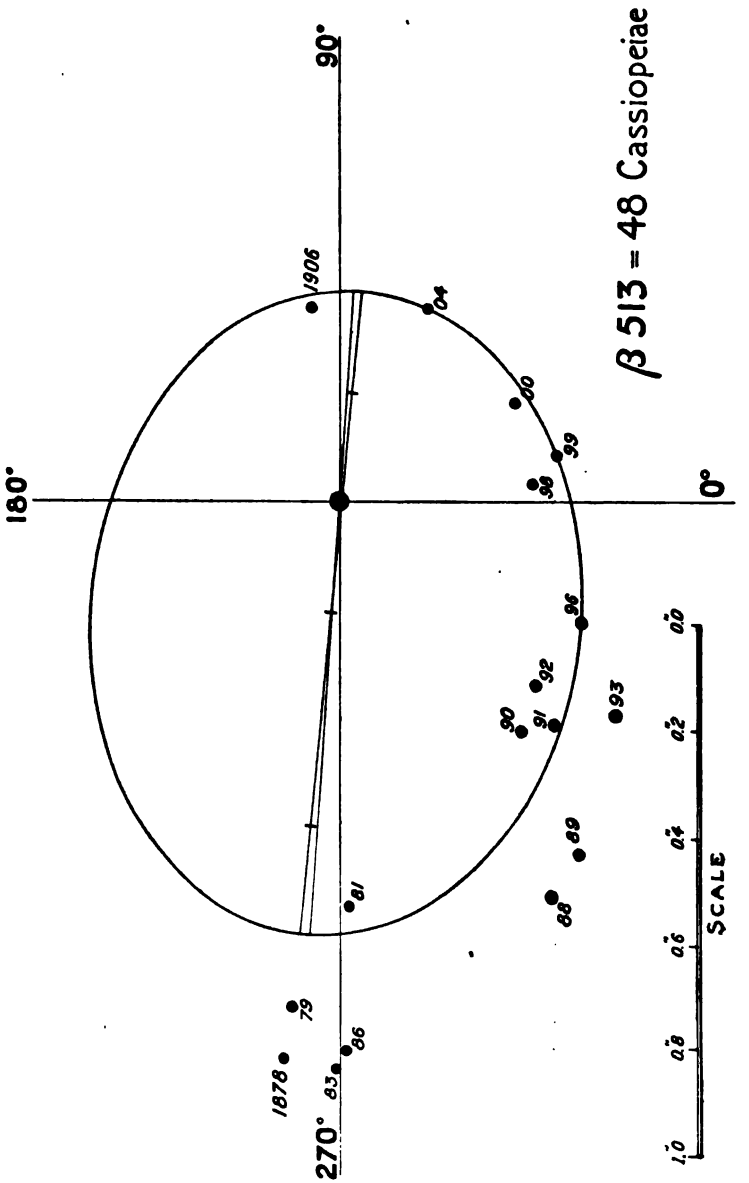
This remarkable binary was discovered by Burnham with the 18½-inch refractor at Chicago in 1878. Its place for the epoch 1880 is $\alpha = 1^h\ 52^m\ 7^s$; $\delta = +70^\circ\ 19'$; and the magnitudes of the components are 5·0 and 7·0. The star is always close, and presents the same difficulty in dividing the components as the systems 99 Herculis, 85 Pegasi, and τ Cygni, which can be well separated only by powerful instruments in good seeing. The measures of such unequal pairs are often vitiated by sensible systematic errors, so that the computer encounters difficulty in reconciling the observations to any orbit.

The orbit herewith presented gives the general character of the motion with essential accuracy, and will be useful to observers for the next ten or fifteen years, while the companion is again receding towards apastron.

The two ends and one side of the apparent ellipse have been described. The principle of areas shows that the orbit has a rather small eccentricity, and that it is not highly inclined. I am indebted to Professor Eric Doolittle, of the Flower Observatory of the University of Pennsylvania, for the last measure, which falls exactly on the orbit here laid down. His note on the measure was, "I am certain this is real; a perfect night."

Comparison of computed with observed places for β 513.

t	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$	Observer, Nights.
1878·70	264°·5	265°·1	− 0°·6	1″·05	0″·82	+0″·23	Δ, 1; β, 3.
1879·40	265·0	267·2	− 2·2	0·95	0·82	+0·13	OΣ, 1; β, 2.
1881·67	271·9	273·6	− 1·7	0·76	0·81	−0·05	β, 1.
1883·78	269·7	280·2	−10·5	1·07	0·79	+0·28	En., 5.
1885·77	284·9	286·6	1·7	...	0·76	...	HΣ, 1.
1886·03	270·1	287·6	−17·5	1·03	0·75	+0·28	T., 4.
1888·70	298·1	297·4	+ 0·7	0·83	0·70	+0·13	β, 4.
1889·52	304·4	300·9	+ 3·5	0·76	0·67	+0·09	β, 3.
1890·62	308·6	305·8	+ 2·8	0·55	0·64	−0·09	β, 4.
1891·61	313·5	310·6	+ 2·9	0·58	0·62	−0·04	β, 4.
1892·99	317·2	316·4	+ 0·8	0·5±	0·58	−0·08	Sp., 1.
1893·47	322·2	321·2	+ 1·0	0·65	0·56	+0·09	Bax., 1.
1896·09	333·7	340·2	− 6·5	0·5±	0·48	+0·02	Sp., 1.
1898·92	5·9	7·3	− 1·4	0·36	0·42	−0·06	A., 3.
1899·72	11·6	16·3	− 4·7	0·42	0·41	+0·01	A., 3-2.
1900·76	29·3	28·2	+ 1·1	0·38	0·40	−0·02	A., 1.
1904·00	65·2	67·3	− 2·1	0·40	0·40	±0·00	A., 2.
1906·32	97·4	95·5	+ 1·9	0·37	0·39	−0·02	A., 2.
1907·93	118·8	115·9	+ 2·9	<0·4	0·39	+0·01	Doo., 1.


β 513 = 48 Cassiopeiae
90°
180°
0°
270°
1906
04
00
99
98
96
92
91
90
93
89
88
81
79
86
83
1878
SCALE
0.0
0.2
0.4
0.6
0.8
1.0

Ephemeris.

t	θ_c	ρ_c	t	θ_c	ρ_c
	°	″		°	″
1908·60	124·3	0·38	1914·60	193·4	0·47
1909·60	137·7	0·38	1915·60	201·2	0·50
1910·60	150·5	0·39	1916·60	208·2	0·54
1911·60	163·0	0·40	1917·60	214·2	0·57
1912·60	174·4	0·42	1918·60	219·8	0·60
1913·60	184·3	0·44			

All the observations are given in Burnham's *General Catalogue of Double Stars* recently published by the Carnegie Institution. It is sufficient, therefore, to add a comparison of the computed with the observed places upon which the apparent orbit was based.

The elements of β 513 are as follows:—

P =	52·95 years	Ω =	81°·4
T =	1905·60	i =	35°·9
e =	0·347	λ =	6°·2
a =	0″·610		

Apparent orbit:—

Length of major axis	= 1″·214
Length of minor axis	= 0″·962
Angle of major axis	= 84°·7
Angle of periastron	= 86°·7
Distance of star from centre	= 0″·21.

For several years to come β 513 will deserve careful attention from observers with powerful telescopes. The motion will be rapid for some ten years, after which the system will widen out, so that observation will be much easier than at present.

U.S. Naval Observatory,
Mare Island, California,
1907 *December* 10.

On the Orbit of β 552 = 11 *Orionis.*

By T. J. J. See, A.M., Ph.D. (Berol.).

This interesting binary was discovered by Burnham with the 18½-inch refractor at Chicago in 1877, but it had been previously suspected with the 6-inch in 1874 as "possibly a close pair." The place for the epoch 1880 is $\alpha = 4^h\ 45^m\ 4^s$; $\delta = +13^\circ\ 27'$; and the magnitudes are 7·0 and 10·0. It is therefore an exceedingly difficult object, and can be well seen only in the most powerful telescopes, under good conditions. On two favourable nights in 1892 Burnham could not see it with the 36-inch refractor of the Lick Observatory. Professor Hermann Struve, with the 30-inch refractor at Pulkowa, had suspected the companion on the opposite side in 1886, when the distance was only 0″·21; but as his angle corresponds to the true place at the time, I have reversed it, on the supposition that the point of the diffraction fringe noted was produced by the companion, which was then too close for separation, but might easily produce an opposite image resembling a real companion.

This system is highly inclined upon our visual ray. The components being so unequal, the observations are somewhat discordant. After a careful study of the movement, I have deduced the following elements:—

P = 56·0 years	Ω = 4°·1
T = 1887·0	i = 56°·7
e = 0·345	λ = 90°·2
a = 0″·528	

Apparent orbit:—

Length of major axis	= 1″·00
Length of minor axis	= 0″·59
Angle of major axis	= 4°·2
Angle of periastron	= 94°·4
Distance of star from centre	= 0″·10

Comparison of computed with observed places for β 552 = 11 *Orionis.*

t	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$	Observer, Nights.
1874·95	340°±	348°·6	− 8°·6	0″·5±	0″·51	−0″·01±	β.
1877·97	360·±	359·5	+ 0·5	0·8±	0·49	+0·31±	β, 1.
1886·19	85·0 !	74·5	+10·5	0·5±	0·21	+0·29±	H $\mathfrak{X}$, 1; angle is here reversed.
1889·19	144·7	141·2	+ 3·5	0·35±	0·25	+0·10±	Sp., 1.
1890·96	156·7	161·4	− 4·7	0·33	0·34	−0·01	β, 3.
1891·13	153·8	162·8	− 9·0	0·33	0·34	−0·01	Sp., 8.
1892·06	...	169·3	...	...	0·38	...	β, 2; apparently single with 36-inch.

t	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$	Observer, Nights.
1893·07	165°·3	175°·5	−10°·2	0″·35±	0″·42	−0″·07	Sp., 1.
1894·15	178·2	181·1	− 2·9	0·38	0·45	−0·07	Sp., 2.
1894·88	177·0	184·3	− 7·3	0·40	0·47	−0·07	Bax., 2.
1896·11	179·9	188·8	− 8·9	0·35	0·49	−0·12	Sp., 7.
1896·86	187·0	192·1	− 5·2	0·42	0·50	−0·08	Hu., 1; A., 3.
1897·63	193·5	194·9	− 1·4	0·37	0·50	−0·13	Sp., A., 5; Hu., 3.
1898·01	189·9	197·7	− 7·8	0·45	0·50	−0·05	Lewis, 2.
1898·43	198·1	197·8	+ 0·4	0·44	0·51	−0·07	A., 6.
1899·76	202·5	202·0	+ 0·5	0·45	0·51	−0·06	See, 1; A., 4.
1899·85	207·3	202·8	+ 4·5	0·42	0·51	−0·09	Lewis, 3.
1900·71	204·3	205·9	− 1·6	0·45	0·51	−0·06	A., 3; Doo., 3.
1901·85	204·6	210·1	− 5·5	0·49	0·50	−0·01	A., 2; Doo., 2.
1902·74	229·3	212·6	+16·7	0·32	0·50	−0·18	β, 1.
1903·66	218·9	216·8	+ 2·1	0·46	0·49	−0·03	A., 4; Biesbroeck, 2.
1905·90	227·9	225·9	+ 2·0	0·46	0·46	±0·00	A, 2.
1907·93	238·7	235·1	+ 3·6	0·55	0·44	+0·11	Doo., 3.

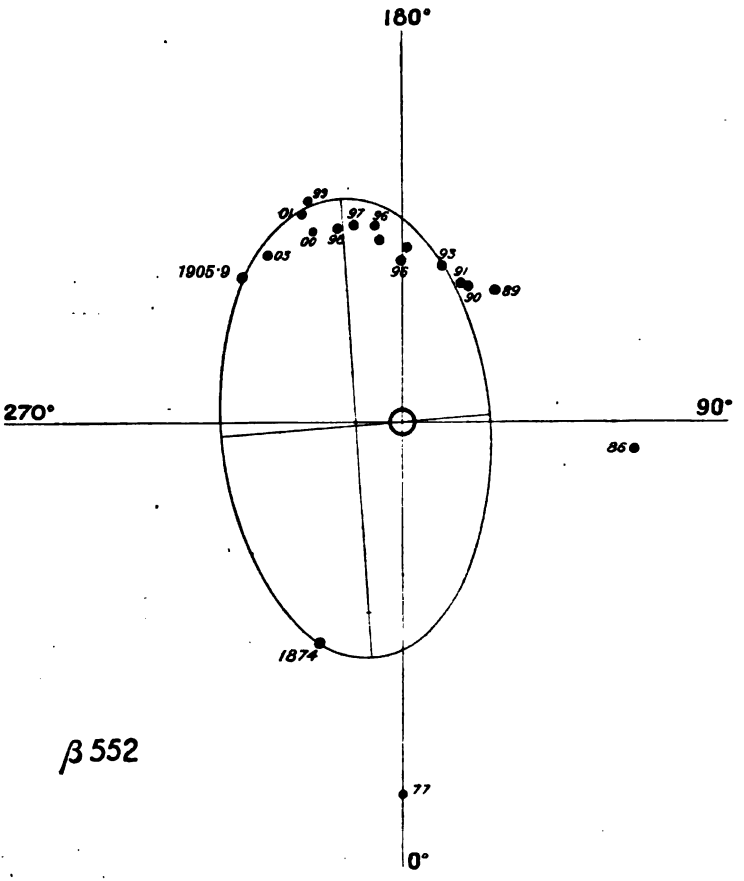
Ephemeris.

t	θ_c	ρ_c	t	θ_c	ρ_c
	°·	″·		°·	″·
1907·0	230·9	0·45	1913·0	262·3	0·39
1908·0	235·5	0·44	1914·0	268·4	0·39
1909·0	240·5	0·43	1915·0	274·5	0·39
1910·0	245·6	0·42	1916·0	280·6	0·39
1911·0	250·6	0·41	1917·0	286·5	0·39
1912·0	256·3	0·40	1918·0	292·4	0·40

The accompanying table shows the comparison of the computed with the observed places. Considering the discordance of the measures, the representation in angle is good. The distances could be slightly improved by reducing the major axis; but as the scale of distance is still somewhat uncertain, I have not thought it worth while to do this. It seems certain that these elements give the essential character of the motion, which is rendered very peculiar by the high inclination, giving a nearly stationary phase at each end of the apparent orbit. The orbit of β 552 has some resemblance to that of γ Coronæ Borealis, the components of which are correspondingly unequal and difficult to measure.

It is clear that the periastron was swept over about 1887·0, and that the period is about fifty-six years. The eccentricity may be higher than that given here, but an increase in this element

would hardly be warranted by the existing observations. The ephemeris will be useful to observers, and it is to be hoped that



they will continue to follow this interesting object for a number of years, till the orbit can be adequately confirmed.

U.S. Naval Observatory,
Mare Island, California,
1907 *December* 12.

Note on the Comparative Eccentricities of Visual and Spectroscopic Binary Stars. By T. J. J. See, A.M., Ph.D. (Berol.).

For some time the writer has been occupied with a critical examination of the comparative eccentricities of visual and spectroscopic binary stars, in the hope of obtaining an observational criterion that will throw light on the evolution of these systems. It is now clear that the investigation will afford the criterion sought; and as he recently had occasion to mention the general result in letters to some friends abroad, it seems likely that a preliminary notice of this work may be of interest to the Society.

If we exclude from consideration all binaries telescopically resolvable in modern instruments, there remain about 30 orbits of purely spectroscopic binaries, which have been worked out from spectrograms taken chiefly at the Lick, Yerkes, and Potsdam Observatories. The eccentricity is well determined, and the average value for these 30 systems is about 0·22. In the case of very rapid binaries, with periods often much less than a day, the orbits are practically circular. Dr. A. W. Roberts and Professor G. W. Myers have investigated the light changes of certain variables, and shown that they are composed of two stars in the process of separation, and therefore revolving in orbits which are without sensible eccentricity.

The average eccentricity of visual binary stars remains about 0·50, as indicated in the work *Researches on the Evolution of the Stellar Systems*, vol. i., published in 1896. The subject has, however, been so much extended and improved during the past eleven years that it has seemed advisable to re-examine each orbit anew. There are about 80 orbits in which the eccentricity may be fairly well determined. I hope to have the final result of this examination ready for presentation to the Society in a short time. The work is already so far advanced that we may confidently state that *the average eccentricity among visual binaries is considerably more than twice that among the spectroscopic binaries.* The latter are stellar systems, with components so close together, owing to the small size of their orbits, or so far removed from us in space, that we cannot resolve them even in the greatest of modern telescopes. In some cases they are closer sub-systems of known visual binary stars. Among such systems we may mention ξ Ursae Majoris, κ Pegasi, α Geminorum, etc., which are spectroscopically subdivided in exact analogy to the many double stars which have been resolved into triple stars by increase of modern telescopic power; but where the spectroscopic binaries stand alone, we can infer the average size of their orbits only from the generalised form of Kepler's law.

For a large number of systems we are justified in assuming that the average mass will be the same among the spectroscopic as among the visual binary stars. And as the periods of the spectroscopic binaries are much shorter than those of the visual binaries, it

follows also that their mean distances are correspondingly smaller. This conclusion might indeed fail here and there in particular cases, but, as applied generally to these two classes of bodies, is entirely valid.

Accordingly, we see that in the universe generally small eccentricity is connected with the small mean distance, and *vice versa*. This confirms, from an observational standpoint, the theory of the development of these systems long ago inferred from other considerations (*cf.* the writer's *Inaugural Dissertation*, Berlin, 1892). Heretofore scarcely anyone has dared to hope that a direct confirmation of this theory would ever be possible; but, unless this reasoning is entirely at fault, such a positive confirmation is near at hand, and is due to the notable advance in our knowledge of the stars arising from the discovery and investigation of spectroscopic binaries, chiefly during the past ten years.

U.S. Naval Observatory,
Mare Island, California:
1907 *November* 30.

Micrometrical Measures of Double Stars (*Fifth Series*). By the Rev. T. E. Espin, M.A.

In the early part of 1907 very few known double stars were measured, pending the arrival of Burnham's General Catalogue. The latter part of the year has been devoted to the completion of measures of new pairs, and to the measurement of such stars of Sir John Herschel's as have not been remeasured since his observation of them. The places are those of the General Catalogue (1880); but in several cases, as explained in the notes, corrections to Herschel's places have been made.

h.	R.A. 1880. h m s	Decl. ′ ″	P.	D.	Mags.	Nights.	Date.
1009	0 9·6	+47 56	26·7	16·77	9·0, 9·1	2	1907·760 AB
			140·9	36·15	11·7	2	·760 AC
1029	25·9	44 16	279·9	11·85	8·8, 10·4	3	·905
1985	28·4	48 11	143·3	17·97	10·0, 10·2	2	·851
1049	38·0	50 6	303·1	15·32	9·2, 11·0	3	·784
1050	38·1	44 23	185·1	13·97	9·7, 10·7	2	·896
1058	51·0	49 34	283·7	11·70	10·2, 10·3	3	·862
1060	52·0	44 16	293·1	9·48	9·5, 10·7	4	·924
2013	59·2	44 8	247·0	23·22	8·5, 11·5	2	·891
2031	1 9·6	43 49	260·6	20·15	8·5, 12·7	2	·903 BC
			342 2	45·52	A= 8·0	2	·903 AB

h.	R.A. 1880. h m	Decl. ° ′	P. °	D. ″	Mags.	Nights.	Date.	
1077	1 10·7	+44 0	295·2	38·05	8·0, 9·6	2	1907·903	
2144	2 25·0	48 20	259·4	26·22	8·6, 9·5	2	·880	
2232	4 22·4	47 2	325·3	13·42	8·8, 10·5	2	·924	
2237	36·1	47 26	123·0	15·90	9·1, 11·6	2	·942	
2239	39·4	45 58	162·2	13·52	9·5, 10·5	2	·942	
2248	58·2	47 11	333·1	14·92	8·8, 11·0	2	·994	
2249	5 1·1	47 21	97·2	14·20	8·8, 9·4	2	·942	
2264	21·7	47 49	125·8	5·70	8·4, 9·5	2	·942	
378	6 0·2	28 58	257·2	12·27	9·0, 9·0	2	·115	
379	1·3	31 17	117·3	9·85	7·0, 11·5	1	·041	
460	8 45·3	28 43	333·7	43·72	7·5, 11·0	2	·183	
803	56·7	28 4	9·4	11·62	9·2, 11·5	2	·232	
495	9 12·9	35 46	135·2	15·81	11·0, 11·0	2	·216	
2857	19 7·0	41 35	177·5	14·40	8·0, 13·5	1	·756	AB
			210·9	36·60	12·0	1	·756	AC
1460	53·6	46 28	87·2	6·87	9·9, 11·0	2	·835	
1463	55·9	45 29	313·1	10·20	9·8, 11·5	3	·897	
1481	20 1·8	49 3	7·2	17·40	10·0, 10·1	2	·842	BC
			88·0	27·62	A= 9·7	2	·842	AB
1584	45·6	47 38	224·0	3·75	9·0, 10·0	2	·770	
1594	50·4	47 6	48·6	8·78	9·0, 10·1	4	·878	
1604	56·1	48 43	135·5	25·67	10·2, 11·0	2	·825	
1651	21 24·5	47 38	230·0	10·95	9·4, 10·1	2	·828	
1657	28	47 54	See Note					
1676	35·1	46 39	135·0	29·22	8·4, 9·7	2	·926	
1681	35·6	47 52	109·9	7·35	9·0, 9·1	2	·770	
1692	42·0	46 39	249·6	11·75	10·5, 11·5	3	·930	
1698	44·9	46 43	325·6	7·62	9·1, 12·0	3	·930	
1701	46·9	46 32	195·3	11·42	9·7, 10·7	2	·951	
1705	49·8	46 29	37·6	3·07	11·9, 13·5	2	·904	
1712	55·3	48 8	176·7	9·30	9·4, 9·5	3	·786	
1738	22 4·5	45 53	178·9	9·57	10·6, 11·7	2	·932	
1762	18·5	47 48	88·4	14·70	9·7, 13·7	2	·770	BC
			354·9	18·35	A= 9·1	2	·770	AB
1766	21·8	49 41	265·2	13·32	9·0, 10·1	2	·806	

h.	R.A. 1880. h m	Decl. ° ′	P. °	D. ″	Mags.	Nights.	Date.
1772	22 23·5	+45 32	103·5	6·05	9·6, 10·2	2	1907·910
1793	33·1	46 25	291·5	13·95	10·0, 11·7	2	·910
1794	33·2	46 22	315·0	21·80	9·4, 11·0	2	·910
1795	33·3	46 43	213·8	12·07	10·7, 11·5	2	·910 BC
			209·8	16·10	A= 10·2	2	·910 AB
1797	34·6	49 30	125·4	14·10	9·7, 11·0	2	·951
1814	44·1	47 57	76·1	14·10	9·0, 10·2	2	·782
1863	23 10·4	48 21	51·1	8·57	11·5, 12·5	3	·813
1878	21·3	49 46	268·2	8·87	10·5, 10·7	2	·858
1883	21·9	45 44	145·4	18·55	9·3, 9·6	2	·903
1893	29·2	46 50	247·0	4·47	8·6, 8·7	3	·795
1917	48·7	45 6	76·2	12·92	8·9, 12·2	2	·935
1923	54·3	50 3	277·6	10·77	11·5, 11·8	2	·918
1927	57·0	44 28	76·6	14·75	8·6, 8·9	2	·951
1931	58·0	49 18	113·1	21·90	8·0, 10·7	2	·759

Notes.

h 1009 is BD + 47°; 40, C not given by *h*.

h 1029 is BD + 44°, 110, *h*'s place one min. too small in R.A.

h 1985 is BD + 48°,172.

h 2013 is BD + 44°,232, *h*'s angle 256°·0.

h 2249 a small star at P 306°, D 119″ is double and is No. 467 of the list of new pairs.

h 2857 the nearer *comes* is not in *h*.

h 1460 $22^{s}\cdot2\ p\ 43''\cdot7$ N of BD + 46°, 2817.

h 1657 *h* says "10° ± 10″ ± in a cluster." I have looked for this on several nights, but have failed to find any such pair. The only pair that bears any resemblance is one at P 16°·3 D 101″ from BD + 47°, 3439 (Aitken 770) which I have measured as follows:—
P 39°·7, D 7″·00, mags. 9·5, 10·7, 2 nts. 1907·884.

h 1705 This is $13^{s}\cdot6\ f\ 2'\cdot5$ N of BD + 46°, 3480, and if identical with *h* 1705 *h*'s place is 1 min. too great. His observation is "P 80°·4, D 3½″, mags. 11–12, 14, difficult." I could not find any other pair here.

h 1762 Star C not given by *h*.

h 1795 *h* says, "? if a 4th star." No such star seen on either night, save a distant brighter one n.p.

h 1863 *h* reversed angle and says "difficult to measure."

h 1893 This is BD + 46°, 4082, and *h*'s place requires a correction in Decl. of + 30′. His angle is the mean of 250°·6 and 252°·6. The star BD + 46°, 4083, is in the field $8^{s}\cdot8\ f\ 73''\cdot45$ N at an angle of 51°·6.

Various Stars.

Name.	R.A. 1880. h m	Decl. ° ′	P.	D.	Mags.	Nights.	Date.
ν Cassiopeiæ ...	0 41·7	+57 11	236·4	5″·67		2	1907·043
			236·1	5·72		2	·990
Espin 44 ...	49·8	56 51	250·1	7·40	8·3, 12·0	3	·341
φ Cassiopeiæ...	1 12·5	57 36	264·8	41·54	7·0, 11·0	2	·107 CD
			231·1	132·93	A= 5·0	2	·107 AC
A.G. (Bonn) ...	51·7	50 1	355·2	13·62	9·0, 9·1	2	·774
A.G. (Bonn) ...	3 17·3	46 36	315·0	10·67	8·5, 9·2	2	·957
Espin 57 ...	4 47·4	47 27	175·9	2·27	9·5, 9·9	3	·942 AB
			61·3	16·52	C= 14·0	2	·940 AC
A 1316 ...	5 54·5	30 0	181·0	4·68	8·5, 11·7	3	·121
Rümker 4 ...	6 12·8	29 43	273·2	5·50	9·0, 9·2	2	·115
6 Cancri ...	7 56·8	28 8	187·5	60·06	5·0, 12·0	2	·232 AB
			81·0	78·63	C= 11·0	2	·232 AC
Σ 1321 ...	9 6·4	53 13	66·5	18·82		2	·202 AB
			282·7	28·44	C= 14·0	2	·202 AC
Küstner 40 ...	11 42·6	34 22	182·6	3·12	8·9, 9·2	2	·216
12 Comæ ...	12 16·5	26 31	54·0	34·91	4·5, 11·5	3	·240 AB
			167·5	64·66	C= 8·0	2	·235 AC
A.G. (Bonn)...	18 54·7	47 2	229·0	2·37	8·3, 8·4	3	·846
Lewis 32 ...	19 56·1	29 36	128·0	3·17	9·8, 10·0	3	·724
Arg. 39 (Espin 92)	20 38·7	48 50	113·2	9·67	8·2, 8·6	3	·778 AB
			125·1	30·98	C= 12·0	4	·714 BC
			232·0	33·47	D= 14·0	2	·893 BD
+48°,3995 ...	23 13·5	48 16	231·6	49·10	7·3, 8·8	2	·799

Notes.

φ Cassiopeiæ CD may be the pair measured by H as 271°·8.

A. G. III. 17·3. The note in Bonn Observations "Dupl. 3″ med." may refer to Σ 371, 7 min. p.

A 1316. Detected and measured before Professor Aitken's results reached me. He calls the *comes* 14·2 mag.

Rümker 4. This is BD + 29°, 1181, 6′ N of Rümker's place.

6 Cancri. A star mentioned in "Celestial Objects" as having two *comites* detected by Birmingham.

Σ 1321. The minute *comes* was first seen 1901 Jan. 22.

12 Comæ. The nearer *comes* previously seen by Burnham.

Lewis 32. This is BD + 29°, 3849.

Arg. 39. After charting all the available measures, I find a motion for B of 0″·216 towards 221°·5. C, measured by Burnham, is almost at right angles to the line of motion. I have added the still fainter star D, as being nearly in the direction of motion.

New Double Stars. By the Rev. T. E. Espin, M.A.

No.	B.D.	R.A. 1900. h m	Decl. ° ′	P. °	D. ″	Mags.		Nights.	Date. 1907.
443	+48°,5	0 0·8	+49 10	32·9	4·27	8·7	9·9	2	·740
444	+44,126	31·4	44 57	192·5	2·93	9·1	9·6	3	·968
445	+44,130	32·6	44 54	349·7	3·32	9·0	11·0	3	·968
446	+49,182	40·2	49 18	256·1	10·12	8·4	9·0	2	·867
447	+49,186	41·1	49 43	277·9	5·60	8·5	11·7	2	·836
448	...	59·1	50 2	81·3	2·50	9·5	9·7	2	·774
449	+49,352	1 13·8	49 58	212·8	11·20	8·5	9·1	3	·876
450	+47,414	21·7	47 37	147·4	9·52	8·0	13·5	2	·759 AB
				268·9	20·10	C =	13·0	2	·759 AC
451	+49,386	21·8	49 57	65·1	1·97	9·1	9·2	3	·886
452	+49,420	31·7	50 8	176·2	2·62	9·4	11·3	2	·881
453	+44,387	48·9	44 50	259·0	4·45	8·5	11·2	2	·994
454	+49,514	53·2	50 7	132·2	8·65	8·5	10·0	3	·786
455	+49,637	2 14·0	49 41	135·6	4·54	9·2	13·0	2	·850
456	+49,665	21·0	50 2	269·8	3·28	9·1	11·4	3	·821
457	+49,671	22·2	49 14	6·7	6·80	7·7	11·5	2	·759
458	+48,701	28·1	48 58	316·8	5·30	9·1	9·9	3	·886 AB
				243·9	24·58	C =	13·5	3	·886 AC
459	+48,708	29·9	48 56	142·7	3·35	8·9	10·2	4	·888
460	+48,711	30·9	48 13	316·8	2·68	9·1	9·3	3	·894 AB
				336·8	18·85	C =	13·5	1	·909 AC
461	+48,756	40·2	48 40	342·7	8·22	8·5	9·6	3	·900
462	...	3 7·9	49 23	172·9	2·55	9·7	10·4	2	·896
463	+49,891	9·5	49 21	257·3	4·95	9·3	11·7	2	·896
464	+47,806	14·2	47 21	65·8	7·15	9·1	9·2	2	·817
465	+49,1015	38·6	50 7	251·1	7·37	8·7	9·2	2	·896
466	+49,1092	56·7	49 32	57·1	3·95	8·5	11·0	2	·896 AB
				123·4	14·22	C =	12·0	2	·896 AC
467	...	5 2·4	47 25	98·2	1·52	11·0	11·3	2	·940
468	+30,3021	17 30·3	30 3	11·5	2·28	9·2	10·0	4	·613
469	+28,2829	42·8	28 1	140·9	4·41	10·0	13·0	3	·650 BC
				267·5	48·12	A =	8·5	2	·658 AB
470	+27,2943	59·3	27 51	206·5	7·17	8·6	9·8	3	·658
471	+27,2948	18 0·7	27 6	271·4	18·23	7·0	14·0	3	·676 AB
				44·1	30·35	C =	10·0	2	·671 AC
472	+27,2980	9·9	27 57	8·5	3·08	9·1	9·6	2	·655
473	+42,3026	10·0	42 49	158·0	9·63	8·5	10·0	3	·703 BC
				100·3	30·78	A =	8·5	3	·703 AB

No.	B.D.	R.A. 1900. h m	Decl. ° ′	P. °	D. ″	Mags.		Nights.	Date. 1907.
474	+42,3054	18 16·8	+42 55	34·5	3·68	9·2	12·3	3	·725
475	+27,3032	24·0	27 2	222·9	9·72	8·8	9·8	2	·671
476	+27,3041	26·7	28 0	347·6	2·52	9·5	10·1	3	·666
477	+26,3298	30·9	26 56	233·1	7·76	8·9	11·5	3	·677
478	+42,3144	40·5	42 30	182·3	8·22	8·7	9·2	2	·699
479	+27,3208	57·9	27 9	139·3	8·05	8·6	9·8	2	·603
480	+27,3218	58·9	27 33	304·2	2·45	9·1	10·1	2	·637
481	+27,3261	19 6·2	27 41	136·0	4·32	9·0	12·7	3	·650
482	+25,3756	9·7	25 36	167·4	4·27	8·9	11·2	2	·609
483	+25,3803	17·5	25 23	358·2	12·62	7·2	12·0	4	·634
484	+28,3322	20·8	28 44	332·6	7·30	8·5	14·0	3	·692 AB
				335·6	22·60	C =	9·0	3	·692 AC
485	+27,3419	27·5	27 55	342·4	2·37	9·2	10·5	2	·655
486	+25,3870	28·6	25 13	287·9	8·35	9·0	14·0	3	·656 BC
				18·2	45·24	A =	8·8	2	·633 AB
487	+27,3431	29·8	27 10	336·4	3·87	9·1	11·5	2	·708
488	+27,3434	29·9	27 16	50·5	2·12	9·1	9·2	2	·708
489	+27,3438	31·4	27 41	243·2	15·40	8·2	10·8	3	·714 AB
				205·5	20·32	C =	11·2	3	·714 AC
490	+43,3305	33·4	43 14	166·8	3·50	9·0	10·2	2	·740 BC
				229·7	16·38	a =	12·0	3	·758 Aa
				223·8	61·70	A =	8·9	2	·740 AB
491	+47,2874	33·5	47 54	57·7	9·67	9·1	9·6	2	·720 AB
				251·0	18·20	C =	14·0	1	·725 AC
492	+25,3902	34·2	25 28	158·1	5·42	9·2	9·3	2	·632
493	+43,3311	34·5	43 14	316·0	4·20	9·3	9·5	3	·758
494	+27,3549	49·6	27 49	190·1	3·65	8·9	10·2	2	·725 AB
				309·0	16·35	C =	12·0	1	·739 AC
495	+28,3553	54·3	28 43	310·5	4·11	11·0	11·8	3	·657 BC
				245·5	39·87	A =	8·0	2	·688 AB
496	+28,3594	58·5	28 42	279·2	2·22	8·9	10·0	2	·688
497	+29,3886	20 1·1	30 4	200·3	5·28	9·1	10·8	3	·658
498	+29,3889	1·7	30 6	241·3	9·38	8·3	11·2	3	·658
499	+45,3080	7·8	45 18	339·8	14·35	8·0	12·0	3	·784 AB
				77·3	26·20	C =	13·0	1	·756 AC
500	+49,3218	8·8	49 25	351·6	6·45	8·8	9·2	3	·891
501	+45,3093	9·0	45 17	125·3	1·32	9·1	10·2	2	·715
502	+48,3059	10·9	48 53	223·2	11·25	7·7	10·0	4	·897
503	...	16·5	47 48	353·1	4·57	9·2	9·5	3	·734
504	+28,3760	24·1	28 40	87·2	6·02	9·2	10·2	2	·783

No.	B.D.	R.A. 1900. h m	Decl. ° ′	P. °	D. ″	Mags.		Nights.	Date. 1907.
505	...	20 26·5	+30 9	13·0	1·92	9·8	10·0	2	·783
506	+28,3779	27·1	28 35	303·1	6·57	8·7	11·2	2	·654
507	+28,3790	28·9	28 31	295·5	7·30	8·5	11·0	2	·603 AB
				340·2	32·73	C =	8·6	2	·603 AC
				224·3	36·52	D =	9·1	2	·603 AD
508	+28,3847	36·9	29 4	230·1	2·25	8·6	10·0	2	·783 AB
				221·9	18·07	C =	10·0	2	·783 AC
509	+48,3187	37·5	48 39	283·6	7·22	8·8	12·0	3	·723
510	+27,3871	43·9	27 31	235·1	6·11	10·5	10·7	3	·704 BC
				343·3	43·45	A =	8·0	3	·704 AB
511	+26,4026	50·9	26 42	51·5	7·42	9·0	9·7	2	·640
512	+46,3201	21 5·5	46 52	18·3	13·27	7·0	12·3	3	·710
513	+45,3474	12·3	45 59	52·2	7·05	9·0	13·5	2	·847
514	+46,3246	12·3	46 54	195·9	3·85	8·9	9·3	3	·695
515	+25,4504	14·0	25 49	302·5	3·50	9·2	12·7	2	·721
	+48,3376	21·4	48 53	161·5	19·16	6·5	12·0	3	·709
516	...	26·7	46 51	140·5	3·31	9·4	11·0	3	·844
517	+28,4122	28·4	29 4	239·4	2·05	9·2	9·9	4	·781
518	+45,3589	30·6	46 3	11·9	4·55	9·2	12·3	3	·666
519	+47,3476	30·6	47 42	304·3	7·62	9·5	9·7	2	·801
520	+27,4125	34·6	27 59	38·3	7·19	9·4	9·9	4	·714
521	+28,4171	39·9	28 19	269·7	14·25	7·6	14·0	2	·628
522	+44,3972	48·7	44 46	174·7	4·35	9·0	11·0	2	·888
523	+26,4294	49·1	26 33	94·9	6·19	9·1	9·2	3	·764
524	+48,3536	49·6	48 52	5·5	5·87	11·5	13·3	3	·776 BC
				188·7	20·37	A =	9·0	2	·711 AB
525	+48,3544	50·9	48 48	149·1	3·82	8·6	10·5	2	·711
526	+25,4650	52·0	25 44	108·4	7·34	8·4	12·5	3	·771
527	+27,4230	56·8	27 21	207·4	3·05	9·7	9·9	2	·674
528	+46,3546	58·6	46 44	74·5	2·47	9·1	11·7	2	·720
529	+46,3555	59·4	46 17	71·6	5·75	8·5	11·0	2	·705
530	+47,3677	59·5	48 11	190·9	8·17	8·0	9·5	2	·703
531	+47,3680	22 0·2	48 10	225·1	2·55	9·2	11·5	3	·706
532	+46,3580	2·4	46 47	245·6	9·82	9·0	9·1	2	·720
533	+45,3848	10·8	45 23	321·5	4·25	9·0	12·0	2	·730
	+48,3665	13·5	48 39	292·4	21·22	7·0	12·0	2	·755
534	+48,3673	14·4	49 10	104·5	6·02	9·0	9·3	2	·789 BC
				248·7	9·37	a =	12·0	2	·789 Aa
				68·0	48·97	A =	8·5	2	·789 AB
535	+25,4724	20·0	25 39	301·9	8·33	9·3	10·8	3	·828

No.	B.D.	R.A. 1900. h m	Decl. ° ′	P. °	D. ″	Mags.		Nights.	Date. 1907.
536	...	22 20·4	+26 54	270·1	2·97	10·2	10·2	2	·673
537	+49,3855	23·2	50 9	19·7	2·27	9·2	9·4	2	·806
538	+46,3716	24·7	46 24	66·8	3·40	9·1	10·5	2	·889
539	+42,4437	28·3	42 36	287·7	6·10	9·1	11·0	3	·836
540	+48,3762	28·7	48 26	281·5	1·30	9·1	9·9	3	·709
541	+25,4787	35·9	26 13	220·3	6·61	8·0	12·4	4	·751
542	+49,3968	48·9	49 29	282·6	5·05	8·7	10·5	3	·710 AB
				83·9	57·88	C =	8·8	2	·708 AC
543	+46,3945	23 3·8	46 58	22·0	3·87	9·2	10·5	2	·783
544	+49,4054	4·2	49 22	245·6	2·30	9·5	9·7	2	·951
	+25,4927	17·5	25 22	264·4	19·95	6·7	13·0	2	·713
545	+48,4024	18·9	48 44	319·7	9·70	8·1	12·0	3	·755
546	+26,4623	20·1	26 24	162·2	2·37	9·1	10·8	3	·683
547	+46,4096	31·9	46 46	237·1	6·95	8·5	12·0	2	·795
548	+46,4139	39·8	46 54	315·9	14·37	8·5	8·8	2	·835
549	+47,4264	40·0	47 57	51·7	8·10	8·5	13·7	2	·704 AB
				238·7	16·07	C =	9·0	2	·704 AC
550	+45,4323	41·8	46 7	167·2	17·35	8·6	8·9	2	·814
551	+47,4313	23 49·5	47 41	311·6	4·72	11·7	12·7	2	·834 BC
				11·4	24·60	a =	10·5	1	·865 Aa
				88·1	30·90	A =	8·6	2	·834 AB

Appearance of Ring-extension during the disappearance of Saturn's Ring, Oct.–Nov. 1907. By R. T. A. Innes.

As Professor Campbell's observation of the appearance of Saturn's ring will give rise to some discussion, I do not delay sending the notes I have made so far. The telescope used is the 9-inch Grubb refractor mentioned in an earlier note. Sir Howard Grubb has supplied it with a double concave eyepiece of high magnifying power, which transmits at least twice as much light as an ordinary eyepiece. This negative eyepiece gives an erect image; its only drawback is an excessively small field.

On several occasions Mr. R. N. Kotze, my son and myself noticed the beaded appearances referred to by Professor Campbell, but we assumed that they were caused by either Mimas or Enceladus, and our curiosity did not lead us further.

I now give, without further comment, the exact notes made by myself at the telescope:—

1907 Oct. 16. Ring seen at moments.

17. Ring all but held steadily. It is very faint and not full length.

1907 Oct. 21. Extension of ring clearly seen on f. side; not so distinct on p. side, but Titan and other satellites near.

Although I refer to the appearance as the "Ring" on the 16th and 17th, this word might convey a wrong impression. What is seen is a ghost-like extension where the ring was. It tapers off sharply, and is perhaps shorter than the ring.

22. to Nov. 20. Between these dates this ghost-like ring was seen on all occasions that the planet was looked at.

Nov. 21. Planet just past meridian. Ring-extension seen to full length on each side. It is brightest close to the planet, and then it is discontinuous. Two cloud-bands on each side of equator. Shadow of ring on planet is a deep brown, and there is a suspicion, but nothing more than a suspicion, of its being divided into two lines. (On Nov. 6 it was noticed that the shadow of Titan was much darker than the ring-shadow.)

25. Ring-extension decidedly discontinuous, and doubtful if full length. The satellites no longer bead the ring-extension, but pass N or S of it.

A rough sketch will give an idea of the relative darkness of the sky, the ring-shadow, cloud-bands, and of the discontinuity of the ring-extension about the dates Nov. 22–25.

Professor Campbell's message reached here on the 28th inst.

Dec. 1. Ghost-like extension, including discontinuities, is about half the diameter of Saturn on each side, and is discontinuous. It leaves the disc on the south side of the ring-shadow.

Johannesburg:
1907 *December 2.*

Observations of Saturn's Ninth Satellite, Phœbe, *from Photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich, in* 1907.

(*Communicated by the Astronomer Royal.*)

Phœbe has been under observation during the recent opposition of Saturn with the 30-inch reflector, photographs having been obtained whenever possible. In all 16 photographs have been secured on 16 nights, between August 10 and December 6.

From one to two hours' exposure was necessary on account of the faintness of the satellite and its low altitude (5° south of the equator).

The positions of the satellite have been measured on the photographs taken with the reflector with reference to three or four faint comparison stars (of eleventh or twelfth magnitude), symmetrically distributed about the satellite. The positions of these faint comparison stars were then measured relatively to the reference stars (of eighth to ninth magnitude) in the Astronomische Gesellschaft Catalogue on photographs (with 30^m and 40^m exposure) taken with the Astrographic 13-inch refractor. The field, sensibly free from distortion, being much larger with this telescope than with the reflector, from 16 to 20 reference stars were available on each plate.

As Saturn moved slowly, it was possible to make one reference plate serve for several photographs which were each referred to it. The constants were determined in the usual manner, all the stars on the plate given in the A.G. Catalogues being used for this purpose. Right Ascensions and Declinations of Phœbe were then determined and compared with the tabular positions of Saturn after applying to the tabular places the corrections

R.A. $+^s\cdot051$ Dec. $+0''\cdot48$

as determined below.

Observations of Phœbe.

Date and G.M.T.				Apparent R.A.			Apparent Dec.			Phœbe − Saturn R.A.		Phœbe − Saturn Dec.		Exp.
	d	h	m	s	h	m	s	°	′	″	m	s	′	″	m
Aug.	10	12	55	35	23	50	53·23	−3	34	29·7	−0	27·31	−4	22·0	105
	11	12	39	47		50	41·08		36	0·6	0	28·71	4	29·5	133
	18	12	39	10		49	7·84		47	44·6	0	38·25	5	39·8	143
	19	12	53	50		48	53·36	3	49	32·2	0	39·62	5	50·1	120
	29	11	4	59		46	19·31	4	8	4·7	0	52·53	7	22·6	93
Sept.	2	10	56	11		45	12·25		15	54·4	0	57·50	7	55·9	54
	9	11	45	18		43	9·46		30	1·9	1	5·68	8	53·6	107
	10	11	47	20		42	51·68	4	32	2·2	1	6·74	9	0·2	70
Oct.	2	10	36	20		36	21·70	5	14	36·1	1	27·68	11	16·1	90
	4	10	52	26		35	47·97		18	7·7	1	29·11	11	24·7	120
	8	10	44	28		34	42·92		24	51·1	1	31·83	11	40·2	53
	12	9	1	36		33	41·99		31	1·8	1	34·22	11	52·4	90
	30	8	33	44		29	55·18		52	45·3	1	40·62	12	18·1	100
Dec.	3	6	34	27		28	4·66		56	56·9	1	33·00	10	45·9	121
	5	6	35	9		28	12·86		55	35·8	1	31·77	10	36·3	120
	6	6	45	38	23	28	17·69	5	54	49·7	−1	31·04	−10	30·0	90

Errors of Tabular Place of Saturn.

The positions of Phœbe given above depend on the positions of certain stars taken from two catalogues of the Astronomische Gesellschaft, and will be affected by the mean error in the places of these stars. To eliminate this it is necessary to determine the position of Saturn referred to the same stars.

For this purpose a series of photographs of Saturn was taken with the Thompson 26-inch refractor, using the occulting shutter. With a rather slow plate an exposure of 2 minutes was given to the stars in the field, Saturn being exposed for about $\frac{1}{100}$th second every alternate second. In this way a good measurable disc of Saturn was obtained, whilst giving sufficient exposure for the stars.

The field of the 26-inch refractor on a 16 c.m. plate being only 1° square, or a quarter the area of the Astrographic plate, only three or four stars were usually available on each plate, but the positions of these were deduced from measures made on the Astrographic reference plates for Phœbe, using all the stars. The mean systematic error of these three or four stars will therefore be the same as if all the stars had been used, and the position of Saturn deduced will be affected by the same error, and will then be comparable with the deduced places of Phœbe.

Eleven photographs were selected for measurement. From two to four images of Saturn, and of each of the stars, were measured on each plate in the Astrographic micrometer, and the following are the results obtained. The observed R.A. and Dec. of Saturn were computed to $0^s{\cdot}001$ and $0''{\cdot}01$ respectively; but as the tabular place in the N.A. is only given to $0^s{\cdot}01$ and $0''{\cdot}1$ respectively, the errors are only given to that order of accuracy.

Errors of N.A. Tabular Place of Saturn.

		Tab.—Obs. R.A.	Tab.—Obs. Dec.
		s	"
July	24	+·01	−0·3
Aug.	18	−·06	·5
	19	·05	·5
	19	·06	·1
Sept.	9	·09	·6
	23	·08	·5
Oct.	2	·07	·4
	4	·04	·3
	12	·05	·7
	15	·05	·6
	30	−·02	−0·8
	Mean	−$^s{\cdot}051$	−$0''{\cdot}48$

Royal Observatory, Greenwich:
1908 *January* 8.

Observations of Occultations of Stars by the Moon, made at the Royal Observatory, Greenwich, in the Year 1907.

(*Communicated by the Astronomer Royal.*)

Day, 1907.		Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation.	Observer.
						h m s	
January	26	Disapp. ν Geminorum ...	Old Altazimuth	100	Dark	6 39 40·12	A. C.
	26	,, ,, ...	Sheepshanks Equatorial ...	100	,,	6 39 39·76	J. E.
February	23	,, ζ Geminorum ...	Astrographic Equatorial ...	225	,,	7 10 30·37	W. S.
	23	,, Piazzi VI. 311 ...	,, ,, ...	225	,,	7 11 40·18	W. S.
	23 (*b*) (*c*)	,, ,, ...	Sheepshanks Equatorial ...	100	,,	7 11 40·46	J. E.
March	20	,, *m* Tauri	Old Altazimuth	100	,,	9 23 21·42	S. E.
	20 (*a*)	,, ,, ...	Astrographic Equatorial ...	225	,,	9 23 20·59	H.
	20	,, ,, ...	Sheepshanks Equatorial ...	100	,,	9 23 21·23	R. C.
	21	,, χ^1 Orionis	Old Altazimuth	100	,,	7 14 42·47	S. E.
	21 (*a*)	,, ,, ...	Great Equatorial	670	,,	7 14 42·57	W. B.
	21	,, ,, ...	Sheepshanks Equatorial ...	100	,,	7 14 42·08	J. E.
	21	,, ,, ...	Astrographic Equatorial ...	225	,,	7 14 41·89	W. S.
	21 (*a*)	,, ,, ...	Merz Refractor	250	,,	7 14 41·92	R. F.
	21	,, B.D. $+19^\circ$, 1146 ...	Astrographic Equatorial ...	225	,,	9 32 7·46	W. S.
	21	,, B.D. $+20^\circ$, 1202 ...	Merz Refractor	250	,	10 6 16·43	C. D.
	21	,, B.D. $+20^\circ$, 1209 ...	Astrographic Equatorial ...	225	,,	10 37 8·75	W. S.
	21	,, B.D. $+20^\circ$, 1211 ...	,, ,, ...	225	,,	10 46 22·22	W. S.
	21	,, χ^4 Orionis	Merz Refractor	250	,,	12 28 49·33	R. F.

Day, 1907.		Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
March	21	Disapp. χ^4 Orionis ...	Sheepshanks Equatorial ...	100	Dark	12 28 51·23	J. E.
	22	,, B.D. +20°, 1578 ...	Astrographic Equatorial ...	225	,,	9 25 16·99	W. S.
	22	,, B.D. +20°, 1590 ...	,, ,, ...	225	,,	11 17 9·63	W. S.
	23	,, Piazzi VII. 44 ...	,, ,, ...	225	,,	7 16 13·93	W. S.
	23	,, B.D. +20°, 1854 ...	,, ,, ...	225	,,	7 30 46·53	W. S.
	23	,, B.D. +20°, 1866 ...	,, ,, ...	225	,,	8 39 26·01	W. S.
	24	,, B.D. +19°, 2021 ...	,, ,, ...	225	,,	8 27 42·81	W. S.
	24	,, B.D. +19°, 2024 ...	,, ,, ...	225	,,	8 57 22·65	W. S.
	24	,, B.D. +18°, 1978 ...	,, ,, ...	225	,,	11 0 43·41	W. S.
	28	,, *b* Virginis	,, ,, ...	225	,,	12 12 27·99	A. L.
April	19 (*a*)	,, 56 Geminorum ...	,, ,, ...	225	,,	9 23 17·04	H. F.
May	24	,, *n* Virginis	Sheepshanks Equatorial ...	120	,,	10 18 19·31	S. E.
June	22	,, B.D. −13°, 4111 ...	Astrographic Equatorial ...	225	,,	9 40 18·98	A. L.
	22	,, ,, ...	Sheepshanks Equatorial ...	100	,,	9 40 16·71	J. S.
	24	,, ξ Ophiuchi	Thompson Equatorial (Hodgson)	100	,,	11 39 48·01	D. E.
	24	,, ,, ...	Astrographic Equatorial ...	225	,,	11 39 48·13	J. S.
	24	,, ,, ...	Sheepshanks Equatorial ...	100	,,	11 39 48·03	R. C.
September	24	,, μ Ceti	,, ,, ...	100	Bright	12 50 34·00	R. F.
	24 (*a*)	Reapp. ,, ...	Astrographic Equatorial ...	225	Dark	13 45 30·90	W.

Day, 1907.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
September 26	Disapp. δ^2 Tauri	Astrographic Equatorial ...	225	Bright	11 57 3·23	S. E.
October 2	Reapp. 8 Leonis	,, ,, ...	225	Dark	17 11 8·27	H. F.
24	,, i Tauri	,, ,, ...	225	,,	10 18 8·18	W. S.
December 11	Disapp. Lalande 45490 ...	Sheepshanks Equatorial ...	100	,,	6 6 18·79	A. C.
11	,, ,, ...	Astrographic Equatorial ...	225	,,	6 6 18·54	A. L.
17	,, δ^1 Tauri	,, ,, ...	225	,,	12 52 16·14	S. E.
17	,, ,, ...	Old Altazimuth	100	,,	12 52 16·20	A. C.
17	,, ,, ...	Sheepshanks Equatorial ...	100	,,	12 52 16·40	R. C.
17	,, δ^3 Tauri	Astrographic Equatorial ...	225	,,	14 8 18·08	S. E.

(*a*) Instantaneous. (*b*) Observation doubtful. (*c*) Very faint, ill-defined.

The apertures of the telescopes used are as follow:—

Great Equatorial	28 inches.	Great Equatorial (Corbett Telescope) ...	6½ inches.
Merz Refractor	12¾ ,,	Thompson Equatorial (Hodgson Telescope)	6 ,,
Astrographic Equatorial (Guiding Telescope)	10 ,,	Old Altazimuth	4 ,,
Sheepshanks Equatorial	6¾ ,,		

The initials, S. E., H., A. C., C. D., D. E., W. B., H. F., W., J. S., W. S., J. E., R. C., R. F., A. L., are those of Mr. Eddington, Mr. Hollis, Mr. Crommelin, Mr. Davidson, Mr. Edney, Mr. Bowyer, Mr. Furner, Mr. Witchell, Mr. Storey, Mr. Stevens, Mr. J. Evans, Mr. Cullen, Mr. Fowler, and Mr. Lyne, respectively.

Royal Observatory, Greenwich:
1908 *January* 7.

LIBRARY OF THE UNIVERSITY OF

Day, 1907.		Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
March	21	Disapp. χ^4 Orionis ...	Sheepshanks Equatorial ...	100	Dark	12 28 51·23	J. E.
	22	,, B.D. +20°, 1578 ...	Astrographic Equatorial ...	225	,,	9 25 16·99	W. S.
	22	,, B.D. +20°, 1590 ...	,, ,, ...	225	,,	11 17 9·63	W. S.
	23	,, Piazzi VII. 44 ...	,, ,, ...	225	,,	7 16 13·93	W. S.
	23	,, B.D. +20°, 1854 ...	,, ,, ...	225	,,	7 30 46·53	W. S.
	23	,, B.D. +20°, 1866 ...	,, ,, ...	225	,,	8 39 26·01	W. S.
	24	,, B.D. +19°, 2021 ...	,, ,, ...	225	,,	8 27 42·81	W. S.
	24	,, B.D. +19°, 2024 ...	,, ,, ...	225	,,	8 57 22·65	W. S.
	24	,, B.D. +18°, 1978 ...	,, ,, ...	225	,,	11 0 43·41	W. S.
	28	,, *b* Virginis	,, ,, ...	225	,,	12 12 27·99	A. L.
April	19 (*a*)	,, 56 Geminorum ...	,, ,, ...	225	,,	9 23 17·04	H. F.
May	24	,, *n* Virginis	Sheepshanks Equatorial ...	120	,,	10 18 19·31	S. E.
June	22	,, B.D. −13°, 4111 ...	Astrographic Equatorial ...	225	,,	9 40 18·98	A. L.
	22	,, ,, ...	Sheepshanks Equatorial ...	100	,,	9 40 16·71	J. S.
	24	,, ξ Ophiuchi	Thompson Equatorial (Hodgson)	100	,,	11 39 48·01	D. E.
	24	,, ,, ...	Astrographic Equatorial ...	225	,,	11 39 48·13	J. S.
	24	,, ,, ...	Sheepshanks Equatorial ...	100	,,	11 39 48·03	R. C.
September	24	,, μ Ceti	,, ,, ...	100	Bright	12 50 34·00	R. F.
	24 (*a*)	Reapp. ,, ...	Astrographic Equatorial ...	225	Dark	13 45 30·90	W.

Day, 1907.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
September 26	Disapp. δ^2 Tauri	Astrographic Equatorial ...	225	Bright	11 57 3·23	S. E.
October 2	Reapp. 8 Leonis	,, ,, ...	225	Dark	17 11 8·27	H. F.
24	,, ι Tauri	,, ,, ...	225	,,	10 18 8·18	W. S.
December 11	Disapp. Lalande 45490 ...	Sheepshanks Equatorial ...	100	,,	6 6 18·79	A. C.
11	,, ,, ...	Astrographic Equatorial ...	225	,,	6 6 18·54	A. L.
17	,, δ^1 Tauri	,, ,, ...	225	,,	12 52 16·14	S. E.
17	,, ,, ...	Old Altazimuth	100	,,	12 52 16·20	A. C.
17	,, ,, ...	Sheepshanks Equatorial ...	100	,,	12 52 16·40	R. C.
17	,, δ^3 Tauri	Astrographic Equatorial ...	225	,,	14 8 18·08	S. E.

(*a*) Instantaneous. (*b*) Observation doubtful. (*c*) Very faint, ill-defined.

The apertures of the telescopes used are as follow:—

Great Equatorial	28 inches.	Great Equatorial (Corbett Telescope) ...	6½ inches.
Merz Refractor	12¾ ,,	Thompson Equatorial (Hodgson Telescope)	6 ,,
Astrographic Equatorial (Guiding Telescope)	10 ,,	Old Altazimuth	4 ,,
Sheepshanks Equatorial	6¾ ,,		

The initials, S. E., H., A. C., C. D., D. E., W. B., H. F., W., J. S., W. S., J. E., R. C., R. F., A. L., are those of Mr. Eddington, Mr. Hollis, Mr. Crommelin, Mr. Davidson, Mr. Edney, Mr. Bowyer, Mr. Furner, Mr. Witchell, Mr. Storey, Mr. Stevens, Mr. J. Evans, Mr. Cullen, Mr. Fowler, and Mr. Lyne, respectively.

Royal Observatory, Greenwich:
1908 *January* 7.

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Ephemeris of Flora near the time of Opposition in 1908. By A. M. W. Downing, D.Sc., F.R.S.

This ephemeris is computed from Brünnow's *Tafeln der Flora*, in combination with the corrected continuation of certain of the tables published in *Monthly Notices*, vol. lxiv., No. 6.

Berlin Midnight, 1908.	Apparent R.A.	Apparent Dec.	Log Distance from Earth.
	h m s	° ′ ″	
May 1	15 56 44·46	−12 5 29·7	0·19103
2	55 49·64	2 26·6	·18968
3	54 53·70	11 59 24·5	·18840
4	53 56·71	56 23·6	·18720
5	52 58·73	53 24·0	·18607
6	51 59·82	50 25·9	·18500
7	51 0·05	47 29·6	·18400
8	49 59·53	44 35·2	·18308
9	48 58·30	41 42·9	·18223
10	47 56·45	38 52·9	·18146
11	46 54·07	36 5·5	·18077
12	45 51·23	33 21·0	·18015
13	44 47·96	30 39·6	·17961
14	43 44·29	28 1·5	·17915
15	42 40·27	25 27·0	·17876
16	41 35·98	22 56·2	·17845
17	40 31·51	20 29·3	·17822
18	39 26·95	18 6·4	·17806
19	38 22·39	15 48·0	·17798
20	37 17·92	13 34·2	·17798
21	36 13·61	11 25·1	·17806
22	35 9·50	9 20·8	·17821
23	34 5·68	7 21·8	·17844
24	33 2·25	5 28·2	·17875
25	31 59·24	3 40·2	·17914
26	30 56·74	1 58·0	·17960
27	29 54·78	11 0 21·8	·18014
28	28 53·47	10 58 51·8	·18075
29	27 52·91	57 28·2	·18143
30	26 53·13	56 11·2	·18219
31	15 25 54·19	−10 55 0·8	·18302

Berlin Midnight. 1908.	Apparent R.A. h m s	Apparent Dec. ° ′ ″	Log Distance from Earth.
June 1	15 24 56·11	− 10 53 57·0	0·18392
2	23 59·00	53 0·3	·18489
3	23 2·97	52 10 9	·18592
4	22 8·05	51 28·9	·18702
5	21 14·27	50 54·2	·18819
6	20 21·64	50 26·9	·18942
7	19 30·25	50 7·1	·19071
8	18 40·18	49 55·0	·19206
9	17 51·47	49 50·7	·19347
10	17 4·13	49 54·1	·19493
11	16 18·17	50 5·2	·19645
12	15 33·62	50 24·0	·19802
13	14 50·56	50 50·6	·19964
14	14 9·03	51 25·2	·20131
15	15 13 29·07	− 10 52 7·7	0·20304

Magnitude at opposition, May 18 = 9·7.

Victor A. Lowinger, Trigonometrical Survey Department, Taiping, Federated Malay States;
Rev. Malcolm Parker Miller McLean, M.A., The Rectory, West Raynham, Norfolk;
Alfred William Porter, B.Sc., Assistant Professor of Physics, University College, London, W.C.;
Harold Knox Shaw, B.A., Trinity College, Cambridge; and
Capt. Eldred Weston White, Brockley Villa, Upton Road, Southville, Bristol,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

John Bullock, M.A., 78 Airedale Avenue, Chiswick, W. (proposed by J. D. McClure);
Arthur Brunel Chatwood, B.Sc., A.M.Inst.C.E., Astronomer to H.H. the Nizam, Hyderabad, Deccan, India (proposed by H. H. Turner);
F. R. Cripps, 22 Hornsey Rise Gardens, N. (proposed by A. C. D. Crommelin);
Henry Zouch Darrah, C.S.I., Indian Civil Service, Junior Carlton Club, Pall Mall, S.W. (proposed by Sir R. S. Ball);
Charles Gaskell Falkner, M.A., Ireton Bank, Rusholme, Manchester (proposed by Sir Andrew Noble);
Henry William Moore, B.A., 64 Curzon Street, Mayfair, W., and New University Club, St. James's Street, S.W. (proposed by J. J. Atkinson); and
Harold Morris-Airey, M.Sc., Lecturer in Physics, Armstrong College, Newcastle-upon-Tyne (proposed by Thomas Thorp).

Report of the Council to the Eighty-eighth Annual General Meeting of the Society.

The following table shows the progress and present state of the Society :—

	Patron	Honorary Members	Fellows		Associates	Total
			Compounders	Annual Subscribers		
1906 December 31	1	3	260	409	48	721
Since elected	...	...	+4	+21	+1	...
Deceased	...	−1	−8	−6	−5	...
Resigned	...	...	...	−3	...	...
Removals	...	...	+3	−3	...	...
Expelled	...	...	...	−1	...	...
1907 December 31	1	2	259	417	44	723

Major Hills' Account as Treasurer of the Royal

RECEIVED.

	£	s.	d.	£	s.	d.
Balances, 1906 December 31 :—						
At Bankers', as per Pass-book	14	1	0			
In hand of Assistant Secretary on Account of Turnor and Horrox Fund	4	6	0			
In hand of Assistant Secretary on Petty Cash Account	0	17	5			
				19	4	5
Dividends :—						
£1,250 Metropolitan 3-per-cent. Stock ...	35	12	8			
£1,964 12 7 Swansea Corporation 3½-per-cent. Stock	65	6	6			
£3,400 East Indian Railway 3-per-cent. Debenture Stock	96	18	0			
£3,200 London and North-Western Railway 3-per-cent. Debenture Stock	91	4	0			
£4,000 Midland Railway 2½-per-cent. Debenture Stock	95	0	0			
£500 Lancashire and Yorkshire Railway 3-per-cent. Consolidated Preference Stock ...	14	5	0			
£1,860 Gas Light and Coke Co. 3-per-cent. Debenture Stock	53	0	2			
£1,650 Commercial Gas Co. 3-per-cent. Debenture Stock	47	0	6			
				498	6	10
Received on account of Subscriptions :—						
Arrears	129	3	0			
Annual Contributions for 1907	573	6	0			
,, ,, in advance	8	8	0			
Admission Fees	52	10	0			
First Contributions	36	15	0			
				800	2	0
Composition Fees...				175	7	0
Sales of Publications, &c. :—						
At Williams & Norgate's, 1906	44	18	1			
At Society's Rooms, 1906	108	5	2			
Sales of Photographs, 1907	54	17	0			
				208	0	3
Income Tax refunded by Commissioners of Inland Revenue				26	14	11
				£1,727	15	5

Examined and found correct.

RICHARD INWARDS.
G. J. NEWBEGIN.
A. E. CONRADY.

Astronomical Society, from 1907 *January* 1 *to December* 31.

PAID.

	£	s.	d.	£	s.	d.
Assistant Secretary: Salary	250	0	0			
,, ,, Editing Society's Publications	50	0	0			
Clerk's Salary	75	0	0			
				375	0	0
House Duty	2	12	6			
Fire and Servants' Insurance	9	12	6			
				12	5	0
Printing, &c.:—						
Monthly Notices (Spottiswoode & Co.)	41	12	3			
,, ,, (Neill & Co.)	327	14	1			
Appendix to *Memoirs* (Harrison & Sons) ...	21	10	0			
List of Fellows (Neill & Co.)	15	16	0			
Miscellaneous (Spottiswoode & Co.)	16	7	6			
Photo-Engraving for *Monthly Notices* (A. E. Dent & Co.)	13	5	1			
				436	4	11
Turnor and Horrox Fund, purchase of books for Library	23	3	6			
Binding books in Library	38	17	4			
				62	0	10
Reproduction of Photographs (Hinton & Co.) ...				41	16	4
Cataloguing Astronomical Literature for the International Catalogue of Scientific Literature ...				30	0	0
Expenses of Meetings	20	7	11			
Lantern expenses	6	5	2			
Time Signal: rental of wire	5	0	0			
				31	13	1
Postage and Telegrams	94	2	10			
Carriage of Parcels	3	19	4			
Stationery and Office expenses	8	7	5			
,, (Spottiswoode & Co.)	2	0	6			
				108	10	1
Album for Photographs of Associates				2	10	0
House expenses: Allowance and sundry expenses	59	0	9			
Coal and Gas	44	19	7			
Electric Light expenses	19	11	11			
Repairing Council-room Chairs (Shoolbred & Co.)	9	15	0			
Sundry Fittings and Repairs	9	13	4			
Sundries	3	18	1			
				146	18	8
Gold Medals (A. Wyon)				52	12	6
Cheque-book and bankers' charges on cheques ...				0	10	9
Balances, 1907 December 31:—						
At Bankers', as per Pass-book	386	7	8			
Cheques not credited till 1908	27	2	0			
In hand of Assistant Secretary on account of Turnor & Horrox Fund	6	2	6			
In hand of Assistant Secretary on Petty Cash Account	8	1	1			
				427	13	3
				£1,727	15	5

Report of the Auditors.

We have examined the Treasurer's account of receipts and expenditure for the year 1907, and have found and certified the same to be correct. The cash in hand on December 31, 1907, including the balance at the bankers', etc., amounted to £427, 13*s.* 3*d.*

The funded property of the Society is the same as at the end of the previous year, but we are glad to report an improvement in the financial position of the Society, as shown by the balance on December 31, which exceeds that of January 1, 1907, by more than £400.

We have examined the Stock Certificates and the Bank of England verification of the Inscribed Stock standing in the name of the Society.

The books, instruments, and other effects in the possession of the Society have been examined, and they appear to be in a satisfactory condition.

We have laid on the table a list of the names of those Fellows who are in arrear for sums due at the last Annual General Meeting of the Society, with the amount due against each Fellow's name.

(*Signed*) RICHARD INWARDS.
G. J. NEWBEGIN.
A. E. CONRADY.

Bequests to the General Funds of the Society.

The Carrington Bequest (1876): A sum of £2,000 Consols. Sold in 1899, and stands now in £1,881 14*s.* London and North-Western Railway 3-per-cent. Debenture Stock.

The McClean Bequest (1905): A sum of £2,000. Invested in Swansea Corporation 3½-per-cent. Stock.

The Farrar Bequest (1906): A sum of £100. Invested in Swansea Corporation 3½-per-cent. Stock.

The cost of reporting Professor Hale's lecture at the special evening meeting on June 26, 1907, was defrayed by Mr. Franklin-Adams.

Trust Funds.

The Turnor Fund: A sum of £464 18*s.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be used in the purchase of books for the Library.

The Horrox Memorial Fund: A sum of £103 6*s.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be used in the purchase of books for the Library.

The Lee and Janson Fund: A sum of £334 10*s.* 9*d.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be given by the Council to the widow or orphan of any deceased Fellow of the Society who may stand in need of it.

The Hannah Jackson (née Gwilt) Fund: A sum of £309 18*s.* 6*d.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be given in medals or other awards, in accordance with the terms of the Trust.

Assets and Present Property of the Society, 1908 January 1.

	£	s.	d.	£	s.	d.
Balances, 1907 December 31 :—						
At Bankers', as per Pass-book	386	7	8			
Country cheques not credited till 1908 ...	27	2	0			
In hand of Assistant Secretary on account of Turnor and Horrox Fund	6	2	6			
In hand of Assistant Secretary on Petty Cash Account	8	1	1			
				427	13	3
Due on account of Subscriptions :—						
4 Contributions of 6 years' standing	50	8	0			
6 ,, 5 ,, ,, ...	63	0	0			
9 ,, 4 ,, ,, ...	75	12	0			
46 ,, 2 ,, ,, ...	193	4	0			
61 ,, 1 ,, ,, ...	128	2	0			
Other sums	6	6	0			
	516	12	0			
Less Subscriptions paid in advance	8	8	0			
				508	4	0
Due for Photographs sold				2	0	0
Due from Williams & Norgate for Publications sold in 1907 ...				23	0	9
£3,400 East Indian Railway 3-per-cent. Debenture Stock, including the Turnor Fund, the Horrox Memorial Fund, the Lee and Janson Fund, and the Hannah Jackson (*née* Gwilt) Fund.						
£3,200 London and North-Western Railway 3-per-cent. Debenture Stock (including the Carrington Bequest).						
£4,000 Midland Railway 2½-per-cent. Debenture Stock.						
£500 Lancashire and Yorkshire Railway 3-per-cent. Consolidated Preference Stock.						
£1,860 Gas Light and Coke Co. 3-per-cent. Debenture Stock.						
£1,650 Commercial Gas Co. 3-per-cent Debenture Stock.						
£1,250 Metropolitan 3-per-cent. Stock.						
£1,964 12 7 Swansea Corporation 3½-per-cent. Stock (the McClean and Farrar Bequests).						
Astronomical and other Manuscripts, Books, Prints, Photographs, and Instruments.						
Furniture, &c.						
Stock of Publications of the Society.						
Four Gold Medals.						

Celestial Photographs.

The following is a list of reproductions of Celestial Photographs published by the Royal Astronomical Society for sale to the Fellows:—

R.A.S. Ref. No.	Subject.	Photographed by
1	Total Solar Eclipse, 1889 January 1	W. H. Pickering
2	Total Solar Eclipse, 1893 April 16	J. M. Schaeberle
3	Total Solar Eclipse, 1886 August 29	A. Schuster
4	Nebulæ in the *Pleiades*	Isaac Roberts
5	Nebula M 74 *Piscium* (N.G.C. 628)	Isaac Roberts
6	Great Nebula in *Orion*	Isaac Roberts
7	Milky Way near M 11	E. E. Barnard
8	Milky Way near Cluster in *Perseus*	E. E. Barnard
9	Comet *c* 1893 IV. (Brooks), 1893 October 21	E. E. Barnard
10	Comet *a* 1892 I. (Swift), 1892 April 7	E. E. Barnard
11	Nebula about η *Argus*	David Gill
12	Portion of Moon (Hyginus-Albategnius)	Loewy and Puiseux
13	Comet *c* 1893 IV. (Brooks), 1893 October 22	E. E. Barnard
14	Comet *c* 1893 IV. (Brooks), 1893 October 20	E. E. Barnard
15	Comet *c* 1893 IV. (Brooks), 1893 November 10	E. E. Barnard
16	Comet *a* 1892 I. (Swift), 1892 April 26	E. E. Barnard
17	Comet *f* 1892 III. (Holmes), 1892 November 10	E. E. Barnard
18	Comet *a* 1892 I. (Swift), 1892 April 18	E. E. Barnard
19	Portion of Moon (Alps, Apennines, &c.)	Loewy and Puiseux
20	Nebula in *Andromeda*	Isaac Roberts
21	*Jupiter*, 1892 September 26	Lick Observatory
22	Cluster M 13 *Herculis* (N.G.C. 6205)	W. E. Wilson
23	Total Solar Eclipse, 1893 April 16 (5 sec.)	J. Kearney
24	Total Solar Eclipse, 1893 April 16 (20 sec.)	J. Kearney
25	The Moon (Age $7^d\ 3^h$)	Lick Observatory
26	The Moon (Age $12^d\ 6\frac{1}{2}^h$)	Lick Observatory
27	The Moon (Age $16^d\ 18^h$)	Lick Observatory
28	The Moon (Age $23^d\ 8^h$)	Lick Observatory
29	The Sun, 1892 February 13	Roy. Obs., Greenwich
30	The Sun, 1892 July 8	Roy. Obs., Greenwich
31	Portion of Moon (Region of Maginus)	Loewy and Puiseux
32	The Moon (Age $14^d\ 1^h$)	Lick Observatory
33	Portion of Moon (Ptolemæus, &c.)	Lick Observatory
34	Portion of Moon (Mare Serenitatis)	Lick Observatory
35	Portion of Moon (Clavius, Licetus, &c.)	Lick Observatory

R.A.S. Ref. No.	Subject.	Photographed by
36	Portion of Moon (Regiomontanus, &c.)	Lick Observatory
37	Portion of Moon (Tycho, Thebit, &c.)	Lick Observatory
38	Portion of Moon (Theophilus, &c.)	Lick Observatory
39	Total Solar Eclipse, 1896 August 9 (3 sec.)	S. Kostinsky
40	Total Solar Eclipse, 1896 August 9 (26 sec.)	A. Hansky
41	Cluster M 56 *Lyræ* (N.G.C. 6779)	
42	Nebulæ M 81, 82 *Ursæ Majoris* (N.G.C. 3031, 3034)	
43	Cluster M 56 *Lyræ* (N.G.C. 6779) (enlarged)	
44	Solar Corona, 1871 December 12, Baikul	H. Davis
45	Solar Corona, 1875 April 6, Siam	Lockyer and Schuster
46	Solar Corona, 1878 July 29, Wyoming	W. Harkness
47	Solar Corona, 1882 May 17, Egypt	Abney and Schuster
48	Solar Corona, 1883 May 6, Caroline Island	Lawrance and Woods
49	Solar Corona, 1885 Sept. 9, Wellington, N.Z.	Radford
50	Solar Corona, 1886 August 29, Grenada, W.I.	A. Schuster
51	Solar Corona, 1887 August 19, Japan	M. Sugiyama
52	Solar Corona, 1889 January 1, California	W. H. Pickering
53	Solar Corona, 1889 December 22, Cayenne	J. M. Schaeberle
54	Solar Corona, 1893 April 16, Fundium	J. Kearney
55	Solar Corona, 1893 April 16, Brazil	A. Taylor
56	Great Nebula in *Orion*	W. E. Wilson
57	Dumb-bell Nebula, *Vulpecula* (N.G.C. 6853)	W. E. Wilson
58	Spiral Nebula, *Canes Venatici* (N.G.C. 5194–5)	W. E. Wilson
59	Ditto (enlarged) (N.G.C. 5194)	W. E. Wilson
60	Annular Nebula, *Lyra* (N.G.C. 6720)	W. E. Wilson
61	Meteor Trail and Comet Brooks, 1893 Nov. 13	E. E. Barnard
62	Total Solar Eclipse, 1898 January 22 (5 sec.)	W. H. M. Christie
63	Total Solar Eclipse, 1898 January 22 (20 sec.)	W. H. M. Christie
64	Solar Corona, 1896 August 9, Novaya Zemlya	G. Baden-Powell
65	Solar Corona, 1898 January 22, Pulgaon, India	E. H. Hills
66	Nebula in *Andromeda*	Roy. Obs., Greenwich
67	Spectrum of Sun's Limb, 1898 January 22	E. H. Hills
68	Annual Nebula, *Lyra* (N.G.C. 6720)	Lick Observatory
69	Dumb-bell Nebula, *Vulpecula* (N.G.C. 6853)	Lick Observatory
70	Spiral Nebula, *Canes Venatici* (N.G.C. 5194–5)	Lick Observatory
71	Spiral Nebula, *Ursa Major* (N.G.C. 5457)	Lick Observatory
72	Trifid Nebula, *Sagittarius* (N.G.C. 6514)	Lick Observatory
73	Great Nebula in *Orion*	Lick Observatory
74	Cluster M 13 *Herculis* (N.G.C. 6205)	Lick Observatory
75	Solar Surface with Faculæ, 1893 August 7	G. E. Hale

R.A.S. Ref. No.	Subject.	Photographed by
76	Faculæ and Prominences, 1892 June 25	G. E. Hale
77	Total Solar Eclipse, 1898 Jan. 22 (⅔ sec.)	W. H. M. Christie
78	Nebula H V, 14 *Cygni* (N.G.C. 6992)	W. E. Wilson
79	Portion of Moon (Theophilus, &c.)	Yerkes Observatory
80	Total Solar Eclipse, 1900 May 28 (30 sec.)	E. E. Barnard
81	Comet 1901 I., 1901 May 4	Roy. Obs., Cape of G. H.
82	Comet 1901 I., 1901 May 6	Roy. Obs., Cape of G. H.
83	Comet 1901 I., 1901 May 9	Perth Obs., W. Australia
84	Solar Surface with Faculæ, 1895 August 18	H. Deslandres
85	Solar Prominences, 1894 April 11	H. Deslandres
86	Nebula about Nova *Persei*, 1901 Sept. 20	G. W. Ritchey
87	Nebula about Nova *Persei*, 1901 Nov. 13	G. W. Ritchey
88	Total Solar Eclipse, 1901 May 18 (10 sec.)	F. W. Dyson
89	Total Solar Eclipse, 1901 May 18 (40 sec.)	F. W. Dyson
90	Comet *b* 1902 III. (Perrine), 1902 Sept. 29	Roy. Obs., Greenwich
91	Portion of Moon (Mare Serenitatis, &c.)	Yerkes Observatory
92	Portion of Moon (Rough Crater Region, Mare Nubium)	Yerkes Observatory
93	Portion of Moon (Tycho, Theophilus, &c.)	Yerkes Observatory
94	Portion of Moon (Bullialdus to Copernicus)	Yerkes Observatory
95	Portion of Moon (Copernicus, enlarged)	Yerkes Observatory
96	Great Nebula in *Orion*	Yerkes Observatory
97	Great Nebula in *Orion* (Central portion)	Yerkes Observatory
98	Nebula in *Andromeda*	Yerkes Observatory
99	Nebula in *Cygnus* (N.G.C. 6960)	Yerkes Observatory
100	Nebula in *Cygnus* (N.G.C. 6992)	Yerkes Observatory
101	Cluster M 13 *Herculis* (N.G.C. 6205)	Yerkes Observatory
102	Cluster M 15 *Pegasi* (N.G.C. 7078)	Yerkes Observatory
103	Solar Surface with Faculæ, 1903 April 27	Yerkes Observatory
104	The Moon, 1900 April 5	P. Puiseux
105	The Moon, 1902 November 13	P. Puiseux
106	The Moon, 1903 February 6	P. Puiseux
107	The Moon, 1903 September 12	P. Puiseux
108	Nebulosity about 15 *Monocerotis*	E. E. Barnard
109	Milky Way about β *Cygni*	E. E. Barnard
110	Nebulosity near α *Cygni*	E. E. Barnard
111	Milky Way near χ *Cygni*	E. E. Barnard
112	Star Cloud in *Sagittarius*	E. E. Barnard
113	Wilky Way in *Cepheus*	E. E. Barnard
114	Milky Way about M 8	E. E. Barnard

R.A.S. Ref. No.	Subject.	Photographed by
115	Milky Way about *θ Ophiuchi*	E. E. Barnard
116	Milky Way near N.G.C. 6475	E. E. Barnard
117	Great Nebula near *ρ Ophiuchi*	E. E. Barnard
118	Milky Way about 58 *Ophiuchi*	E. E. Barnard
119	Milky Way near *Omega* nebula	E. E. Barnard
120	Star Cloud in *Sagittarius*	E. E. Barnard
121	Nebula about *ν Scorpii*	E. E. Barnard
122	Sun, 1905 January 30	Roy. Obs., Greenwich
123	Sun-spot, 1905 January 30	Roy. Obs., Greenwich
124	Sun, 1905 January 31	Roy. Obs., Greenwich
125	Sun-spot, 1905 January 31	Roy. Obs., Greenwich
126	Sun, 1905 February 2	Roy. Obs., Greenwich
127	Sun-spot, 1905 February 2	Roy. Obs., Greenwich
128	Sun, 1905 February 3	Roy. Obs., Greenwich
129	Sun-spot, 1905 February 3	Roy. Obs., Greenwich
130	Sun, 1905 February 5	Roy. Obs., Greenwich
131	Sun-spot, 1905 February 5	Roy. Obs., Greenwich
132	Sun, 1905 February 8	Roy. Obs., Greenwich
133	Sun-spot, 1905 February 8	Roy. Obs., Greenwich
134	Nebula near *ψ Eridani*, 1905 January 8	Max Wolf
135	Nebula M 33 *Trianguli* (N.G.C. 598)	Isaac Roberts
136	Nebula in *Perseus* (N.G.C. 1499)	Isaac Roberts
137	Nebula in *Monoceros* (N.G.C. 2237–9)	Isaac Roberts
138	Nebula ♅ V. 24 *Comæ* (N.G.C. 4565)	Isaac Roberts
139	Nebulæ ♅ V. 42, &c., *Comæ* (N.G.C. 4631)	Isaac Roberts
140	Nebulæ ♅ V. 37 *Cygni* (N.G.C. 7000?)	Isaac Roberts
141	Nebula Index Cat. 405 *Persei*	Isaac Roberts
142	Clusters in *Perseus* (N.G.C. 869, 884)	Isaac Roberts
143	Cluster ♅ VI. 30 *Cassiopeiæ* (N.G.C. 7789)	Isaac Roberts
144	Eclipse, 1905 August 30 (5 sec.)	W. H. M. Christie
145	Eclipse, 1905 August 30 (20 sec.)	W. H. M. Christie
146	Eclipse, 1905 August 30 (7 sec.)	W. H. M. Christie
147	Eclipse, 1905 August 30 (20 sec.)	W. H. M. Christie
148	Eclipse, 1905 August 30 (Portion)	W. H. M. Christie
149	Region of Nebula *ρ Ophiuchi*	E. E. Barnard
150	Nebula *ρ Ophiuchi* (enlarged)	E. E. Barnard
151	Region of *θ Ophiuchi*	E. E. Barnard
152	Great Rift near *θ Ophiuchi*	E. E. Barnard
153	Great Star Cloud in *Sagittarius*	E. E. Barnard
154	Small Star Cloud in *Sagittarius*	E. E. Barnard

R.A.S. Ref. No.	Subject.	Photographed by
155	Region of Cluster M 11	E. E. Barnard
156	Sun-spots, 1906 July 6	Roy. Obs., Greenwich
157	Sun-spot, 1906 July 31	Roy. Obs., Greenwich
158	Sun-spot, 1906 August 30	Roy. Obs., Greenwich
159	Eclipse, 1905 August 3 (large prominence)	De la Baume Pluvinel

Nos. 44–55, Nos. 64, 65, and No. 147 form a series of corona photographs, oriented and reduced to the same scale.

The above photographs are now on sale to Fellows as prints, either platinotype or aristotype, mounted on sunk cut-out mounts, measuring 12 inches by 10 inches; also unmounted, and as lantern slides. Nos. 44–55 and Nos. 64 and 65 are also supplied as transparencies, 6¼ inches square.

Price of prints, mounted 1*s.* 6*d.* each, unmounted 1*s.* each; lantern slides, 1*s.* each; packing and postage extra.

Transparencies, 6¼ inches square (Nos. 44–55 and Nos. 64 and 65), 3*s.* 6*d.* each.

Orders to be addressed to W. H. Wesley, Burlington House, London, W. In ordering prints or slides the R.A.S. Reference No. only need be quoted, but in the case of prints it should be stated whether platinotypes or aristotypes are required, and whether mounted or unmounted.

The Gold Medal.

The Council have awarded the Gold Medal of the Society to Sir David Gill, K.C.B., for his Contributions to the Astronomy of the Southern Hemisphere and his other Astronomical Work. The President, in his Address to the Society, gives the grounds upon which the award has been founded.

Publications of the Society.

During the past year vol. lxvii. of the *Monthly Notices* has been issued.

The following paper, reprinted from the *Philosophical Transactions* of the Royal Society, has been issued as an appendix to the *Memoirs*:—

F. W. Dyson, Determination of Wave-length from Spectra obtained at the total solar eclipses of 1900, 1901, 1905.

R.A.S. Ref. No.	Subject.	Photographed by
115	Milky Way about θ *Ophiuchi*	E. E. Barnard
116	Milky Way near N.G.C. 6475	E. E. Barnard
117	Great Nebula near ρ *Ophiuchi*	E. E. Barnard
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126	Sun, 1905 February 2	Roy. Obs., Greenwich
127	Sun-spot, 1905 February 2	Roy. Obs., Greenwich
128	Sun, 1905 February 3	Roy. Obs., Greenwich
129	Sun-spot, 1905 February 3	Roy. Obs., Greenwich
130	Sun, 1905 February 5	Roy. Obs., Greenwich
131	Sun-spot, 1905 February 5	Roy. Obs., Greenwich
132	Sun, 1905 February 8	Roy. Obs., Greenwich
133	Sun-spot, 1905 February 8	Roy. Obs., Greenwich
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154	Small Star Cloud in *Sagittarius*	E. E. Barnard

expedition of his father. He went (1851–55) to the famous Clapham Grammar School, where the Rev. C. Pritchard was doing pioneer work in the way of a scientific education for boys; and thence to Trinity College, Cambridge (1855–59), and to the Royal School of Mines (1861–5). He was then (1866) appointed Professor of Mechanical and Experimental Physics in the University of Glasgow, a position which he resigned in 1871 in favour of a corresponding Professorship at Durham; and from this he retired in 1886. From 1888 till his death in 1907 he lived with brother and sisters at Observatory House, Slough, the former residence of his grandfather.

Alexander Herschel was best known for his work on luminous meteors. He was himself a diligent observer, spending long hours in the open in the watch for meteors. His post of astronomical observation at Slough was necessarily different from that of his grandfather, for during the half-century following the death of Sir William trees had grown up in the garden where his great telescope used to stand; and when four of his grandchildren returned to Observatory House in 1888 (after half a century of occupation by strangers) they found a pleasantly shaded garden, but no horizon visible. Over the wall, however, in the kitchen-garden, where Sir William's forges and optical works used to be, there were no trees; and there Alexander Herschel would lie on his back and watch the broad sky for meteors. His observations are contained in a long series of small note-books, and every observation is numbered, starting each new year with unity again. The last observation is dated 1907 February 13. A hasty glance at these books with unskilled eyes suggests that probably there is a great deal of valuable material of which no use has yet been made. But a proper estimate of the value of his work in this field can only be given by a meteor observer, and Mr. W. F. Denning has very kindly written the following notes:—

"Professor Herschel's meteoric work was characterised by its remarkable accuracy, its comprehensive detail and large amount, extending over something like half a century. It not only embraced his personal observations, but comprised the comparison and reduction of a great number of materials sent in to him by various other observers in England and abroad. During a long series of years he collected as many descriptions as possible of the fire-balls which appeared from time to time, and calculated their real paths in the atmosphere.

"As a result of long practice, he acquired great precision in noting the flights of individual meteors amongst the stars, and could reproduce them on his charts with unusual fidelity. He observed the great meteoric showers of 1866, 1872, and 1885, and collected together the different values for the positions of radiation, and obtained their mean places. It was from his averaged radiant for the November Leonids of 1866 that Professor Schiaparelli was successfully enabled to deduce his conclusions on the virtual identity of the orbits of this meteoric stream and Tempel's comet of 1866.

"An important paper by Professor Herschel appeared in the *Monthly Notices* for 1872, in which he called attention to certain showers presumably connected with Biela's periodical comet, and he pointed out the probability of a recurrence of the shower at the end of November 1872, advising observers to maintain a watch of the sky at this particular period. His anticipations were realised in a very notable manner by the occurrence, on November 27, of one of the grandest meteoric exhibitions of modern times, which fully substantiated the theory of close association existing between comets and meteors, and certainly demonstrated that the slow meteors returning at intervals at the end of November really represented the *débris* of the lost comet of Biela.

"Valuable work was accomplished by Professor Herschel in the calculation of the radiant points of comets. A complete list of his results, forming a valuable summary for reference and comparison, was published in the British Association Report, 1875, and in the same volume and also in *Monthly Notices*, vol. xxxviii. p. 369, he gave lists of known accordances between cometary and observed meteor showers. In collaboration with Mr. R. P. Greg, he formed several catalogues of the radiant points of meteors observed by the British Association members, including one giving 88 positions in 1868, and another of 63 positions in 1875 (British Association Reports, 1868, 1872, 1875)."

But though the work on meteors engrossed the greater part of his scientific activity, Alexander Herschel had many other interests. One of the chief of these was photography. His father was a keen and original photographer—the first man, it will be remembered, to use glass as a support for a photographic film (and the first picture he took in this way was of *his* father's great telescope). Alexander's devotion to photography was thus hereditary; and here, again, he not only worked assiduously himself, but stimulated activity in others. In 1893 his portrait was presented to the South Kensington Museum by the Amateur Photographic Association, and among his treasured possessions was a beautiful album of photographs presented to him by the Newcastle-on-Tyne and Northern Counties Photographic Association (of which he was President, 1885–7) as a "slight acknowledgment of services rendered."

His brother, Colonel J. Herschel, has remarked on his extraordinary vitality; he nearly always *ran*, rather than walked, to his miscellaneous occupations, and his mental activities had two unfortunate results: he shunned the interruptions of society so far as to become practically a recluse, taking all his meals alone, and he became impatient of his bodily needs until the neglect of them shattered his health. He died on June 18, 1907, at the age of seventy-one, and on June 22 was buried in the grave at St. Laurence's Church at Upton, where his famous grandfather had been laid eighty-five years before.

He was elected a Fellow of the Society 1867 March 8.

H. H. T.

WILLIAM THOMSON, BARON KELVIN of Largs, was born at Belfast, June 24, 1824. He was the second son of James Thomson, who, descended from a small farmer in the north of Ireland, in 1832 became Professor of Mathematics in the University of Glasgow. The James Thomson who in 1872 succeeded Rankine as Professor of Engineering at Glasgow was Lord Kelvin's elder brother. In 1834 he became a matriculated member of the University, and at the age of 15, during a fortnight's visit to Germany, he read Fourier's treatise to such purpose that soon after his return he wrote a series of papers in which he successfully defended Fourier from a charge of unsoundness brought against him, through some misconception, by a very competent mathematician. In 1841 he proceeded to Peterhouse, Cambridge; he was very youthful-looking and was known as "the pretty boy," but, despite his youth, he won the Colquhoun Sculls in 1843, which, it is interesting to note, had been won the previous year by the Hon. G. (afterwards Mr. Justice) Denman. He was also one of five Peterhouse men who founded the University Musical Society, the only survivor being Bishop Cridge of Victoria, B.C., who is now 86. While still an undergraduate he published papers in the *Cambridge Mathematical Journal* on the Motion of Heat, and the Mathematical Theories of Electricity and Attractions. In 1845 he graduated as Second Wrangler and first Smith's prizeman; he was almost immediately elected a Fellow of his College, and then he spent a short time in Regnault's laboratory in Paris. The next year he was appointed to the Chair of Natural Philosophy at Glasgow, which he continued to hold for fifty-three years: on resigning the Professorship he applied to be appointed as a research student; and in 1904 he was installed as Chancellor of the University.

To give even the titles of the papers he published on various branches of mathematics and physics, and the names of the instruments he invented, would require more space than could be allotted to this notice. His great strength lay in the wonderful faculty he possessed for directly applying the results of the most abstruse mathematical investigations to the solution of practical problems, and in the mechanical ingenuity which enabled him to invent instrument after instrument which satisfied, almost to perfection, the requirements indicated by the most refined analysis. It was Professor Newcomb who said, on the occasion of Lord Kelvin's jubilee in 1896: "It has been the general—I do not know but that it has been the almost universal—rule that the men who have by their studies and thought promoted our knowledge of nature, have not been those who have applied that knowledge to the direct benefit of mankind. I am not sure but that Lord Kelvin is the single, solitary exception to this rule."

His direct contributions to Astronomy were not numerous compared with the rest of his work, but he has told us that he learnt from Stokes before the end of 1851 that the existence of the D line in the solar spectrum proved the presence of sodium in the Sun, and from that date he regularly taught this fact in his

lectures; Kirchoff's work was not published until 1859. In 1862 he presented a paper to the Royal Society of Edinburgh in which he showed from the equations representing the conduction of heat in a solid that the present distribution of underground temperature necessarily implied a finite limit to the age of the Earth, and he fixed the date of its consolidation at between twenty and four hundred millions of years ago. An address on Geological Time, given to the Geological Society of Glasgow in 1868, led to a lively controversy with Huxley, in which he maintained that the fact that the oblateness of the Earth corresponds with the present length of the day shows that the Earth could not have been rotating with much greater velocity when it solidified, and that the retardation due to tidal friction therefore enables us to assign a limit to the time within which this must have happened. He also endeavoured, by means of the conservation of energy, to fix a limit to the time during which the Sun could, by its radiation, have kept the Earth at a temperature at which organic life is possible. It should, however, be mentioned, with regard to the argument from the figure of the Earth, that Sir George Darwin believes the Earth to have a greater power of adjusting its figure to its rate of rotation than Lord Kelvin thought possible. It was just after the conclusion of this controversy that Huxley introduced Sir William Thomson (as he then was) as his successor in the Presidency of the British Association with the words, "gentler knight never broke lance." Returning to the same subject thirty years later, with a more complete knowledge of underground temperature, Lord Kelvin placed the date of solidification at "more than twenty and less than forty millions of years ago, and probably much nearer twenty than forty." The discovery of the properties of radium has, however, diminished the force of these arguments, whether derived from the conduction of heat in the interior of the Earth, or from the age of the Sun's heat.

These investigations, as well as the principle of the dissipation of energy, which was stated in a paper presented to the Royal Society of Edinburgh in 1852, and in the *Philosophical Magazine* for the same year, have a direct bearing on cosmical physics and on the evolution of stellar systems.

Lord Kelvin also gave much attention to the rigidity of the Earth, as evidenced by the phenomena of precession and the tides. He took up the former problem where it was left by Hopkins, and, by amending his argument, he showed that it led to the conclusion that either the globe is solid throughout, or else the crust yields with nearly the same freedom to external forces as if it were liquid. He then compared the heights of oceanic tides on a perfectly rigid globe with those on globes possessing the rigidities of glass and steel respectively, and, from the best data available, concluded that the average stiffness of the Earth is greater than that of glass, but perhaps not greater than that of steel. His work on the Tides in general and the construction of his "Tide Predictor" afford a very good illustration of his

power of first dealing with theory and then translating it into practice.

He effected many improvements in navigation. He published a set of tables for facilitating the use of Sumner's method for determining the position of a ship. In 1876 he patented a compass in which the single, large, strongly magnetised needle formerly used is replaced by eight small parallel magnets of comparatively feeble intensity; he was thus able to secure lightness and long vibrational period, giving greater steadiness to the compass; the short needles allowed the correction to be accurate on all courses of the ship, for the place where the adjustment was made, and the small magnetic moment allowed the correction of the quadrantal error to remain accurate for all parts of the world. As an illustration of his attention to minute details, it may be noted that 32 radial slits were given to the card to prevent warping. This compass is now in almost universal use, as is also the deep-sea sounding apparatus, which is frequently the only means by which a sailor can determine his position when neither land nor sky is visible.

One of the greatest of the indirect benefits which Lord Kelvin conferred upon Astronomy was through ocean telegraphy, which has rendered possible the accurate determination of longitudes. To this achievement he contributed more than any other person, whether by demonstrating its theoretical possibility in opposition to Sir George Airy, by showing that the shortness of the life of the first Atlantic cable of 1858 was due to the intensity of the currents used in signalling, or by his invention of the mirror galvanometer and siphon recorder, which rendered possible the use of comparatively feeble currents.

He was knighted in 1866 as a recognition of his services in connection with the first Atlantic cables of 1858, 1865, and 1866. He had been elected a Fellow of the Royal Society in 1851; he subsequently received a Royal and a Copley medal, and was elected President in 1890. He was President of the Royal Society of Edinburgh 1887–90, and again from 1895 to the time of his death, and President of the British Association at the Edinburgh meeting in 1871. In 1892 he was created a Peer, taking his title from the small stream at the foot of the hill on which the University of Glasgow stands. He was one of the first recipients of the Order of Merit at its creation in 1902, and he became a Privy Councillor in the same year.

Nothing could show more clearly the estimation in which Lord Kelvin was held, both at home and abroad, than the celebration of the jubilee of his professoriate at Glasgow in 1896. Delegates from all parts of the world presented addresses; those from the Institute of France brought the Arago medal, which had been bestowed upon only three previous recipients. Congratulations were received from the Queen and the Prince of Wales, and telegrams from Europe, Asia, Africa, America, and Australasia.

Lord Kelvin was twice married: first to Margaret, daughter of Mr. Walter Crum, of Thornliebank; and secondly, to Frances Anna,

daughter of Mr. Charles R. Blandy, of Madeira. There was no issue of either marriage. He died at Netherhall, Largs, on December 17, and was laid to rest in Westminster Abbey on December 23; his grave is by the side of that of Sir Isaac Newton.

He was elected a Fellow of the Society 1868 November 13.

S. A. S.

David Joseph Kennelly was born at Cork, Ireland, on October 1, 1831, and was the son of David Nugent Mountjoy Kennelly and Zaida Teresa de la Romana Kennelly, *née* Marquesa de la Romana, of Las Palmas, Minorca. He went to sea as midshipman in 1846, and was rapidly promoted in the service of the Royal East India Company's Navy. He was made second sailing-master of H.M.S. *Meteor* in 1849, and, as master of the frigate *Ferooz*, in 1853 conveyed the 10th Lancers from Cananore to Suez, on their way to the Crimea. In the same year he served on board the frigate *Akbar* on an expedition to the Persian Gulf and the Euphrates, bringing to Bombay Layard's Nineveh slabs now in the British Museum. He was accustomed to compute the daily longitudes as well as latitudes mentally, with the aid of the regular tables, but without pencil or paper. The science of astronomy especially interested him in relation to navigation.

At the outbreak of the Indian Mutiny in 1857 Captain Kennelly was Naval A.D.C. to the Governor of Bombay. He was immediately sent, as master of a frigate, with despatches to the General commanding at the Cape of Good Hope, calling for such troops as South Africa could spare. He brought back to India H.M. 89th regiment of infantry, and landed them into action at the Gulf of Cambay. Returning to Bombay, he assisted in the disarmament of three mutinous sepoy regiments. He was twice mentioned in despatches in recognition of services.

After the Indian Mutiny, Captain Kennelly was appointed a Joint Commissioner of Bombay, and master-attendant in charge of H.M. dockyard at that port. He held this appointment until he retired from Indian service in 1868 and returned to Great Britain. He studied law in London from 1872 to 1876, and was admitted to the bar at the Inner Temple, pleading for some years in the Admiralty Court at Westminster. Later he became a barrister of the Supreme Court of Nova Scotia, and was appointed a K.C. of the province of Cape Breton. He made his home in Louisburg, Cape Breton, and was for many years connected with the coal-mining industries of that province.

Captain Kennelly received a diploma from the University of Bombay, and was at the time of his death the senior honorary corresponding member of the Royal Geographical Society, whose diploma he received for Oriental geographical research. He died at Sydney, Cape Breton, on August 27, 1907, bequeathing a fund for the maintenance and preservation of a portion of the old French citadel at Louisburg, Cape Breton, and for the protection of its historical relics. He had already presented the land for this pur-

pose to the town of Louisburg. He is survived by ten children. He was elected a Fellow of the Society 1864 January 8.

The Rev. THOMAS PERKINS was educated at Blundell's School, Tiverton, and at Christ's College, Cambridge. He took his degree in 1865, when he was bracketed 20th wrangler, Lord Rayleigh being the senior wrangler that year. He was ordained Deacon in 1868 and Priest in the following year by the Bishop of Durham, and after holding mathematical masterships at Durham and Reading Schools he was in 1878 appointed Headmaster of the Shaftesbury Grammar School. In 1893 he was presented by the Bishop of Salisbury to the living of Turnworth, Dorsetshire, which he held until his death on March 21, in the 65th year of his age. He took great interest in architecture, and was the author of *A Handbook to Gothic Architecture* and of several volumes in "Bell's Cathedral Series." At the time of his death he had just completed a book on Romsey Abbey Church, and was engaged in editing, and partly writing, a book to be entitled *Memoirs of Old Dorset*. He was an enthusiastic photographer, and enjoyed a well-earned reputation for the artistic merit of his work.

Mr. Perkins was elected a Fellow of the Society 1885 June 12.

JAMES GEORGE PETRIE was a native of Kirkcaldy, Fifeshire. He came to London at an early age and engaged in financial journalism. He soon made himself highly respected in the City, and for the last sixteen years had been chief of the reporting staff of the *Financial Times*. He was for a time President of the Shorthand Society in London. He was an ardent volunteer, and on coming to London joined the London Scottish; he left that regiment as a non-commissioned officer to take a commission as second lieutenant in the 21st Middlesex, from which he retired quite recently as a major. He received the V.D. for long service, and was one of the most popular officers in the battalion. He was an original member of the British Astronomical Association, served on the Council 1892–1894, and was Secretary from 1894 until his death, which occurred on September 22, after a long illness. He was in his 60th year. He was twice married, and leaves a widow with four children.

Mr. Petrie was elected a Fellow of the Society 1889 May 10.

THOMAS ROBSON was born at Lisburn, Co. Antrim, on 19th June 1868. He served as a pupil teacher at the Spring Grove Board School, Huddersfield, from 1883 to 1887, as assistant master at Rastrick Grammar School from 1887 to 1889, and at the Ulster Provincial School, Lisburn, from 1890 to 1892. In 1893 he took his B.A. degree at London University, and the next year he returned to his previous post at the Ulster Provincial School. Here he remained until 1897, when he again came to London as a student at the Royal College of Science. He passed the intermediate examination for the B.Sc. in 1899, and the next year he was appointed Science

Master at Doncaster Grammar School, a post he held until his death, which occurred on April 5.

He married Eliza Williams, of Randall's Mills, Co. Wexford, in August 1904, but leaves no family.

He was elected a Fellow of the Society 1903 November 13.

EDWARD JOHN ROUTH was born at Quebec in 1831, and was the son of Sir Randolph Isham Routh, K.C.B., Commissary-General to the British Forces from 1826 to his death in 1858, by his second wife Marie Louise, sister of Cardinal Taschereau, Archbishop of Quebec. The boy came to London at the age of 11, and was educated, first at University College School, and later at the College itself, under De Morgan. He matriculated at London University in 1847, and won two scholarships and a gold medal. He went up to Peterhouse, Cambridge, in 1850; was Senior Wrangler in 1854 (Clerk Maxwell being second, and the two being bracketted for the Smith's Prizes); was elected a Fellow of Peterhouse, appointed Lecturer, and ultimately became a most successful private tutor. In 1864 he married Hilda, eldest daughter of Mr. (afterwards Sir) G. B. Airy, Astronomer Royal, and by the statutes of the time he thus vacated his fellowship, but in 1883 he was elected to the first honorary fellowship at Peterhouse. He gave up private tuition in 1888, though he continued to lecture. His health broke down about the beginning of 1907, and on Friday, June 7, he passed peacefully away.

It may not be easy for future generations to understand the position Routh occupied in Cambridge life, or the debt Cambridge owes to him.

There are, roughly speaking, two ways of giving instruction, one by lectures to large audiences, the other by personal interviews with small groups or individuals. The private tutor was called into existence in Cambridge by the failure of the Colleges and of the University to give any teaching beyond lectures, and sometimes very inadequate lectures. The importance of such "coaching" had been rendered manifest by the success of those who were able to pay for it, and it was easy to recognise that some coaches were better than others by comparing the achievements of their pupils. Routh was practically the successor of a great coach, William Hopkins, with whom he had himself read, and who was able in 1849 to say that in twenty-one years he had had among his pupils nearly 200 wranglers, 17 of them senior wranglers. Routh's own success was even greater, for between 1858 and 1888 he had between 600 and 650 pupils, including 27 senior wranglers. In fact, in the twenty-five years 1861–85 he claimed the senior wrangler in every year but one. His success was so great that at one time more than a quarter of the undergraduates studying mathematics were practically learning all their mathematics from him. His pupils were naturally divided into four "years" by their time of entering the University; and each "year" was subdivided into three or four classes, so that he

had perhaps a dozen classes going at once; and to each class he would talk without hesitation for the allotted hour, rarely forgetting to begin just where he had left off, and seldom making even a slip in working on the blackboard. When it is remembered that he was thus covering the whole range of subjects for the Tripos, including ultimately the mathematical physics which had so rapidly developed during his own lifetime, one cannot but marvel at the astonishing memory which retained such a mass of facts in so orderly an arrangement. If at any time in his career all living senior wranglers could have been induced to compete in an Olympic Tripos, Routh would surely have been easily first of them all. Some of the lessons he gave were doubtless old and had been repeated many times, but many were new, for Routh kept fully abreast of the times, reading, digesting, and reducing to the form of "a little manuscript, which you had better copy out in the other room," such memoirs as were appearing from day to day. He was teaching almost uninterruptedly from 7 or 8 A.M. till 2 P.M.; after which he took a walk for two or three hours with the utmost regularity, devoting the evening to setting or looking over examination papers for his pupils. This busy life went on through July and August (the "Long Vacation") as well as through the regular terms; but immediately he was free, he got away with his family abroad, or to some complete change of scene. How he managed to find time to write his comprehensive treatises on Statics and Dynamics, and to win the Adams Prize in 1877 by a masterly essay on the "Stability of Motion," must remain a mystery to most of us.

Looking to-day at Routh's notes on Astronomy, one finds little to modify; indeed they have been consulted many times during a dozen years of teaching, and always with profit. The details of a transit-circle have become more familiar to his former pupil than they were when Routh sketched his diagram on the board and briefly enumerated the chief points, but years of added experience do not suggest any essential improvement on Routh's description, which has often been repeated almost verbatim. His quaint little touches of humour are often the quickest route to an explanation. "There is a wheel with sixty teeth," he would say in describing the chronograph, "but one is removed, so there are only fifty-nine." After explaining the method used for planetary aberration by antedating the observation—"Why cannot this method be used for a star?"—he would ask, and then reply, "Because light may take a thousand years to reach us from the star, and during that time the path of the earth is sensibly curved." Those who looked up from their notes at this would catch the little twinkle which impressed the point permanently on their memory. There were quaint expressions in other departments of mathematics which successive generations of pupils learnt to look for. All floating bodies were called ships: "Let us now consider the case of a spherical ship," he would say. His pupils did not see much of him outside the class-room until they had taken their degrees, but some of them were then privileged to form lifelong

friendships with their former tutor. To accompany him on one of his daily walks was to realise a new pleasure in walking. It is perhaps worth recording, though a trivial matter, that he seldom failed to draw attention, when opportunity offered, to the transit-circle mark on Grantchester steeple, pointing out to his companion how it had been placed so as to minimise unsightliness.

When he gave up private tuition in 1888 his former pupils asked permission to have his portrait painted by Herkomer as a present for Mrs. Routh. Mr. Justice Stirling made the presentation in the company of a large number of the subscribers, including 13 senior wranglers.

Dr. Routh was elected a Fellow of the Royal Society in 1872, and served on its Council, 1888–90.

He was elected a Fellow of the Society on 1866 April 13.

H. H. T.

By the death of Mr. Henry Chamberlain Russell, C.M.G., B.A., F.R.S., the State of New South Wales has lost its former Government Astronomer and Meteorologist, and Australia one of its most eminent sons. Mr. Russell was the son of the Hon. Bourn Russell, and was born in West Maitland in 1836. He graduated at Sidney University in 1858, and obtained the Deas-Thomson Scholarship for Physics and Chemistry. Immediately on leaving the university he was appointed assistant to Mr. Scott, the Government Astronomer. At the age of 25 he temporarily held this latter post after the death of Mr. Scott, and before Mr. Smalley was appointed. On the death of the latter in 1870, Mr. Russell succeeded to the post and held it for thirty-five years. He at once set about re-organising and re-furnishing the observatory, so that within seven years he had added seven new rooms and a dome, and had replaced the old instruments by modern ones, and throughout his long tenure he kept the observatory in an efficient state. He also turned his attention to meteorology. In 1870 there were 12 meteorological observing stations in the whole State of New South Wales, and the Government could not afford any large contribution towards an increase. Mr. Russell, by his activity, persuasion, and influence, induced the squatters and farmers to make observations of rainfall and evaporation; and by his mechanical ingenuity furnished the stations with the necessary apparatus, with the smallest possible outlay. The result was that when he resigned his post in 1903 there were 1800 stations, and over 90 per cent. were voluntary.

Mr. Russell was chief organiser of the Australian observers of the transit of Venus in 1874. Five Australian observatories took part, and Mr. Russell furnished the report to the Royal Astronomical Society.

By 1876 he had collected all the observations of rainfall and discussed them, with the result that he believed there was a periodicity. In 1878 he had a large number of returns from the observing stations, and after careful analysis was enabled to com-

mence the publication of weather maps in the papers. In 1887, in conjunction with Sir Charles Todd of South Australia and Mr. Ellery of Victoria, he succeeded in establishing a system of weather forecast, the weather changes occurring with an amount of regularity which permits of 82 per cent. of the forecasts being correct. Mr. Russell was greatly interested in the project of charting the heavens by means of photography, and attended the first meeting of the Congress in Paris. The zone -54° to -62°, requiring 1400 plates, was allotted him.

Such are the chief landmarks in two branches of scientific work, and they convey but a poor idea of Mr. Russell's genius and general activity. The picture may be filled in from the following general remarks.

Considering him as an inventor and mechanic, we find a list of 23 instruments designed and in great part made by him, generally with the object of ensuring success in his meteorological endeavours. We can mention only a few:—

A self-recording Anemometer and Pluviometer.

A self-recording Tide Gauge.

An electric Barograph.

Various self-recording Pluviometers.

An Anemometer in which the motion of the vane and cups is conveyed 75 feet vertically to the recording parts, and which also permits rain to drop 65 feet down the centre of a large tube and be recorded on the same sheet as the wind.

A Governor for driving clocks—the governor floating in mercury.

The application or combination of the two pendulums to produce uniform rotary motion.

An Equatorial Stand for the $11\frac{1}{2}$-inch refractor.

The Equatorial mounting for the 15-inch reflector.

We find that he contributed 130 papers to various societies; and a general idea of the breadth of his interests may be gathered from a selection of his astronomical papers contributed to our Society. To the *Memoirs* he contributed—

"Report of the Transit of Venus," 1874, vol. xlvii. "Measures of Double Stars," in Sir J. Herschel's Cape Catalogue, together with a list of 351 new double stars, vol. xlvii. "Measures of Double Stars made at the Sydney Observatory in the years 1882–1889," vol. l.

And to the *Monthly Notices*—

"On the Australian Eclipse Expedition 1871," vol. xxxii. "Note on the Nebula surrounding η Argus," vol. xxxii. "Observations of Comet Brorsen," vol. xxix. "Observations of the Great Southern Comet of 1880," vol. xl. "Some Celestial Photographs taken at the Sydney Observatory." "An Electric Control for a Driving Clock," vol. li., "Transits of Mercury 1881 and 1894," vols. xlii., lv.

Mr. Russell took a very active part in initiating technical education, and served as a member of the Board of Technical Education, his services being recognised when in 1891 he was

made Vice-Chancellor of the University of Sydney. He was for several years President of the Royal Society of New South Wales, and we may mention that he was the only graduate of Sydney ever elected to the Royal Society, of which he became a Fellow in 1886. In 1891 Mr. Russell was made Companion of the Order of St. Michael and St. George. In 1903 he had a severe illness, from which he was scarcely expected to recover, and ever since he had suffered from indifferent health; finally he succumbed, in his 71st year, on 1907 February 22. He leaves a widow, with one son and four daughters.

He was elected a Fellow of the Society 1871 February 10.

T. L.

ASAPH HALL was born at Goshen, Connecticut, on the 15th of October 1829. At one time wealthy, the family had become poor, and Asaph Hall was apprenticed to a carpenter. Intending, however, to become an architect, he studied mathematics in his spare time. Finally, in 1856, he entered the University of Michigan and studied astronomy under Dr. Brünnow. Leaving Michigan, he became an assistant at Harvard Observatory, under Bond, where he remained until his transfer to the Naval Observatory at Washington in 1862. In the following year he was made Professor of Mathematics. He was amongst the earliest astronomers to appreciate the value of observations of Mars for determining the parallax of the Sun; his investigations, in 1862, of the solar parallax by this means is his first paper of note. From observations made at Upsala, Santiago, and Washington, he deduced the value of 8″84 for the solar parallax. In these early years at Washington he also published *A Catalogue of 151 Stars in the cluster of Præsepe*, and *A Study of Comets Faye and Winnecke*, showing that there was no retardation due to a resisting medium. In 1870 he published in the *American Journal of Science* an interesting paper on the "Secular Perturbations of the Planets." An important paper on the "Determination of Longitudes by Moon Culminations" appears in vol. xxxiii. of our *Monthly Notices*. About this time he was giving great attention to minor planets, both by observation and computation. Thus in the *Astronomische Nachrichten*, 1875, will be found a discussion of the Washington observations of Flora, and in the same publication for 1874 a determination of the orbit of Alceste. Throughout this period, too, he contributed numerous mathematical papers to the *American Journal of Science* and the *Messenger of Mathematics*.

In 1875 he took charge of the 26-inch Washington refractor, and made numerous measures of double-stars, diameters of planets, and distances and positions of satellites, which, with his drawings of Mars and Saturn, appear in the volumes of *Washington Observations*. Sir W. Herschel in 1794 found the rotation period of Saturn to be $10^h\ 16^m$, and from that time to 1876 no one appears to have made an attempt to verify this time. Professor Hall set this down as one of his tasks, and on December

7, 1876, he was fortunate enough to find near Saturn's equator a bright spot. This he followed for sixty rotations, and was enabled to fix its period of rotation as $10^h\ 14^m\ 24^s$, a value which has subsequently been found by other observers. It will thus be seen that Asaph Hall was an excellent observer, and fully alive to the needs of astronomy. It is not surprising, therefore, that, being equipped with so powerful an instrument as the 26-inch Washington refractor, he should decide in 1877 to begin a systematic search for possible satellites of Mars. On August 11, the second day of his work, he discovered an object which he suspected to be a satellite. Bad weather intervening, he did not obtain another chance till August 16, when his suspicions were confirmed; on the following night, while watching for the satellite, he found a second moon nearer to Mars. The quick movements of this inner satellite were perplexing, and, to quote Professor Hall's words, "it would appear on different sides of the planet on the same night; and at first I thought that there were two or three inner moons, since it seemed to me at that time very improbable that a satellite should revolve around its primary in less time than that in which the primary rotates." His observations of August 20 and 21, however, satisfied him that there was but one inner moon. In 1878 he published the orbital elements of these satellites, showing that Deimos completed its revolution in $30^h\ 17^m\ 54^s$, and Phobos in $7^h\ 39^m\ 14^s$. For his discoveries he received in 1878 the Lalande Prize of the Paris Academy of Sciences. In 1879 Lord Lindsay, as President, presented him with the Gold Medal of our Society for "his discovery and observations of the satellites of Mars, and for his determination of their orbits." In the same year he received the degree of LL.D. from Yale College.

In 1882 Nyrén gave reasons for supposing Struve's constant of aberration should be increased from $20''{\cdot}445$ to $20''{\cdot}492$, which, combined with Newcomb's determination of the velocity of light, gave $8''{\cdot}794$ as the solar parallax. Loewy, in 1891, had made investigations tending to show that Struve's value should be retained, and at this juncture Professor Hall, from observations of α Lyræ since 1862, had found the value $20''{\cdot}454$, upholding Struve, and making the corresponding parallax $8''{\cdot}81$.

Professor Hall devoted much time to the measurement of double-stars; his measures of the companion of Sirius from 1872 to 1888 form a fine series, and give the place of the satellite till its separation from Sirius was only $5''{\cdot}3$. The whole of his double-star measures are published in two volumes, one containing his work during the years 1875–1880, the other 1881–91. This work led to investigations of the parallax of α Lyræ and of 61 Cygni. He also gave special attention to measuring the diameters of the various planets, and of the distances and positions of their satellites, and, as was the case with all his observations, he followed up the measures with investigations of elements, particularly of Mimas, Enceladus, Tethys, Dione, Rhea, and Titan, which are in Appendix i. of the

Washington Observations, 1883; of Oberon and Titania, the outer satellites of Uranus, in 1881, Appendix i.; while in Appendix ii. he gives a determination of the orbit of the satellite of Neptune, and deduces the mass of Neptune. Finding that most of the measures of Hyperion had been made near one elongation, he made measures in 1875 and 1876 with the 26-inch, and computed an orbit. His mathematical papers were numerous, and covered most astronomical investigations.

Professor Hall had taken part in several observational expeditions, such as—the solar eclipse of 1869 (Behring Straits), of 1870 (Sicily), and of 1878 (Colorado); the transit of Venus of 1874 (Vladivostock) and of 1882 (Texas). In 1891 he retired from the Navy, and in 1896 he was elected to one of the Professorships of Astronomy at Harvard. His death occurred on 1907 November 22. He was elected an Associate of the Society 1879 Jan. 10. T. L.

Pierre Jules César Janssen was born at Paris on the 22nd of February 1824. He was descended from a family which originally dwelt at Liége. His father was a well-known musician, and his mother the daughter of an architect of some celebrity in Paris. An unfortunate accident when young, through the carelessness of a nurse, rendered him lame for life. The boy seems to have been endowed with considerable talent, which he at first exercised in drawing and painting. At the age of sixteen he entered into business as a clerk at a bank, in which he remained for seven or eight years, but during this period he devoted his spare time to the study of mathematics, which so absorbed him that he determined to quit business and to study science. He entered the Collège Bonaparte, and about 1850 took his degree as Bachelier-ès-Sciences. Subsequently at the Sorbonne he attended the lectures of Chasles, Cauchy, Lefébure, Le Verrier, and Sturm, and in 1852 received the degree of Licentiate of Mathematical Science, and later on that of Physical Science.

At the early part of Le Verrier's direction of the Paris Observatory, Janssen was with him as a computer; but the occupation not being very congenial to him, he did not long remain at the observatory, and for the next two years he became Assistant Professor of Mathematics at the Lycée Charlemagne.

In 1856 he made a tour in Turkey, Asia Minor, and Egypt, and in 1857–58 he undertook a scientific expedition to Peru to investigate and determine the position of the magnetic equator. In this he was assisted by his pupils Ernest and Alfred Grandidier. Unhappily he was seized with serious illness, from which he nearly died, and which compelled him to abandon the enterprise, and to sail for Europe to recover his health.

Soon after his return to France he commenced those researches that have made the name of Janssen illustrious in the domain of Solar Physics.

His first published scientific paper appeared in the *Comptes Rendus* for 1860, under the title "Sur l'absorption de la chaleur

rayonnante obscure dans les milieux de l'œil," in which he showed the relative proportion of heat rays which are absorbed by the cornea, the aqueous humour, the crystalline lens, and the vitreous humour, and proved that only a small percentage of such rays reach the retina.

At this time the brilliant researches of Kirchoff and Bunsen had strongly attracted the attention of physicists, and Janssen determined to devote himself to some of the problems that had been indicated by spectrum analysis. In 1860 Brewster and Gladstone had demonstrated the existence of atmospheric bands in the solar spectrum. But so far back as 1832 the former had found that certain dark lines which were invisible with a high sun became conspicuous as it approached the horizon; and in 1856 Crookes had called attention to the fact that the atmosphere absorbed a great part of the more refrangible rays.

In 1862 Janssen established an observatory at Montmartre for the purpose of investigating the absorptive influence of the atmosphere. By means of a powerful spectroscope composed of five prisms he succeeded in resolving the dark bands into fine lines, which he found to vary in intensity directly as the altitude of the Sun, being darkest at sunrise and sunset. This established their atmospheric origin, and Janssen accordingly named them "telluric lines." The results were published in the *Comptes Rendus* for 1862.

In order to confirm these important observations he determined in 1864 to make experiments from the summit of the Faulhorn, when he was able to convince himself that the telluric lines were fainter there than at sea-level. Subsequently, at Geneva, on examining with the spectroscope the light of a bonfire at Nyon, a distance of thirteen miles, he at once perceived several of the telluric lines which he and others had observed in the spectrum of a low sun. Further researches on this point in 1866 enabled him to establish the fact that most of these absorption lines are produced by aqueous vapour.

In 1867 Janssen was sent by the Bureau des Longitudes to Italy to observe the annular eclipse of the Sun, his object being to make careful comparison of the spectra of the limb and centre of the Sun. As the light of the limb has to pass through a greater thickness of the solar atmosphere, it was expected that the absorption lines there would be strengthened, but the results obtained were purely negative.

In 1868 he was delegated by the Académie des Sciences and the Bureau des Longitudes to proceed to India to observe the total solar eclipse of August 18 in that year.

He was provided with four telescopes of 6 inches aperture with spectroscopes of different optical power, and he established his observatory at Guntoor. On mounting his instruments he found that the case of spectroscopes had been carried off by mistake and could not be available for the eclipse, but his foresight had provided one supplementary spectroscope, which was all he had at his disposal on the critical day.

Several expeditions had been organised for this eclipse, for the purpose of making spectroscopic observations of the solar prominences. All were successful in observing bright lines in the prominence spectra, and thus establishing their gaseous nature.

In his report on the eclipse, Janssen wrote: "Pendant l'obscurité totale je fus extrêmement frappé du vif éclat des raies protubérantielles; la pensée me vint aussitôt qu'il serait possible de les voir en dehors des éclipses." Pursuing this idea on August 19, the day following the eclipse, he succeeded in making those memorable and historic observations which gave astronomers the new method of observing the solar prominences without the aid of an eclipse. With the slit of the spectroscope placed radially, partly on the Sun's disc and partly beyond, he swept a portion of the limb, observing the C line. His words are: "J'étais depuis peu de temps à étudier la région protubérantielle du bord occidental quand j'aperçus tout à coup une petite raie rouge, brilliante de 1 à 2 minutes de hauteur, formant le prolongement rigoureux de la raie obscure C du spectre Solaire. En faisant mouvoir la fente du spectroscope de manière à balayer méthodiquement la région que j'explorais, cette ligne persistait, mais elle se modifiait dans sa longueur et dans l'éclat de ses diverses parties, accusant ainsi une grande variabilité, dans la hauteur et le pouvoir lumineux des diverses régions de la protubérance."

Writing on September 6, 1868, he says: "c'est le 19 août que j'ai fait cette découverte, aussi la *véritable éclipse* a eu lieu pour moi le 19 et non le 18. Depuis j'ai pu tracer jour par jour la figure, la place, la composition des protubérances du Soleil visibles seulement jusqu'ici pendant les éclipses. Je lis dans un livre fermé jusqu'ici pour tous."

Janssen's report, dated September 19, was read at the meeting of the Académie des Sciences on October 26, 1868, and immediately following it was read a communication from the present Sir Norman Lockyer announcing his independent successful discovery of the same method of viewing solar prominences without an eclipse as that made by Janssen. The Académie des Sciences struck a medal in honour of the discovery, bearing the effigies of both astronomers, and the eloquent words of M. Faye on the occasion may be fitly quoted: "Mais au lieu de chercher à partager, et par conséquent à affaiblir le mérite de la découverte, ne vaut-il pas mieux en attribuer indistinctement l'honneur entier à ces deux hommes de science qui ont eu séparément, à plusieurs milliers de lieues de distance, le bonheur d'aborder l'intangible et l'invisible par la voie la plus étonnante peut-être que le génie de l'observation ait jamais conçue?"

The new field thus opened for the study of Solar Physics at once led the French Government to consider the question of establishing a suitable physical observatory for Janssen. The question was seriously delayed by the war, and it was not till September 1874 that the Académie des Sciences could report on the subject. A provisional observatory was first established at

Montmartre, but eventually it was decided to utilise the Château de Meudon, where Janssen took up his residence in October 1876. In the meantime Janssen had undertaken several solar eclipse expeditions. In 1870, during the siege of Paris, he made his memorable escape in a balloon to observe the eclipse of December 22 at Oran, where the weather, however, prevented all observations. In 1871 he went to India and established himself in the Neilgherry Hills. He employed a specially constructed reflecting telescope, the mirror of which was 15 inches diameter, and had a focal length of only 56 inches. A direct vision spectroscope of ten prisms was employed. With the brilliant spectrum such an instrument gave he observed some of the Fraunhofer lines in the continuous spectrum of the corona, and was also able to determine that the green 1474 line in the corona was not visible in the spectrum of the prominences. He also went to India to observe the eclipse of 1875 April 6.

In 1874 he undertook an expedition to Japan to observe the transit of Venus.

In order to secure a photograph of the actual geometrical contact he invented the "Revolver Photographique," an instrument which takes a series of photographs at short intervals of time precisely on the principle of the cinematograph, of which it is indeed the forerunner.

It is from this period that those researches of Janssen's in solar photography date which have culminated in the remarkable "Atlas de Photographies Solaires," published in 1903. At the commencement of work at the Meudon Observatory, Janssen erected a special instrument for photographing the solar surface. It consisted of a telescope of 5 inches aperture, with a suitable enlarging lens which allowed him to obtain an image of the Sun from 12 to 18 inches diameter. In the optical construction of it he utilised the fact that there exists in the spectrum, near the Fraunhofer line G, a narrow band of rays which possess a photographic activity upon the salts of silver much more intense than that of any other portion of the spectrum; so that if the exposure be very short and properly regulated, the effect is practically the same as if the sunlight were monochromatic, consisting of these rays alone, and any defect in the correction for chromatic aberration in the lens is rendered almost harmless. With this telescope, and with exposures so small as from $\frac{1}{1000}$ to $\frac{1}{3000}$ of a second and even less, Janssen obtained that marvellous delineation of the solar surface which is so well known from the published results. He described all his methods in a very complete paper on the subject in the *Annuaire du Bureau des Longitudes* for 1879. The first significant discovery he was able to announce was that of the "Réseau Photosphérique," by which term he designated certain hazy, smudgy patches in the granular surface of the photosphere, which, in his opinion, showed continual agitation and change in the solar atmosphere. Employing this apparatus in photographing a partial eclipse of the Sun, he showed that not the slightest distortion

in the solar granules could be detected up to the limb of the Moon, and by this most delicate test any evidence of the existence of a lunar atmosphere was completely negatived.

Though the Château de Meudon had been in a great measure destroyed by the siege of Paris, yet the stables, nearly 100 metres long, remained intact, and were formed by Janssen into a very suitable laboratory for the study of absorption spectra, the divisions of the stalls forming excellent supports to the lengthy tubes he employed.

Jules Janssen was endowed with great energy, courage, and determination, which in no case are shown more conspicuously than in the ascents he made of Mount Blanc for the purpose of making spectroscopic observations in the higher strata of the atmosphere. He made two ascents, in 1893 and 1895. His lameness prevented him from walking, and consequently he was carried by porters from Chamounix to the summit—a quite unique performance. But this enabled him to arrive at that high altitude with a mind and body unwearied by physical exertion, and thus able to devote himself under the most favourable circumstances to the observations he proposed to make, and also to plan out an observatory on the summit. · One of his principal objects was to determine the presence or not of oxygen in the Sun. He accordingly made his ascent in winter, so that the influence of aqueous vapour in the atmosphere might be at a minimum. His observations negatived the existence of oxygen in the solar atmosphere, though recent observations are of a more positive character.

His unwearied activity throughout his life is shown by the numerous contributions to the *Comptes Rendus*, the *Annales de Chimie*, and many other scientific publications.

In 1873 Janssen was elected a member of the Académie des Sciences and the Bureau des Longitudes. In 1875 he became a Foreign Member of the Royal Society, who in 1877 gave him the Rumford Medal. The Academies of Rome, St Petersburg, Edinburgh, Brussels, and Washington did honour to him by electing him as one of their corresponding members.

A cold caught last December developed into congestion of the lungs, from which he died on the 23rd of that month, leaving a devoted wife and daughter to mourn the loss of one who will always rank as one of the most eminent scientific men of his country. He was elected an Associate 1872 Nov. 8. E. B. K.

Maurice Loewy was born of Jewish parents at Vienna on April 15, 1833. He was educated at the Polytechnic School and at the University of Vienna, and received his astronomical training at the Imperial Observatory under the directorship of von Littrow. His first contribution to astronomical literature is in the *Sitzungsberichte* of the Vienna Academy for April 16, 1857, and consists of a determination of the elements of the minor planet Leda then just discovered. Other computations of a similar character followed, including one of the elements of Donati's comet.

In August 1860 he accepted an invitation of Le Verrier's to the Paris Observatory, and in the volume of Paris observations for 1861 the initials M. L. are frequently found against the observations made with the meridian circles, the circle of Gambey, and the equatorials. In 1864 he became naturalised, and in 1870 served his adopted country on the ramparts of Paris.

His interest in minor planets and comets continued. In particular he made several investigations of the orbit of Eugenia, allowing for the perturbations. In 1872 he published a theoretical paper for simplifying and expediting the computation of orbits, with a number of subsidiary tables.

M. Loewy's attention, as was to be expected in so skilful an observer, was attracted to the details of the meridian circle. By means of an optical apparatus in the central cube he devised a method of determining the flexure of the telescope at various zenith distances. With the same apparatus he was able to investigate the errors of the pivots. He also developed a method of observation of close polar stars at various points in their circular paths, which he applied to various problems of fundamental astronomy, such as the determination of latitude without using the declinations of fundamental stars, and the determination of right ascensions without relying on the right ascensions of the circumpolar stars. This interest in the details of meridian work continued to the end of his life, a paper which he passed for press shortly before his death being an account of a new method he had devised for determining the errors of a divided circle.

In 1871 M. Loewy prepared his new form of equatorial, to which the name of "Equatorial Coudé" was given. Delaunay, who was at that time Director of the observatory, was much impressed by the possibilities of this form of instrument, but owing to his death the project of constructing one was not carried out. The first instrument of this type was completed in 1882, and had an object-glass of 10½ inches. This form of mounting is now so well known that it is not necessary to say more than that by reflection at two plane mirrors light from any part of the sky is directed into a fixed direction—that of the polar axis. The inconveniences which an observer with an equatorial must put up with are avoided. The observer can direct the instrument to any object without moving from his chair, and his observations are made under the most favourable conditions for his own comfort, similar to those under which the microscope is used by the student of natural history.* The possible drawbacks of this form of mounting are the loss of light at the two reflections and the possible flexure of the mirrors. The latter was avoided by making the mirrors thicker than was usual at that time. The performance of the coudé in the separation of double stars, and still more in the beautiful series of photographs of the Moon taken by Loewy and Puiseux, shows that the definition does not suffer appreciably by the two reflections.

* See Address delivered by the President, Mr. W. H. M. Christie, on presenting the Gold Medal of the Society to M. Loewy, *M.N.*, vol. xlix. p. 248.

M. Loewy's method of determining the constant of aberration is developed in a series of papers to the Academy of Sciences in the years 1886 and 1887. The peculiar liability of this constant to be affected by systematic error in the observations makes the differential method devised by M. Loewy of special value. By the ingenious device of a double mirror formed by silvering two faces of a prism placed in front of the object-glass, and capable of rotation about the axis of the telescope, he was able to view simultaneously two stars in widely different parts of the sky. The distance between two such stars is affected by aberration, and by observing this distance when the stars are at the same altitude, and again, three months later, when they are in the same position, a determination of the constant is obtained. The advantages of the method are that a comparatively large coefficient is involved, that errors due to precession and nutation are absolutely eliminated, and those due to refraction reduced to a minimum. By varying the conditions the same method of observation can be applied to determine the refraction, and it was for this purpose the instrument was originally constructed. The complete theory of the instrument, including the effect of temperature, change in the position of the axis round which the double mirror turns, the most suitable angle of the prism, and the choice of stars, are all carefully dealt with by M. Loewy. His theoretical conclusions as to the freedom of the instrument from systematic error were justified by experience; and though the results which were anticipated have not been realised as yet, the method constituted a new departure of great value. In this instrument, as well as in the equatorial coudé, M. Loewy has placed at our disposal methods of observation based on entirely new principles, and calculated to give astronomers improved and quite independent means of attacking several of the most important problems in our science.*

M. Loewy was elected a member of the Academy of Sciences in 1873, succeeding Delaunay. In the previous year he had been appointed a member of the Bureau of Longitudes. With Admiral Mouchez he instituted the Observatory of the Bureau of Longitudes in the Park Montsouris for the instruction of sailors and explorers. The determination of the difference of longitude between Paris and the cities of Berlin, Vienna, Algiers, and Marseilles are his own personal contributions to Geodetic Astronomy. For thirty years he edited the *Connaissance des Temps* and the astronomical part of the *Annuaire du Bureau des Longitudes*.

In 1878, when Admiral Mouchez became Director of the observatory, M. Loewy was made Assistant Director, and he held the same position under Tisserand whom he succeeded as Director in 1896. During this time the project for the International Photographic Chart and Catalogue of the Heavens took shape, a project to which the Paris Observatory contributed the optical skill of the brothers Henry and the organising ability of Admiral Mouchez. M. Loewy

* See Address delivered by the President, Mr. W. H. M. Christie, on presenting the Gold Medal of the Society to M. Loewy, *M.N.*, vol. xlix. p. 248.

interested himself in the details of this work from the start, and conducted researches on the best methods of measurement of the plates, and the derivation from the measures of the accurate positions of the stars. He also concerned himself with the reproduction of the chart plates, and inaugurated the publication on paper of enlarged copies of the original negatives. In his later years the two pieces of work to which he devoted greatest attention were the photographic atlas of the Moon, and the organisation and execution of the observations of Eros for the determination of the solar parallax. The photographs of the Moon, taken in collaboration with M. Puiseux, are of the greatest beauty and value. The coudé telescope being of long focus, gave an image of the Moon on a large scale, and this was subsequently further enlarged. An immense amount of care and labour was required to obtain photographs with such fine definition, not above one plate in ten, though taken in apparently good atmospheric conditions, being considered by the critical authors as sufficiently good for publication. This atlas gives a permanent record of the minutest details of the Moon at the present epoch; and should any changes occur, they will be detected by comparison of photographs taken in the future with those contained in the atlas of Loewy and Puiseux.

The Eros campaign, which was decided upon by the Astrographic Conference which met at Paris in 1900, brought a large amount of work, which he did not in the least shirk, upon M. Loewy. He undertook and carried out the preparation of ephemerides, the selection of reference stars, and the collection and publication of the results obtained at the different observatories. At the same time he obtained results of the highest degree of accuracy from the photographs taken at the observatory which he directed. He made extensive investigations of the precision which could be obtained by repeating the measures under different conditions; and if he erred at all, it was in having the plates measured with more care rather than with less than they required.

M. Loewy was a corresponding member of the Academies of St Petersburg, Vienna, Berlin, Rome, and Washington. He received the Gold Medal of the Royal Astronomical Society in 1889 for his invention of the equatorial coudé and his method of determining the constant of aberration. The conferences at Paris brought him in contact with astronomers from all countries. All who met him were impressed by his kindly disposition and his zeal for his work.

M. Loewy died suddenly on October 15 while speaking at a meeting of the Council of the French Observatories. He leaves a widow, two sons and four daughters. He was elected an Associate of the Society 1886 November 12. F. W. D.

M. Charles Trépied, Director of the Algiers Observatory, died suddenly on 1907 June 10, a few days after his return from the *Conférence Internationale des Études Solaires* held at Meudon in May last. On eclipse expeditions, at the successive meetings of

the international conferences for the execution of the Astrographic Chart and Catalogue, and at the recent Meudon conference, he had made many friends among English astronomers, who would wish to join in the tribute paid by the late Monsieur Loewy to the high qualities of heart and mind which gained him "les sympathies de tous ceux qui avaient eu l'occasion de l'approcher, la chaude amitié de tous ceux qui avaient eu le privilége de vivre dans son intimité."

Born in 1844, M. Trépied was associated early in his scientific career with the *Bureau des Longitudes* and its observatory at Montsouris, and he was nominated a member of the Bureau in 1877. About this time the French Government was actively engaged in the organisation and equipment of the six *Observatoires astronomiques de province*, and in 1880 they confided to M. Trépied the directorship of the Algiers Observatory, with the task of re-creating it upon a new site. The annual reports to the Ministry of Public Instruction from 1880 to 1896 give a very interesting account of the enterprise. Starting with inadequate instruments and a small budget which allowed only one assistant, M. Trépied devoted himself at first to the observation of the places of the Moon and of moon-culminating stars, and to solar spectroscopy. In 1882 he travelled to Egypt to observe at Sohag the total eclipse of short duration in which "les astronomes anglais, italiens, et français avaient resolus de partager fraternellement les opérations."

In 1885 the Algiers Observatory was transferred to Bouzaréah on a height overlooking the city, and in the course of a few years it was equipped with a new meridian circle, equatorial coudé, and astrographic telescope. The volumes of the *Comptes Rendus* bear continual testimony to the activity of the observatory on many sides: its most conspicuous enterprises include a zone of the Southern A.G. Catalogue, a zone of the Astrographic Chart and Catalogue, and a very considerable share in the photographic observations of Eros.

In 1900, when the line of totality passed over his observatory, and again in 1905, when it passed over Guelma, M. Trépied earned the gratitude of several British eclipse observers by his whole-hearted devotion to their help. And the same kindly qualities which were so conspicuous on these occasions made him the man upon whom was naturally thrust the burden of acting as Secretary to every meeting of the Astrographic Conference; to the Conference of Directors of National Ephemerides which met at Paris in 1896; to the Meudon Conference of 1907; and to many others. In this position he was able on many occasions to exercise his power of polite but acute criticism. "La regle est précise; cependant il est aisé de voir que la précision est toute dans les termes, non dans le fond," was his verdict upon the resolution which bound the observatories to get fourteenth magnitude stars upon their charts at a date when there was even less possibility of defining exactly the fourteenth photographic magnitude than there is now.

A like elegance distinguished his mathematical investigations

in geodesy, where he extended Legendre's theorems on the treatment of large spherical triangles; in celestial dynamics; and especially in the theory of precise celestial photography, to which he contributed formulæ of great generality, and at the same time great convenience.

The untimely death of M. Trépied is one of the heaviest of the great misfortunes which have robbed France within the last few years of nearly all the men who bore so distinguished a part in the inception and development of the great enterprises in which French astronomy has been involved.

He was elected an Associate of the Society 1901 November 8.

A. R. H.

Hermann Carl Vogel, the announcement of whose death on August 13, 1907, was everywhere received with profound regret, will always occupy a high place in the history of astronomy as one of the pioneers in the new fields of inquiry opened up by the application of the spectroscope to the study of the heavenly bodies. He was born at Leipsig on April 3, 1841, and, while yet a student at the university, was appointed as assistant at the observatory in that city in 1865. During the preceding three years a great impetus was given to spectroscopic astronomy by the striking results which had been obtained by Rutherfurd in America, Secchi in Italy, and Huggins in England, and there can be little doubt that at this time Vogel was especially attracted by the great possibilities which the work of these observers suggested. He was probably still further influenced at a little later stage by the discovery of the method of observing solar prominences without an eclipse which was made by Lockyer and Janssen in 1868, and by the first researches of Huggins on the movements of stars in the line-of-sight. At all events, after taking his degree in 1867, we find him to have been actively engaged with Zöllner in 1869 in the observation of solar prominences, and from this time to his death his energies were chiefly directed along astrophysical lines of research.

The record of Vogel's work at the private observatory founded by von Bülow at Bothkamp, of which he was in charge from 1870 to 1874, is the best possible testimony to his great skill and industry in the earlier part of his career. From the well-known publications of the observatory during this period we see that his outlook was already a wide one; the spectra of the Aurora, the Zodiacal Light, and lightning received his careful attention, no less than those of the Sun, stars, star clusters, nebulæ, comets, and planets. For these observations he employed a spectroscope which he had specially designed to secure the stability which is essential to useful work on the motions of the stars, and for the determination of wave-lengths with reasonable accuracy. The results of some of these observations have naturally been superseded by others which have since been made with more powerful instruments, but they were of the highest order for the period, and Vogel was himself always among the first to devise and employ improved methods of observation.

Among the more notable of Vogel's achievements at Bothkamp were the spectroscopic determination of the Sun's rotation, and his classical work on the spectra of planets. Employing a "reversion spectroscope" of the form designed by Zöllner, he indeed obtained the first spectroscopic proof of the Sun's rotation by the opposite displacements of the solar lines at the east and west limbs, and at the same time established the truth of the Doppler principle on which the determination of velocities is based.

The successful work at Bothkamp, in conjunction with that of Spörer, was chiefly instrumental in securing the foundation by the Prussian Government of the present Astrophysical Observatory at Potsdam, the erection of which was commenced in 1874. Vogel was immediately appointed as an observer, and, in 1882, on the observatory passing from the control of the commission at first in charge of the work, he was promoted to the responsible post of Director. It is generally acknowledged that his administration of this important observatory, intended for the advancement of what was practically a new science, and bound by no traditions, has been completely successful in establishing and maintaining a high standard of accuracy in all its numerous undertakings.

Vogel early recognised the importance of preparing a spectroscopic catalogue of the stars on a more extensive scale than had been previously attempted, and, with the assistance of Dr. Müller, in the years 1880 to 1882, he catalogued the spectra of all the stars down to magnitude 7·5 in the zone -1° to $+20^{\circ}$, the total number amounting to 4051. The classification adopted was that proposed by Vogel in 1874, being an extension of Secchi's system, and definitely introducing Zöllner's idea that the different spectra exhibited by the stars depend upon the temperatures which they have reached in a general evolutionary process. In 1895 this classification was further extended so as to take account of the additional features revealed by photographic spectra; and though not universally adopted in detail, its leading principle is embodied in other classifications which have been put forward.

No material advance in the investigation of stellar motions was made until the introduction of the photographic method by Vogel at Potsdam in 1887. The various sources of error which were liable to influence the minute displacements of the lines were thoroughly investigated, and during the next three years the labours of Vogel and Scheiner set this department of astrophysics on a firm foundation. The results were consistent and accurate to an unexpected degree, as was shown by the agreement between the displacements found in the spectra of the Sun and Venus with those resulting from direct calculation. The velocities of 51 stars thus determined were published in 1892 (see *M.N.*, vol. lii. pp. 87 and 541), and an important deduction was that the average line-of-sight velocity — 16·5 kilometres per second — was much smaller than had been previously supposed.

Two results of special interest followed from the spectrographic work during this period, namely, the well-known con-

firmation of Goodricke's explanation of the variability of *Algol* by periodic eclipses produced by a revolving dark companion, and the discovery that *Spica* is a similar system in which the plane of revolution does not pass through the Sun. The interesting facts relating to the dimensions and masses of these systems have passed into the general literature of astronomy and need no further comment, except to recall that, according to Campbell, no less than one in seven of the stars which have been sufficiently observed have since been found to be similarly accompanied by relatively dark bodies. The provision for such investigations at Potsdam was greatly augmented in 1899 by the erection of a telescope of 31½-inches aperture; and though Vogel's health failed in the year following, the work has been energetically carried on with marked success.

Dr. Vogel's name is associated with other pieces of work of great magnitude which have been carried out at Potsdam under his superintendence. He took an important part in the initiation of the International Chart of the Heavens in 1887, and undertook the zone $+31^{\circ}$ to $+40^{\circ}$ as the Potsdam contribution to this great work. He did not live to see the completion of this task, but all the necessary photographs have been secured and four volumes of results have been published.

Another notable undertaking at Potsdam was the precise photometric determination of the brightnesses of all the stars not recorded as fainter than magnitude 7·5 in Argelander's *Durchmusterung*. In the capable hands of Doctors Müller and Kempf this has been brought to a successful conclusion, and is characterised throughout by the extreme care which has been taken to secure the greatest possible accuracy. The General Catalogue, issued in 1907, refers to no less than 14,199 stars.

Dr. Vogel also made valuable contributions to our knowledge by his careful studies of "New Stars," and by his investigations of the absorption of the solar atmosphere for light of different wavelengths. Mention may also be made of an important series of observations of the positions of nebulæ and clusters which he made at Leipzig, with a special view to their future use in the investigation of possible changes.

No less than nineteen learned societies in various parts of the world showed their appreciation of Dr. Vogel's services to science by electing him as honorary member, and, in addition, he was awarded the Gold Medal of the Danish Academy in 1874, the Valz Prize of the Paris Academy in 1891, the Henry Draper Gold Medal of the U.S. National Academy of Sciences in 1892, the Gold Medal of the Royal Astronomical Society in 1893, and the Bruce Gold Medal of the Astronomical Society of the Pacific in 1905. In 1895 he received the honour of the Prussian Order of Merit, and was nominated Privy Councillor of Germany four years later.

This brief notice of a great career cannot be more fittingly closed than by quoting some of the concluding remarks made by Mr. Knobel on the presentation of the Society's Gold Medal to Dr.

Vogel in 1893: "It is a record of unwearying perseverance to accomplish great work; moreover it is the chronicle of a masterly attainment of success. . . . Not only is astronomy enriched by a series of investigations of the greatest value, but a distinct, and indeed marvellous, advance is made in our knowledge and conception of the stellar universe."

He was elected an Associate of the Society 1882 November 10.

A. F.

Proceedings of Observatories.

Royal Observatory, Greenwich.
(*Director, Sir William Christie, K.C.B., Astronomer Royal.*)

Transit-circle.—During the year 8648 observations of transits and 7877 of meridian zenith distances have been obtained. The Sun has been observed 167 times and the Moon 107 times. The lunar crater Mösting A has also been observed 55 times. There have also been observed 262 reflexion observations of stars.

The reductions for the Second Nine Year Catalogue for 1900 are nearly completed and the copy for press is nearly ready.

Altazimuth.—This instrument has been used, as before, as a meridian transit instrument during the second and third quarters of the lunation, for the observation of the Sun, Moon, planets, and the stars in Newcomb's Fundamental Catalogue. The instrument is reversed every two months, alternately by reversing in its Ys, and by turning the instrument through 180° of azimuth.

The total number of meridian transits obtained during the year is 1192, including 77 of the Moon's limbs and 43 of the lunar crater Mösting A. The number of extra-meridian observations of the Moon obtained during her first and last quarters is 27.

A new method of illuminating the field was introduced in June. The small prism cemented to the centre of the object-glass reflecting the light of an electric lamp placed at one side of the glass was replaced by an elliptical reflector with a matt surface of opal glass cemented in the same position, making an angle of 45° with the object-glass; the minor axis of the ellipse is 1·5 inches, the major axis $\sqrt{2}$ times this. This gives uniform illumination over the field, and the wires are much better defined, the small pencil of light from the prism having given rise to diffraction effects.

Reflex zenith tube.—During the year 1638 double observations and 37 single observations have been obtained. γ Draconis was observed on 80 days, β Draconis on 57 days, and ι^2 Cygni on 48 days.

It is proposed to discuss the observations of the year 1903 to 1907 together, for determination of the mean places of the stars and the variation of latitude.

Occultations.—During the year 28 disappearances and 2 reappearances of stars occulted by the Moon have been observed by one or more observers. The results have already been communicated to the Society.

28-inch Refractor.—The working catalogue of double stars was that brought into use in 1906. This catalogue contains all the G. W. Hough stars, about 500, within the working zone, and a number of miscellaneous pairs.

An analysis of the observations gives—

G. W. Hough Stars.

18 under	0″·5 separation
20 between	0″·5 and 1″·0
25 „	1″·0 „ 2″·0
90 over	2″·0 separation

Miscellaneous Stars.

56 under	0″·5 separation
52 between	0″·5 and 1″·0
54 „	1″·0 „ 2″·0
91 over	2″·0 separation

Of the 500 Hough stars within the limit of declination, 325 have been measured in 1906 and 1907.

κ Pegasi was observed on 15 nights, δ Equulei on 13 nights, and 70 Ophiuchi on 16 nights in 1907.

The equatorial and polar diameters of Jupiter were measured on 13 nights with the bi-filar micrometer, and also with the double-image micrometer.

Thompson Equatorial.—With the 26-inch refractor the following photographs have been obtained:—

Neptune and Satellite,		42	photographs on	18	nights.
Jupiter for position,		11	„	8	„
Saturn	„	27	„	17	„
Mars	„	8	„	5	„

With the 30-inch reflector the following photographs have been obtained:—

J vi. Opposition,	1906–7,	25	photographs on	13	nights.
	in 1907,	8	„	6	„
J vii.	„ 1906–7,	7	„	4	„
	in 1907,	3	„	3	„
Saturn *ix. Phœbe*,		16	„	16	„
Comet *d* 1906,		1	„	1	„
g 1906,		1	„	1	„
d 1907,		15	„	13	„
e 1907,		12	„	8	„

In addition to the photographs of Comet *d* 1907 mentioned above, 17 photographs with long exposure for the tail were taken, several of which are of great interest.

104 photographs of 38 minor planets were also obtained. These photographs were all taken for determination of position.

Besides these, 3 long exposures were made in an unsuccessful search for Halley's Comet, and a few others for instrumental adjustments.

As regards the measurement of photographs and reductions,—

The photographs of Neptune and Satellite are measured and discussed as far as the end of the last opposition, and the results published. The photographs of *J vi.* and *J vii.* for the opposition 1906–7; Saturn *ix. Phœbe* 1907; the photographs of Jupiter and Saturn for positions; Comets *d* 1906, *e* 1906, *g* 1906, *d* 1907, and the minor planets photographed in 1905 and 1906, have all been measured and published. The measures of minor planets photographed in 1907 are in hand.

Astrographic Equatorial.—Work with this instrument has still mainly been confined to replacing chart plates, which, though satisfactory in other respects, are, owing to slight photographic defects, unsuitable for production of enlarged prints. During the year, 164 chart plates, 7 catalogue plates, 5 of the field round Saturn, 3 of the Pleiades, 8 of minor planets, and 4 of Comet *d* 1907 were taken. Of the chart plates, 72 were rejected, 33 of them because the exposure was interrupted by cloud, or because they did not come up to the standard in showing faint stars; the large number of other rejections is due partly to an accident to the réseau printing frame, on account of which the réseau lines were printed askew on the plates.

Apart from the long exposure plates, the main work of the branch during the year has been the computation of constants for the plates which form the catalogue. Standard co-ordinates deduced from the places of the reference stars in the Greenwich Second Nine Year Catalogue 1900, now in course of preparation, have been computed for the plates of the zones declination 72° to 75° and 81° to 86°. The scale value and orientation having been computed for these plates, and a mean scale value adopted for the whole series, final adopted constants were computed for all the plates of the zones 72° to the pole, 645 in all.

These constants are included in the Introduction to volume ii. of the Greenwich Astrographic Catalogue, which has been prepared during the year and is in type. This volume will be distributed shortly. The recomputation of the constants of the plates in volume i., with standard co-ordinates deduced from the final places of the reference stars in the Greenwich Catalogue above mentioned, has been begun. The computation of scale value und orientation of the 204 plates in zones 69° to 71° is completed.

Enlarged reproductions of 206 chart plates have been made during the year. The total reproduced to December 31 is 744, the ten zones 65° to 74° being complete, and more than a quarter of the four zones 75° to 78°.

Photoheliograph.—Photographs of the Sun have been taken on 214 days during the year 1907, the Thompson photoheliograph of 9 inches aperture having been used alone on 206 days, the Dallmeyer photoheliograph of 4 inches aperture alone on 3 days, and both instruments on 5 days. Of the photographs taken, 486 have been selected for preservation, including 16 with double images of the Sun for determination of the zero of position of the wires.

Photographs have been received through the Solar Physics Committee from Dehra Dûn, India, up to 1907 December 25, and these have been further supplemented by photographs from the Observatory at Kodaikánal, India, and from the Royal Alfred Observatory, Mauritius. The daily record thus made up is complete, for the year 1907, up to 1907 December 25, the date of the last photograph received from Dehra Dûn. The last photograph received from Kodaikánal is dated 1907 August 12, and the last from Mauritius 1907 January 26. The photographs taken at Greenwich have been measured up to 1907 May 21, those from Dehra Dûn up to the end of May, those from Kodaikánal and from Mauritius as far as they have yet been received.

The solar activity throughout 1907 has shown no diminution as compared with the previous year, the months of January and February being indeed two of the most active during the whole of the present maximum. The work of measurement and reduction, therefore, still continues to be very heavy.

The copy for press of the photoheliographic results for 1906 is complete for the daily record, and is in the hands of the printer as far as the end of June. The copy for press of the sun-spot ledger is nearly ready.

During the past year the volume of supplementary photoheliographic results, 1874 to 1885, has been completed and published. The ledgers of the spot-groups, and the computation of the areas of spots and faculæ as "projected," as well as corrected for foreshortening, are therefore now complete from the beginning of the Greenwich photographic record of the state of the solar surface.

Printing.—The copies of the volume of Greenwich Observations for 1905 were distributed in July 1907. The Heliographic Results 1874 to 1885 have been printed, and will be distributed with the second volume of the Astrographic Catalogue and the Measures of Photographs of Eros for determination of the Solar Parallax, which are just on the point of completion. The printing of the 1906 observations is in progress.

At the request of the Hydrographer, Captain Monro, R.N., and Lieutenant Gibson, R.N., were attached to the observatory from September 16 to October 14 to practise observations with one of the portable transits, in view of a determination of the longitude of the Island of Ascension, and indirectly of the Cape Observatory, to be made in February.

Royal Observatory, Cape of Good Hope.
(*Director, Mr. S. S. Hough, H.M. Astronomer.*)

After twenty-eight years' service as H.M. Astronomer, Sir David Gill retired from the observatory on February 20, and was succeeded by Mr. S. S. Hough, Chief Assistant. The vacancy in the staff thus created was subsequently filled by the appointment

of Dr. J. K. E. Hahn, of the Royal Observatory, Edinburgh, to the post of Chief Assistant, who arrived at the Cape on July 1.

Transit-Circles.—The new reversible transit-circle has been in use throughout the year, principally in observations of the list of fundamental stars commenced in January 1905. To this list were added a number of circumpolar stars, and throughout the winter months consecutive transits of the latter, above and below pole, were observed as frequently as circumstances permitted. The method of observing employed has been the Repsold-Struve method, with travelling wire driven by hand.

The discussion of the observations of circumpolars in conjunction with the observations for azimuth determination in the preceding years, while exhibiting unmistakable evidence of the stability of the azimuth marks, indicated a significant and apparently increasing discordance between the azimuth determinations made with Clamp East and those made with Clamp West. Attempts to account for these discordances led to the detection of a slight mechanical defect in the end-bearing of the micrometer-screw, which has now been rectified. Since September 13, observations have been directed to the further testing of the instrument under its altered condition, and to the determination of the personality of observers in observing slow-moving circumpolar stars. These observations appear to indicate that the defects previously shown have now been completely remedied. Regular observation of the Sun and minor planets have been made throughout the year.

The old transit-circle has been employed in observing miscellaneous stars used in connection with the Geodetic Survey of S. Africa, those of which occultations have been observed, or of which places were required for various purposes.

The total number of observations secured with both instruments was as follows:—

Observations of Right Ascension:—

	Reversible T.C.	8-in. T.C.
Number of observations { day	726	
Number of observations { night	5226	1788
Collimation by collimators	70	52
,, reflexion and reversal	36	
Level	807	121
N. Azimuth Mark	973	
S. Azimuth Mark	1006	

Observations of Zenith Distance:—

Number of observations { day	726	
Number of observations { night	4153	1023
Zenith Distance of N. Mark	214	
,, ,, S. ,,	214	
Runs	318	82
Nadir	716	80
Flexure	51	14

In each case the day observations include observations of the Sun taken on 116 days, observations of Venus on 89 days, and of Mercury on 55 days. The lunar crater Mösting A has been observed on 13 nights with the old transit-circle and on 31 nights with the new transit-circle.

Heliometer.—The systematic observation of the major planets at the time of opposition has been continued and the following observations secured:—

Opposition of	Neptune (1907)	40 obs. on	5 nights.
,,	Mars	50 ,,	7 ,,
,,	Uranus	44 ,,	7 ,,
,,	Saturn	45 ,,	6 ,,
,,	Neptune (1908)*	11 ,,	2 ,,

In connection with the triangulation of the comparison stars used for the planetary observations, 478 measures of distance have been made during the year, and the following zodiacal areas have now been completely triangulated:—

Name of Triangulation.	Limits of R.A.		Limits of Dec.	
	h m	h m	° ′	° ′
Jupiter 1904	1 28 to	1 43	+ 6 12 to	+10 21
Jupiter 1905	3 49	4 10	+17 54	+21 2
Neptune 1897–1908, Jupiter 1906	5 19	7 4	+20 15	+25 22
Mars 1899	7 54	8 22	+23 16	+25 49
Mars 1901	10 11	10 41	+13 7	+17 15
Jupiter 1898, Mars 1903	12 14	12 45	− 2 8	+ 1 6
Jupiter 1899	14 8	14 23	−12 55	−11 13
Mars 1905	14 43	15 15	−18 59	−15 37
Uranus 1898–1907, Saturn 1898–1901, Jupiter 1900, 1901	15 47	19 8	−25 7	−18 13
Saturn 1902	19 40	19 55	−22 29	−19 28
Saturn 1903, 1904, Jupiter 1902	20 24	21·34	−21 14	−14 42
Jupiter 1903	23 10	23 30	− 7 30	− 4 30

During the transit of Mercury of November 14 a series of measures of the distances of Mercury from the Sun's limbs, comprising in all 48 pointings, were secured, and in connection therewith observations of 5 standard pairs for the determination of instrumental corrections were made on 3 nights.

The Victoria Telescope.—This instrument has been almost exclusively used throughout the year in the photography of stellar

* Still under observation.

spectra for the determination of radial velocities and the solar parallax. The instrument was in use on 103 nights, and 231 plates of the spectra of 18 stars were secured. 246 stellar spectra have been measured, mainly in connection with the determination of solar parallax.

The radial velocities of the following stars, specially selected for the determination of solar parallax, are completely reduced and await final discussion:—

α Tauri	25	plates	covering	3	quadratures.
α Orionis	35	,,	,,	3	,,
α Can. Min.	34	,,	,,	3	,,
β Geminorum	14	,,	,,	3	,,
α Boötis	46	,,	,,	3	,,
α_2 Centauri	44	,,	,,	3	,,
α Scorpii	39	,,	,,	4	,,

A considerable number of experimental plates for the examination of the focal adjustments have also been secured and carefully measured. A discussion of these plates has resulted in a distinct improvement in the focal adjustments, while their permanence has been further secured by an improved arrangement for circulating the air surrounding the prism box. Whereas formerly a practically perfect temperature and focal control was secured so long, but only so long, as the instrument remained undisturbed in one position, under the improved circulatory system no appreciable change in focus occurs as the result of moving the instrument from one position to another.

Astrographic Telescope.—This instrument has been chiefly used in the photography of selected regions, in accordance with the plan proposed by Professor Kapteyn. The number of plates secured is as follows:—

Description of Plate.	No. of Plates.	No. of Exposures.	Duration of Exposures.
Kapteyn areas, proper motion plates	56	112	30^m, 30^m
Kapteyn areas, parallax plates (1st max.)	184	184	30^m
,, ,, ,, (2nd max.)	24	48	30^m, 30^m
Parallax pilot plates	7	...	various
Catalogue plates	9	27	6^m, 3^m, 20^s
Adjustment plates	8	...	various
Comet Metcalf region	1	1	40^m

During the year 199 Catalogue plates, containing 101,136 stars, including 2282 standard stars, have been measured, both in direct and reversed positions of each plate. The total number of plates now measured is 1285, containing about 700,000 star-images, corresponding to nearly 300,000 different stars.

The actual state of the work on December 31 was as follows:—

	Measures of Co-ordinates. No. of Plates.		Rectangular Co-ordinates computed and examined. No. of Plates.	
	Completed.	Outstanding.	Completed.	Outstanding.
−41°	144	...	144	...
42	144	...	144	...
43	144	...	144	...
44	132	12	132	12
45	143	1	143	1
46	142	2	141	3
47	144	...	143	1
48	144	...	80	64
49	118	2	1	119
50	30	90	3	117
−51°	0	120	0	120
	1285	227	1075	437

Plate constants have been derived for plates in zones −41°, −42°, and −43°, and applied to all rectangular co-ordinates in zone −41°, and to 112 plates in zone −42°.

Occultations.—Twenty-four separate phenomena of occultation have been observed, viz.—

Disappearances at dark limb . . .	19
Reappearances „ „ . . .	5

The observations up to November 14 have been completely reduced and results communicated in advance to Professor Newcomb.

Transit of Mercury.—On the occasion of the transit of Mercury of November 14, in addition to observations with the transit-circle and heliometer, the times of ingress and egress were observed by six different observers with the various equatorials.

Comets.—Comet 1907 *a* was observed with the Victoria telescope on 6 nights, and the results have been communicated to the *Astronomische Nachrichten*. The results of observations of Comet Finlay (*d* 1906) and Metcalf (*b* 1906), made during 1906, have likewise been communicated to the *Ast. Nach.*, and published during the year.

Publications:—

Vol. ii. part 5 of the Cape Annals, containing "Meridian Observations of the Sun, Mercury, and Venus during the years 1884–1892," and vol. ii. part 6, containing "Results of Occultations observed during the years 1896–1906," have been passed through the press and issued during the year.

Vol. viii. part 1, containing "Heliometer Observations of the Major Planets during the years 1897–1904," has been passed through the press and will shortly be issued.

The Cape Catalogue 1905–6, based on meridian observations made with the 8-in. transit-circle during the years 1905–6, and reduced to the equinox 1900, has been printed and issued.

The Cape day-numbers for 1910 have been printed, and copies received at the observatory for distribution.

Mr. Lunt has contributed a note to the Royal Astronomical Society on "The presence of Tin in Stellar Atmospheres."

Meteorological observations, comprising daily records of air-temperature, pressure, wind, rainfall, etc., have been continued throughout the year, and the results, as in previous years, forwarded to the Meteorological Commission, Cape Town.

Seismograph.—The records of the seismograph have been forwarded to Professor Milne, while a duplicate copy, at the request of the German Government, is forwarded to the Consul-General for Germany in Cape Town, for transmission to the central station for the investigation of earthquakes at Strasburg.

Longitude operations.—In connection with a proposed telegraphic determination of the longitude of the Island of Ascension, both from Greenwich and the Cape of Good Hope, during the visit of H.M.S. "Mutine" to Ascension in February 1908, Mr. Pett, after leave of absence in England and before his return to the Cape, proceeded to Greenwich and observed on 11 nights for comparison of his personal equation, both with the Greenwich observers and with Captain Monro, R.N., who will conduct the field operations at Ascension.

Geodetic Survey of South Africa.—The field data of the triangulation connecting the northern end of the 30th meridian arc in the Transvaal with the existing triangulation in Rhodesia, executed by Captain Gordon, R.E., between October 1906 and January 1907, were received early in the year. The work has been completely reduced at the observatory, and the results forwarded to Sir D. Gill for publication.

The field work in Northern Rhodesia, after being carried as far as Mpange, S. lat. 9° 41′, or about 73 miles south of Lake Tanganyika, was abandoned in December 1906, and all field books containing the geodetic data were forwarded to the observatory for reduction purposes. The horizontal and vertical angles throughout the chain have been reduced and adjusted so far as is possible until a definitive length of the base measured in the Loangwa valley is arrived at.

The measurements of this base, as contained in the field books, have been completely reduced and discussed, and the length of the base has been derived in relation to that of the Russian standard bar used in the measurements; this bar has been forwarded to the "Bureau International" for standardisation, and the derivation of the actual length of the base is deferred until the results of this comparison are available.

The reduction of the astronomical observations, comprising latitude observations at 18 stations and astronomical azimuths at 2 stations, is well in hand, the field book data having been completely reduced and all star corrections computed. The discussion of the places of the 581 stars involved is now proceeding, but its completion awaits the re-observation of several of the stars with

the transit-circle, which will be secured during the early months of the current year.

Royal Observatory, Edinburgh.
(*Director, Prof. F. W. Dyson, Astronomer Royal for Scotland.*)

The recording micrometer referred to in last year's report as having been applied to the transit-circle has worked very satisfactorily, and the screw shows no appreciable wear. The observation of zodiacal stars has been pushed on vigorously, more than 4000 observations being made in the year. It has been found necessary to leave out of the observing list a number of stars of low declination which cross the meridian before midnight in the short summer evenings. With these omissions only about 200 more observations are required. As regards the computations, the observations as far as September 30 are reduced to mean place, and to apparent place as far as October 31.

Spectroscopic observations of the rotation of the Sun have been continued on the same lines as in previous years. From July the observations have been made photographically instead of visually. Sanction has been given for the purchase of a larger objective, and a 9-inch object-glass of 36 feet focal length is being constructed.

Forty photographs of nebulæ have been taken with the 24-inch reflector. Difficulties have been experienced with this instrument owing to the shifting of the mirror, which have not as yet been quite overcome.

Seismographic observations have been carried on continuously, and the results communicated to the Committee of the British Association.

Meteorological observations have been regularly made. Two platinum resistance thermometers have been obtained and placed, one horizontally, the other vertically, in the rock at a depth of 3 ft. 9 in. The cracks in the rock allow of percolation of water, and experiments with two thermometers at the same depth was advised by Professor Callendar before proceeding to place thermometers at varying depths. Observations have been made daily since November 15.

Arrangements have been made to assist the Perth (Western Australia) Observatory in the measurement of the Astrographic plates. A micrometer has been obtained, and 100 plates have been received from Mr. Cooke.

The observatory has lost the services of Dr. Halm, who was appointed Chief Assistant at the Cape Observatory, and left Edinburgh in May. Mr. Clark was promoted to succeed him. The assistants' posts at the observatory have been placed on a more satisfactory footing, having been made permanent Civil Service appointments, a position formerly only occupied by Mr. Heath, the First Assistant.

Cambridge Observatory. (*Director, Sir R. S. Ball.*)

1. *Meridian Circle.*—The Syndics of the University Press have undertaken the printing of the large volume of meridian circle results 1872–1900 mentioned in the last report, and the work is making good progress. At the end of the year printing was completed up to R.A. $2^h\ 30^m$.

Advance copies of the results of the re-observation and examination of all the single observation stars of the Cambridge A.G. Zone Catalogue (to which reference was made last year) have been communicated to Dr. Ristenpart for incorporation in the *Geschichte des Fixsternhimmels.* All points of interest which arose have been thoroughly examined, and it is believed that very few discordances remain outstanding.

Observation of Gill's Zodiacal Stars has made good progress during the year, 1532 stars having been observed on 50 nights, with the necessary observations of fundamental stars and instrumental errors. The reductions to apparent place have been much interrupted by the work mentioned above, but are complete in R.A. to 1907 March 19, and in Decl. to 1907 September 10; and the reductions to mean place are complete to 1907 February 7.

Mr. Hartley has been in charge of the meridian circle, and has carried out the greater part of all the above work during the year.

The meridian circle clock, by Hardy, which has been in continual use since 1828, was thoroughly overhauled in 1907 January; with the exception of slight wear on one of the spring pallets, which should not affect the time-keeping, the clock is in perfect condition.

2. *Sheepshanks Equatorial.*—Two hundred and thirty-eight plates, each containing four or more exposures, have been taken by Mr. Hinks during the year, in continuation of the series for the determination of stellar parallax begun in 1904. On December 31 sixty fields were on the working list as partly observed, and thirteen had just been removed as completely observed at four or more epochs.

Sixty plates taken at Cambridge, which had been lent to Mr. H. N. Russell, of Princeton, U.S.A., were returned in 1907 April. Further consignments were sent to him of 28 plates in 1907 April and 13 plates in 1907 October, completing the series unfinished on his leaving Cambridge which he has undertaken to measure and reduce.

The new form of colour screen for bright stars has been in use since early in the year and appears to be quite satisfactory.

In preparation for the transit of Mercury on 1907 November 14, temporary alterations were made in the rectangular slides of the breech-piece, so that it became possible to take a series of 64 photographs of a segment of the Sun's limb at intervals of 3 or 4 seconds upon one plate. The sky was completely overcast on the day of the transit.

Mr. Stratton has finished his investigation of proper motions of faint stars in the Pleiades from plates taken at Greenwich, Oxford, and Cambridge; the results were contributed to the Society in 1908 January.

Mr. H. Knox Shaw (Trinity College: Sheepshanks Exhibitioner 1907) became a student at the observatory in 1907 October, and has begun work with the Sheepshanks telescope.

A new measuring machine, to take plates up to 10 × 8 inches, constructed by the Cambridge Scientific Instrument Company, has been brought into use during the year. It is a simplified form of the original Cambridge machine constructed by the same company in 1900, which has given complete satisfaction during its seven years of use.

3. *Reduction of Eros photographs.*—The first part (to 1900 December 31) of the photographic standard system of comparison stars is now complete, and is discussed in *Solar Parallax Papers*, No. 6 (in *M.N.*, 1907 December.) The second part is approaching completion. The system responds well to the tests which have been made upon it, and seems to be well adapted for use in the definitive reduction of the parallax plates.

Definitive places of most of the Crossley reflector comparison stars have been furnished to Professor Perrine. For a certain number the published photographic material was unsatisfactory, and many of these stars were too faint to be measured on Cambridge plates. The Astronomer Royal kindly undertook to measure these stars at Greenwich; he has already communicated the places of a first list of 48 stars, and a second list is under observation at the close of the year. The places of 33 very faint comparison stars used in micrometer observations of the planet have been measured on the Crossley reflector meridian series, and communicated by Professor Perrine.

Systematic corrections have been derived for most of the published series of photographic places of the planet, to reduce them to the new standard system. Printed forms have been prepared for the formation of the equations of condition in the final general solution, and a good deal of the preparatory formation of coefficients is done. A preliminary collection of the micrometric observations has also been made in order to see how far these may be used to strengthen the weak places in the photographic series.

The completion of the standard system of comparison stars made it possible to resume the measurement of the Cambridge photographs of the planet. Thirty-two plates, containing in all 281 images of the planet, have been measured during the year; the number of comparison stars measured on each exposure was about nine. The reduction of these measures is nearly complete. A number of star places required for the completion of the standard system were also derived from these plates.

Mr. Hinks has been aided in this work throughout the year by Miss Julia Bell; and Mr. S. E. Bowd has been engaged on the card catalogue and miscellaneous computing. The expense of this

assistance has been defrayed by a grant from the Government Grant Fund of the Royal Society.

4. *Floating Zenith Telescope.*—As noted in last year's report, Mr. Cookson has found it necessary to extend the observations with his zenith telescope over two years: enough material has now been gathered, the last photograph having been taken in July. The number of photographs available for determining the constant of aberration is 120. All of these plates have been measured in one position of the plate in the micrometer: about half have also been measured in the reversed position. The rest are now being measured. The reduction of all the measurements is in hand and is being carried out with the assistance of Mr. A. D. Campbell.

5. *Reduction of Lunar Observations.*—Mr. Stratton has continued his new reduction of Schlüter's observations of the Moon for comparison with Hayn's determination of the constants of the Moon's libration and of the selenographical co-ordinates of Mösting A, and the work is making steady progress.

6. Courses of instruction in practical astronomy, and in field astronomy and trigonometrical survey, have been given by Mr. Hinks during the year.

The Newall Telescope, Cambridge Observatory.
(*Mr. H. F. Newall.*)

The 25-inch equatorial was used in the early part of the year 1907, with the four-prism spectrograph, in photographing the red end of the spectra of the brighter stars. Special attention was paid to the spectrum of α Orionis in its relation to the spectrum of sun-spots. A note on some of the results was communicated to the Society in May (*M.N.*, lxvii. 482).

Experiments have been made and are being continued in photographing star spectra with a diffraction grating, in order to decide whether the lines of the telluric B group can be satisfactorily utilised for determinations of stellar velocities.

Measurement and reduction of stellar spectra was carried on chiefly in the earlier part of the year.

The solar work has been carried on with the Littrow spectroscope and with temporary arrangements of cœlostat and image lenses. Meanwhile the permanent mounting for the cœlostat has been constructed, and was delivered at the observatory in December. It is expected that the new image lens of long focus which is to be used for the solar observations will be completed in time for the work in the spring or early summer.

The Littrow spectroscope has been used for the study of the cyanogen absorption-bands at wave-length 3883 in the solar spectrum. A note on some of the results was communicated to the Society in November (*M.N.*, lxviiii. 2). The instrument has been found very convenient to handle.

The same instrument has been used by Mr. Hubrecht in

preliminary work on the determination of the rotation of the Sun by the spectroscopic method.

A good deal of work has been devoted to the study of the spectrum of the electric arc under varied conditions in the laboratory.

Dunsink Observatory.
(*Director, Prof. E. T. Whittaker, Royal Astronomer of Ireland.*)

The staff consists, as at the close of the previous year, of the Director and Mr. C. Martin, assistant; the instruments in regular use are the Pistor and Martins meridian circle and the Roberts photographic reflector.

The reduction of the meridian observations of the red stars in the working-list prepared by the late Director, Professor Joly, has been completed during the year. A new working-list for the meridian circle has been prepared, forming an extension of the previous list, and consisting of stars of a similar character and variable stars; observations of these stars are now in progress.

The time service to Dublin has been continued as in previous years.

In the course of the meridian observations it was suspected that some of the red stars which have not hitherto been recognised as variables are such. Accordingly, the Roberts photographic reflector is now being employed in a determination of the photographic magnitudes of the red stars which are not known to be variables, with a view to the detection of variability.

The observatory library has been catalogued during the year.

The professorial courses of lectures delivered in Dublin by the Director have been entitled "Theory of Energy and Radiation" and "Theoretical Spectroscopy." Demonstrations in practical astronomy are given at the observatory by the Assistant Astronomer, and the south equatorial has been employed on the first Saturday in each month in showing objects of interest to visitors. The observatory was visited in July by His Excellency the Lord Lieutenant of Ireland.

Durham Observatory. (*Director, Professor R. A. Sampson.*)

The greater part of the year has been given to the theory and tables of Jupiter's satellites. A new theory has been developed, based upon the constants derived from the Harvard eclipses; it is hoped that this theory will prove of interest for its own sake; it has been much advanced in all its details during the year. The plan for the tables has been laid down, and some parts of them have been computed.

Observations with the Almucantar have been continued as heretofore: 602 stars were observed upon 40 nights.

Glasgow Observatory. (*Director, Professor L. Becker.*)

Good progress has been made with the reductions of observations obtained in former years. The observing work has been continued with transit-circle and spectrograph, but though all favourable occasions were utilised, the output is small owing to the few clear nights. The University authorities have under consideration a scheme which proposes to erect an observing station at some distance from this large manufacturing district.

The time service and the meteorological observations have been carried on as in former years.

Liverpool Observatory. (*Director, Mr. W. E. Plummer.*)

During the past year there has been little change in the instrumental equipment, the staff, or the work of the observatory. The meridian instruments and chronograph are kept at work regularly in order to ensure the distribution of correct time to the port of Liverpool. The apparatus for signalling has worked without failure throughout the year. As in past years, the equatorial has been devoted to the observation of comets, measures being made as long as the comet is bright enough to be seen. Measures of the position of the components of a few binary systems are made with regularity. These observations are confined to those stars that have long been observed, and for which the aperture of the telescope is suitable. In some cases the measures have been used to determine the orbits of binaries, employing as far as possible only those measures that have been made with instruments of similar size. It was hoped that more accordant results might be obtained if the influence of aperture were eliminated, but the accidental errors apparently mask any systematic descrepancies which might arise from this cause.

Some slight additions have been made to the instruments in the meteorological department, and the observations for climatology, etc. are continued as heretofore. The record of unfelt earthquakes with the Milne seismograph is regularly maintained. The results are freely communicated to all who are interested in these inquiries. The testing and rating of chronometers and the examination of other apparatus form part of the routine work of the observatory. Lectures in connection with the University of Liverpool are regularly given.

Radcliffe Observatory, Oxford.
(*Director, Dr. A. A. Rambaut, Radcliffe Observer.*)

During 1907 the work of this observatory has been continued on the same lines as in recent years, but its effective strength has been reduced by death and other causes. On January 21 Mr. E. E. McClellan, who had held the post of third assistant since 1892, died rather suddenly. Mr. H. Barrett, late computer at the Royal

Observatory, Greenwich, was appointed to succeed him on June 1. Towards the end of January, Mr. Reuben Harris, who had been employed as computer at the observatory since March 1903, resigned.

The investigation of stellar parallax has been carried on as vigorously as the weather would permit, but although no opportunity has been lost of photographing Professor Kapteyn's selected regions—the particulars of which were published in last year's report—cloud and fog in the early mornings have much interfered with the completion of this work. There still remain five plates of the region around Nova Persei, five plates of B.D. $+15^{\circ}$, 651 (Hyades), and two plates of B.D. $+31^{\circ}$, 1684, which require for their completion a third season's exposure. It is hoped that these exposures may be successfully carried out by the time this report appears. Five plates of the region surrounding Lalande 5761 have been sent to Professor Kapteyn at Groningen, and there measured and reduced under his direction, with satisfactory results.

Photographs of Daniel's Comet 1907 *d* were obtained on August 18, 20, and 26; and in preparation for the transit of Mercury on November 13–14 the 24-inch telescope was fitted with a short exposure shutter, and photographs of the Sun were taken in the principal focus. Clouds prevented any photograph being taken during the transit.

The 18-inch visual refractor was employed in examining the visibility of Saturn's rings on September 24, November 6, 26, 28, December 5, 6 and 14.

In the autumn of 1906 the heliometer, which has long been out of use, was dismounted, and the 10-inch Barclay equatorial erected in its place in February 1907. To this instrument has been adapted the Grubb bifilar micrometer made for the 18-inch. This combination proves very convenient for observations of double stars and comets. Micrometer measures of the position of Daniel's Comet have been made with it on July 30, August 11, 13, 18, 20, 26, September 8 and 11. Other miscellaneous observations made with this instrument include—

(1) Observations of Mellish's Comet 1907 *e*, on October 18, with the ring-micrometer.

(2) Comparisons of the magnitude and colour of Mira Ceti and α Ceti.

(3) Spectroscopic observations of solar prominences, especially of the great outburst of November 15, an account of which was published in *Nature*, No. 1986, vol. lxxvii.

(4) Transit of Mercury, seen for a few moments through clouds.

(5) Examination of Saturn and its rings on October 4 and November 23.

Naked-eye observations of Mira Ceti and comparison stars were made on January 2, 3, 6, 11, 17, 18, 22, 23, 29, 31, February 2, 10, 13, and 16. The results of the whole series of observations, continued from last year, were communicated to the Society, and published in the *Monthly Notices*, April 1907.

The computing strength of the establishment has this year been mainly directed to the reduction of the meteorological observations, but work in this department has been much interrupted.

A considerable amount of time towards the end of the year has been devoted to the examination of the original records and reductions of the observations made with the transit-circle under the direction of Main, 1862–76, with a view to elucidating a long list of discordances found by Herr Osten between the places published in the annual volumes and those of the *A.G. Catalogue*. This work was undertaken, at Dr. Ristenpart's request, for the *Geschichte des Fixsternhimmels*. The result of the examination has been to account for most of the discordances by errors in reduction, proper motion, or other causes.

Meteorological and Earth temperature observations have been made regularly as heretofore. It is intended to discontinue the latter (which have been carried on without interruption since 1898 November 1) at the end of 1909.

University Observatory, Oxford.
(Director, Professor H. H. Turner.)

Volume ii. of the Oxford Astrographic Catalogue was issued early in the year. Volume iii. (160 plates, centres $+29°$: 62713 star images) was passed for press on July 10, and a few copies have been distributed. Volume iv. (160 plates, centres $+28°$: 65,808 star images) is completely printed except the Introduction, so that the rate of two volumes per year is being maintained, thanks to the steady and devoted work of Mr. Bellamy and Mr. Plummer, and to the important assistance rendered by the Government Grant Committee of the Royal Society.

During the concluding months of the year the Equatorial was dismounted for various repairs, rendered necessary by its long and continuous service. The alterations and cleaning have been carried out under the superintendence of Mr. A. B. Chatwood, B.Sc., A.M.I.C.E., who accepted a modest and temporary post at the Observatory while awaiting confirmation of his appointment as Astronomer to H.H. the Nizam of Hyderabad. Special thanks are due to Mr. Chatwood for the great energy with which he has worked during these few months, and the number of suggestions for improvements in the instrument, which will remove various discomforts experienced in the past. He has also done work of a similar kind in other departments of the observatory.

Mr. H. C. Plummer applied, with the sanction of the Director, for a one year's Fellowship of the Lick Observatory, commencing last September. He will thus have an opportunity of acquiring familiarity with spectroscopic work, which has not yet been organised at this observatory.

The Variable Star observations of Pogson and the Baxendells have been put into the hands of the Director with a view to

publication. Those of Pogson were prepared for press by Mr. C. L. Brook, and are being printed in the *R.A.S. Memoirs*, a subsidy of one-third of the cost having been generously promised by Mr. Brook himself, and another third by the Indian Government. The observations of the elder Baxendell require much more work than those of Pogson, owing to frequent changes of the comparison stars; but, with the help of ungrudging volunteer labour on the part of Miss M. A. Blagg, of Cheadle, Staffordshire, considerable progress has been made with the arrangement in ledger form. Here also the Government Grant Committee have aided the work by a small grant for the more mechanical copying.

The Director attended the meeting of the International Union for Solar Research at Meudon, May 21–25. He has been asked to undertake the formation of a Computing Bureau, and, with a small subsidy from a private source, a beginning has been made by an investigation of the position of the Sun's axis from sun-spot records, on which preliminary note was published in the *Monthly Notices* for December last.

The Director also attended the meeting of the International Association of Academies at Vienna, May 28 to June 1, as a delegate of the Royal Society. He was nominated on the Committee appointed by the Association to consider the question of Lunar Nomenclature, and, at the request of the Chairman (the late M. Loewy), is acting as Secretary to the Committee.

Temple Observatory, Rugby. (*Director, Mr. G. M. Seabroke.*)

This observatory was used during the past year for teaching boys the use of instruments, and attempting to give them some taste for astronomy. The spare time has been taken up with measuring double stars.

Solar Physics Observatory, South Kensington.
(*Director, Sir Norman Lockyer, K.C.B.*)

Observations of Sun-spot Spectra.—The Sun was seen on 243 days during 1907, and observations of sun-spot spectra were made on 107 days, permitting the examination of 211 spots in the region F–D. The records continue to indicate that the lines affected are due to Vanadium, Titanium, Scandium, and Magnesium, in the presence of Hydrogen.

For the photographic record, with large dispersion, of the spectra of sun-spots a special instrument has been erected. It consists of a 12-inch siderostat, feeding a 12-inch concave mirror of 72 feet focal length, combined with a spectroscope, which is of Littrow form, using a 6-inch Cooke lens of 20 feet focal length, and, until a larger grating becomes available, a small plane Rowland grating with a ruled surface 1·9 × 1·3 inches (4·8 × 3·3 cms.), 14,438 lines to the

inch. The results already obtained are very promising, though its situation near the traffic of the adjoining street exposes the instrument to considerable disturbance.

Sun Photographs.—Daily photographs (glass negatives) of the Sun's disc are received from Dehra Dûn (India) and from Mauritius, the gaps in the Indian record being filled up as far as possible by negatives from Mauritius and Greenwich. The negatives are forwarded to Greenwich for measurement and reduction as they are required. Positives on paper are also received. These are mounted on cartridge paper and bound up into half-yearly volumes for ready reference. Since the beginning of 1906 enlargements of spectroheliograms for the same dates have been mounted alongside the paper prints for comparison.

Spectroheliograph.—The weather conditions were distinctly less satisfactory for the purposes of this instrument than during 1906, though fine enough on 142 days to warrant attempts being made to obtain monochromatic photographs of the Sun in "K" light. The instrument cannot be used very satisfactorily during the winter months on account of its position among high buildings. During the period January 3 to December 13, 1907, 351 negatives were secured, showing the distribution of "K" radiation on the Sun's disc. With the addition of an occulting disc over the primary slit plate 62 negatives were obtained, showing the prominences round the solar limb.

By arrangement with the Indian Government, photographs taken with the spectroheliograph at South Kensington and Kodaikánal are exchanged, so that the records may be as complete as possible for the year. During the period January 1, 1906, to September 30, 1907, 465 spectroheliograms have been received from Kodaikánal, while during the years 1906 and 1907, 111 positives, showing the Sun's limb in "K" light, have been forwarded to Kodaikánal.

The measurement of the spectroheliograms is now proceeding. Though, for many reasons, the work is extremely difficult, the consistency of the results, so far obtained, has been very satisfactory.

Stellar Spectra.—The weather conditions for night observations have been less favourable than during 1906. Fifty-two photographs of fifteen stellar spectra have been obtained with the 6-inch Henry prismatic camera with two prisms, and eight stellar spectra with the 9-inch prismatic reflector. The 2-inch quartz-calcite prismatic camera has been employed in photographing twenty pairs of stellar spectra, under conditions as nearly constant as possible, for recording extensions of the spectra in relation to the positions of the stars on the temperature curve of the Kensington classification. The weather has been extremely unfavourable for the photography of faint objects with the 36-inch reflector. Experimental work with plates specially sensitised for the green region showed a satisfactory advance. Among other photographs, a good spectrum, in the green region, of the Orion nebula was obtained.

Publications.—Papers dealing with the pressure data of 73 selected stations over the Earth's surface, the report of the eclipse expedition to Palma, Majorca, August 1905, the presence of sulphur in some of the hotter stars, the spectroscopic comparison of metals present in certain celestial and terrestrial light sources, observations of Sun and stars made in some British stone circles, prominence and coronal structure, and a note on the permanency of some photo-visual lenses, have been completed during the year, and others on allied subjects are in course of preparation.

Royal College of Science, South Kensington.
(*Assistant Professor A. Fowler.*)

Observations of the spectra of sun-spots have been continued in accordance with the scheme of the International Union for Solar Research. The spectra of the umbræ showed no perceptible differences as compared with those recorded in the preceding year, and dark D_3 was very frequently observed in the circumpenumbral regions. Solar prominences were also observed when time permitted, and particular attention was given to the bright lines visible in metallic eruptions.

Further investigations of terrestrial spectra have been made, with special reference to the interpretation of solar and stellar phenomena. A detailed account and table of wave-lengths of the titanium oxide flutings, which are characteristic of the Antarian (third type) stars, has been published (*Proc. Roy. Soc.*, vol. lxxix.A p. 509). An interesting result of the experimental work was the identification of the well-known sun-spot bands between E and F, with the component lines of the flutings of "magnesium hydride," a preliminary account of which was communicated to the Society in June. The continued investigation of this spectrum has shown that many other spot-bands in the yellow, green, and blue are also accounted for by magnesium hydride.

Stonyhurst College Observatory.
(*Director, Rev. W. Sidgreaves, S.J.*)

The solar surface has been observed as usual on all available days; 198 drawings of spots and faculæ have been made, and the greater spots have been examined with the grating spectrometer and the 12-prism spectroscope.

The new heliostat, mentioned in our last report, and also a 6-inch portrait lens—the gift of Mr. Whitelow, F.R.A.S.—have been mounted, but regular work with both instruments has been delayed by impediments experienced in the running of the driving-clocks. Sir Howard Grubb is surmounting those of the heliostat; and the Director, with the friendly help of Mr. Parkinson, a mechanician of Blackburn, has so far improved the equatorial clock gear and the

adjustments of the Whitelow camera that probably no further difficulty will be experienced in a long and repeated exposure of the same photographic plate.

These and other mechanical improvements have occupied a considerable part of the year, and many of the clear nights have been devoted to experimental work in connection with them.

Work with the stellar spectrographs has been practically discontinued, pending an examination of the collection of plates already made. But this examination has been rendered impossible during the past year by the very long and laborious work, now complete, of re-casting all the meteorological computations of the last sixty years.

Some trials of Wratten and Wainwright's pan-chromatic plates on the brightest stars have been made with the Hilger compound prism. These carry the spectrum to Fraunhofer's C, and with them it is intended to complete the photographic records of the stronger spectra in our collection.

Wolsingham Observatory (Rev. T. E. Espin's).

Micrometrical measures of new pairs and neglected double stars have been continued throughout the year, and the results presented to the Society.

Mr. Franklin-Adams' Astrographic Laboratory.

In order to push on with the "Chart of the Heavens," all other work has been suspended and a second assistant engaged. Extra help has also occasionally been employed.

At the turn of the year a week-end conference upon several matters in abeyance was held at Mervel Hill with Sir David Gill, Professor Kapteyn, and Mr. Eddington, in order to make the work as useful as possible in the investigations now being carried on by them.

Messrs. Troughton & Simms have delivered the enumerating machine, and experimental investigations of numbers and magnitudes of stars upon the 15-inch plates are in hand.

Preparations for publication are in hand, but the scarcity of nights with three hours' spells of brightness makes it impossible to foretell the date of commencement.

A new workshop has been built for the production of 15-inch positives.

Sir William Huggins' Observatory, Upper Tulse Hill.

Photography of spectra, and experimental work in several directions, are being carried on in the observatory and in the laboratory.

Sir Wilfred Peek's Observatory, Rousdon, Lyme Regis, Devon.
(Mr. C. Grover, Observer in charge.)

The observatory and instruments are in good working order. The very changeable and unsettled weather of 1907 has been a great hindrance to observations, which were only made on 136 nights; this is less than usual. August, September, and October were the three best months, with 47 fine nights. The observations of long-period Variable Stars have been continued with the 6·4-inch Merz equatorial refractor, and 368 magnitude determinations have been made. Argelander's method has been followed, as in the previous twenty-two years this work has been in progress, during which a total of 10,075 magnitude determinations have been recorded.

Transits of stars are taken as often as required for regulation of the sidereal clock, which maintains a good steady rate.

The planet Mars was observed on 15 nights, and 13 drawings of the disc were made between July 1 and August 9; the presentation of the disc was much the same as in 1892, when a fine series of drawings were made by Professor Keeler, which appear in vol. li. of the *Memoirs R.A.S.* Many of these well-known features were well seen at the recent opposition.

Comet Daniel (*d* 1907) was observed on many nights between July 2 and August 21, at which latter date the comet was a fine naked-eye object with a tail 14° in length—the brightest comet which has been seen for several years. The disappearance of Saturn's ring was well seen. It was visible as a fine hair-line of light on either side of the planet on October 2, and the next night it had totally disappeared, no trace of it being seen with powers 80 to 310. On October 12 the air was so clear that six of the satellites were visible, but no trace of the ring.

Mr. Saunder's Observatory, Crowthorne, Berks.

The principal work has been in continuation of the measurement and reduction of the two Yerkes negatives of the Moon. The reductions of the plate dated 1901 November are completed, and the resulting places of 1800 points copied on to the card index. On that dated 1901 August about 800 points have now been completely measured and reduced; some 700 more are at present measured once only.

Dr. W. E. Wilson's Observatory, Daramona, Co. Westmeath.

During the last year practically nothing has been done with the 2-feet reflector. The weather still remains remarkably cloudy. Numerous experiments have been made with the radio-integrator, and that instrument now gives excellent results.

Kodaikánal and Madras Observatories.
(*Acting Director, Mr. J. Evershed.*)

The first five months of the year were, as is usual at Kodaikánal, extremely favourable for solar observations. September and December were also favourable, but the remaining five months were unfavourable.

Photoheliograms were taken on 339 days, and it was found possible to send to Greenwich all the negatives required to fill in the gaps in the Greenwich and Dehra Dûn sets of daily photographs.

Spots were observed on 352 days, and their positions approximately determined by projection. A total of 301 new groups was recorded, and the mean daily number of groups visible varied from 2·7 in June to 7·1 in February, the average for the year being 4·6.

Spectroscopic work.—Prominences were observed and their positions charted on 305 days. Detailed observations were also made of the bright lines in metallic prominences. Spot spectra were observed on 129 days, and a photographic investigation of the spectra of some of the larger spots of the year was successfully carried out, using a parabolic and a plane grating. Photographs were also obtained of the spectrum of Comet Daniel with a prismatic camera.

Spectroheliograph.—After the completion of the new sliding roof covering the siderostat, work was started on February 1. In April a new primary slit was fitted, and the design of the secondary slit was considerably modified. It was also fitted with a device for automatically recording the setting after each exposure. The instrument has since worked quite satisfactorily, and in the eleven months from February 1 photographs of flocculi were taken on 300 days and of prominences on 253 days. Altogether, 1840 photographs were taken.

Publications.—Bulletins Nos. viii. to xi. were published during the year.

At Madras, astronomical observations were confined to those necessary for the time service.

Perth Observatory, Western Australia.
(*Director, Mr. W. Ernest Cooke.*)

This year has been a memorable one for all the Australian State observatories, because it marks the definite separation of the two departments of Astronomy and Meteorology. Hitherto each observatory has had complete charge of the meteorological work of its State, but a Central Commonwealth Weather Bureau has now been established, and from 1908 January 1 each State Astronomer will be able to devote the whole of his time to Astronomy.

At the Perth Observatory the year 1907 has been marked by considerable progress. All the Catalogue plates have now been

taken, and measurement has at last been commenced. In June two lady assistants were appointed for this purpose, and during the latter half of the year they have measured 62 plates, containing altogether 25,157 stars, each one having been measured in duplicate.

In addition, Professor Dyson, Astronomer Royal for Scotland, has very kindly offered to assist in the measurement of the Perth plates. Arrangements have accordingly been made to send him a regular supply of plates and reference star positions from the most southern portion of our region (40°), whilst in Perth a start has already been made from the other end (32°).

It has been decided to measure only one image of each star, to duplicate each measurement with plate reversed, to disallow and repeat all measurements which differ in the two positions by 0·005 (1″·5) and upwards, and to publish results to three figures only. In justice to Professor Dyson, it should be stated that he wished to take the mean of the two images as was done at Greenwich, and the Director accepts the responsibility of limiting the measurements to the first image only (6 minutes' exposure) He was guided in this decision by Professor Turner's remarks in the Introduction to the Oxford Astrographic Catalogue, vol. i., top of page 25, also page 34, where it is shown that the effect of taking four instead of two measures is to reduce the total probable error of place from ±0″·39 to ±0″·38; and under the present circumstances the extra labour does not appear to be justified.

In the determination of the positions of the reference stars practically a new epoch has been inaugurated this year. The observation of the 422 standard stars was completed in March, and a Catalogue giving their positions for 1905·0 was printed and distributed a few months later. This is the first published Catalogue of the Perth Observatory, and it forms the basis of all subsequent work, in the manner therein described. At the end of March reobservation of the reference stars in zone 33°–4°, which had been interrupted for a little over a year by the work of the Standard Catalogue, was renewed upon the fresh basis; that is to say, that instead of determining clock error from Nautical Almanac stars, and N.P.D. from nadir point readings, both clock error and equator point are now determined from the catalogued positions of at least six of the standard stars situated within the zone of observation.

The winter was, unfortunately, a phenomenally bad one, by far the cloudiest and most continuously wet that has ever been experienced here. But notwithstanding this, the transit observers made excellent progress, and succeeded in completing the zone 33°–4° on 1908 January 2; and, had there not been an exceptional rush of meteorological work in December, the reductions also would be up to date; as it is, they are only about a fortnight behind.

It has been considered advisable to apply corrections to the transit results obtained in previous years (zones 31°–2°, 39°–40°, and the first part of 33°–4°) in order to reduce them to the present

adopted standard, viz. that of the first Perth Catalogue. These corrections will be applied before the places are used to determine the plate constants. Considerable progress has been made with zone 39°–40° which is required for Professor Dyson, but it has not been possible to touch the zone 31°–2°, which is required for the plates now being measured here.

One piece of astronomical field work was undertaken during the year at the request of the Surveyor-General, viz. the determination of the longitude and latitude of Carnarvon, on the N.W. coast. It may, perhaps, not be out of place to call attention to the results obtained, as the method of observation (almucantar) is only just now coming into favour. The instrument was a 12-inch theodolite by Troughton & Simms. The method of using it was described in *Monthly Notices, R.A.S.*, vol. lxiii. p. 156, with an essential correction in vol. lxiv. p. 70. The weather was very bad. A strong gale was blowing all the time, whirling the sand and dust about, and there was a considerable amount of cloud. An ordinary chronometer was used, without electric attachment. Transits were much interrupted by the following causes:—inability to follow chronometer ticks on account of the wind's howling, clouds, flickering of the light, head being blown away from eyepiece by gusts of wind, sand or insects being blown into observer's eye. Time observations were taken on five nights and latitude on four, with the following results:—

Diff. of long. 8^m. $44^s{\cdot}06$, $44^s{\cdot}12$, $44^s{\cdot}07$, $44^s{\cdot}03$, $44^s{\cdot}03$.
Latitude 24° 53′ 9″·5, 8″·9, 10″·1, 9″·5.

The results for latitude depend on only three pairs of stars, and for longitude on from six to ten pairs.

The following is a résumé of the work during the year:—

Nights on which time was determined		197
Observations of zone stars in R.A.		4240
,, ,, in N.P.D.		4240
Observations of clock stars in R.A.		1230
Separate determinations of	Collimation error	11
	Level error	258
	Azimuth error	224
	Nadir point	165
Readings on meridian mark		71

Meteorological.—As already stated, this department has now been handed over to the Federal Government, but during 1907 the work proceeded as usual.

Melbourne Observatory. (*Director, Mr. P. Baracchi.*)

The astronomical work done at this observatory during the year 1907 was as follows:—

Meridian Observations:—

	Observations in R.A.	Observations in N.P.D.
Clock stars	585	—
Azimuth stars	315	59
List stars	1334	1331
Total ...	2234	1390

The list stars were selected from the Melbourne plates of the Astrophotographic Catalogue, to serve as fundamental points of reference for the reduction of these plates. The total number of this class of stars now completely observed not less than three times is 5965.

The Annual Catalogue for 1905 has been completed, and the Annual Catalogue for 1906 is nearly ready. The General Catalogue for 1900, including all stars observed since 1894, has been further advanced.

Astrophotographic Operations.—The Director regrets to report that the Assistant Observer, Mr. W. J. Wallace, who was connected with this work, died on 28th December last, after a long period of ill-health, which extended over the greater part of the year.

The following table shows the number of plates exposed during the year:—

	Passed as satisfactory.
Chart plates with triple exposure of 30^m each ...	13
Catalogue plates (duplicate series)	19
Test plates on South Polar regions	4
Plates for trails, adjustment of centre, etc.	9

The state of the Astrographic work allotted to this observatory is as follows:—

The first series of 1149 plates, covering twice the entire region south of Dec. $-64°$, and the series of chart plates with single exposure of one hour, whose centres lie on the parallels of even degrees of declination from 66° to the South Pole, were completed some years ago. A duplicate Catalogue Series is in course, of which 354 regions have been photographed.

In the series of chart plates with three exposures of 30^m each, having their centres on the parallels of odd degrees of declination, 532 plates have been taken. Thus, the original programme, so far as actual photographic operations are concerned, has been practically carried out.

It is intended, however, to complete the second Catalogue Series, and to extend the triple exposure chart plates to the regions of even degrees of declination, as the chart plates with single exposure of one hour are not considered satisfactory.

The measurement of plates will be dealt with in a separate report.

The following routine duties and other miscellaneous work were carried out for local public requirements as in former years:—

The Time Service, including daily time signals, control of public clocks, and rating of chronometers for the shipping.

The testing of nautical, meteorological, and surveying instruments.

The technical operations of the Bureau on Standard Weights and Measures according to law.

The Weather Service, including the control of some 1000 reporting country stations.

The continuous registration of the variations of sea-level, atmospheric elements, seismic disturbances, and the elements of terrestrial magnetism, including the reduction of magnetic records of past years.

An Australian Weather Bureau has been established by the Federal Government with headquarters in Melbourne, but not in any way connected with this observatory. The new institution assumed official control of the weather service of the various States of the Commonwealth on 1st January last, thus relieving the existing Australian observatories of all their public duties in connection with meteorology.

Joint Report of the Directors of the Melbourne and Sydney Observatories on the Measurement of the Plates of the Astrophotographic Catalogue.

The Catalogue plates of the Sydney and Melbourne Zones, covering the whole region south of $-51°$, are being measured at the Melbourne Observatory by a special Bureau established for this purpose towards the end of 1898, and maintained jointly by the Governments of New South Wales and Victoria.

The work of the Bureau during the year 1907 was carried on by a staff of six ladies, under similar conditions, and using chiefly the same pair of Repsold micrometric instruments as in former years.

The numbers of plates completely measured in the direct and reversed positions, in the course of the year, are—

138 Sydney plates, containing 97,881 stars.
56 Melbourne plates, containing 15,870 stars.

The total aggregate numbers of plates measured up to 31st December 1907 are as follows:—

700 Sydney plates, containing 423,859 stars.
892 Melbourne plates, containing 284,584 stars.

Sydney Observatory. (*Director, Mr. H. A. Lenehan.*)

In the early part of the year it was announced that the Government required the site of the observatory for other purposes, and intended to remove the instruments to a new position.

In May an Advisory Committee was appointed to act with the Government Astronomer, and many sites suitable for an observatory were inspected. The one finally selected was on or near the Canoblas mountain, in the vicinity of the town of Orange, 130 miles west from Sydney, where there is clearness of definition, freedom from city lights, electrical disturbances, and other troubles which are encountered in the present position of the observatory. Nothing has yet been decided as to the future, and the observatory is still fixed in the original site. The report of the Advisory Committee has not been acted upon.

The transit-circle piers and the graduated circles have been cased in, to shield them from air-currents which caused expansion and contraction. This was a very necessary work, and since it has been completed the instrumental corrections have been more constant. There has also been less tarnishing of the graduated silver bands.

Other instruments have been in the hands of the mechanician and placed in working order. A 6-inch Grubb equatorial telescope in the north dome required many alterations to make it effective. The alteration of the double pendulum escapement of the cylinder chronograph to a rotary one was much appreciated by the First Assistant. A new direction recording frame has been made for the anemometer. During the year 1908 a new sidereal clock is to be placed in the basement, where it will be free from air-currents and excessive changes of temperature. A 15-inch equatorial telescope is also to be ordered from Sir Howard Grubb.

New magnets have been received from Kew Observatory to use at the Red Hill branch, where the magnetic work is carried out. During December three members of the Astronomical Staff left to join the McClean and Lick Observatory parties for solar eclipse observations at Flint Island—two of them joining Mr. McClean. Should fine weather conditions prevail on the eventful day, they will obtain experience that should be of use to them when they are called upon to observe the solar eclipses in Tasmania in 1910 and in Australia in 1911.

Mr. Olden has qualified as an observer with the transit-circle, and is of much assistance during the absence of the members of the staff.

Improvements have been made in the buildings at the Red Hill branch. A large dark-room has been constructed for the developing of the Astrographic plates, and a new pier erected for the magnetometer.

The following observations have been made with the transit-circle :—

Number of	R.A. observations of stars . . .	1986
„	N.P.D. „ „ . . .	1665
„	R.A. „ of Sun . . .	447
The 9 A.M.	Collimation, Nadir and Level readings .	306
9 P.M.	Level readings	108
„	Nadir determinations	108
„	Azimuth determinations . . .	118

The computations are in a forward state; the abstract for 1906 will shortly be completed, and a portion of the 1907 work is well in hand. The accumulated observations with the transit-circle for years back are being prepared for printing and publishing. Visitors were numerous, those in the daytime numbering 768, and during the evenings for lectures, etc. 534.

Two comets, *c* and *d* 1907, were observed in July and August, and photographs taken. This to a certain extent interfered with the Astrographic work, and the preparations for the departure of the party to observe the solar eclipse at Flint Island caused a further interruption towards the end of the year.

The readings of the magnetic declination at the Red Hill branch showed 9° 28′ 17″ east, making for Sydney 9° 32′ 17″ east—practically a constant determination. The intensity, vertical and horizontal forces have also been determined.

Tidal registrations on the coast and harbours of New South Wales have been continued.

The seismograph has been in use constantly during the year, and 80 earth tremors have been recorded on the photographic band. These have all been measured and tabulated, and the results forwarded to the Secretary of the Seismological Committee, London. The library has received about 750 books, pamphlets, etc., which shows a falling off in the number received during the previous year. This is probably accounted for by the separation of the two branches of the observatory. Meteorological books which were previously sent to this observatory may have been diverted to the Central Bureau, Meteorological Department, Melbourne.

Lovedale Observatory, South Africa.
(*Dr. Alex. W. Roberts.*)

The Director returned to South Africa in the end of March, and regular observations were carried on during the remaining nine months of the year.

The work done was, as heretofore, the observation of certain well-known southern variable stars.

Observations were made on 121 nights during April to December, and the following measures were secured:—

Algol variables (8 stars) . .	952	observations.
Short period variables (23 stars) .	2052	,,
Long period variables (69 stars) .	2408	,,
	5412	,,

During the past fifteen years the guiding aim of the work attempted at Lovedale has been to secure such a continuous series of observations of typical Algol and short period variable stars as might in the remote, or probably immediate, future give data for a determination of the orbital movements or physical conditions which produce or modify stellar variations of the eclipse and short period character.

It is hoped that this aim has to a certain extent been realised.

In the future the work done at Lovedale will be the examination of suspected variables, and of such long period stars as indicate secular variation in their light curves and periods.

During the year a determination was made of the value of the constant of atmospheric absorption.

Mr. Tebbutt's Observatory, Windsor, New South Wales.

Measures, several times repeated, of thirteen of the most interesting binary stars of the southern hemisphere constitute the only astronomical work at this observatory for 1907. The results will shortly be sent to the Society. Daily observations of two rain-gauges at different heights, and also observations of the various monthly maximum and minimum temperatures, have been secured during the year.

Notes on some Points connected with the Progress of Astronomy during the Past Year.

Planets and Satellites.

The following 98 planets have been discovered or first announced during 1907. The year of discovery is 1907 unless otherwise noted.

Letter and Number.	Date.	Discoverer.	Letter and Number.	Date.	Discoverer.	Letter and Number.	Date.	Discoverer.
XJ 623	Jan. 22	L_1	YU ...	Apr. 4	K	UN[a]	'06 July 16	W
XM 624	Feb. 10	K	YV ...	4	K	UN[b]	'06 July 16	W
XN 625	11	K	YW ...	5	K	ZT ...	July 19	L_1
XO 626	11	K	YX 632	5	K	ZU ...	Aug. 8	K
XP ...	8	M	UM[a] '06	June 20	L_2	ZV ...	8	K
XQ ...	8	M	UM[b] '06	June 20	L_2	ZW ...	29	K
XR ...	4	L_2	VE[a] '06	Sept. 22	L_2	ZX ...	Sept. 8	W
XS 627	Mar. 4	K	VE[b] '06	Sept. 22	L_2	ZY ...	8	W
XT 628	7	K	WT[a] '06	Dec. 6	L_2	ZZ ...	8	K
XU 629	7	K	YY ...	Apr. 7	L_2	AA ...	7	K
XV ...	7	K	YZ ...	7	L_2	AB ...	11	K
XW 630	7	K	ZC ...	Mar. 20	M	AC ...	11	K
XX ...	6	M	ZD ...	Apr. 17	M	AD ...	11	K
XY ...	7	M	ZE ...	18	M	AE ...	11	K
YA ...	Feb. 15	M	ZF ...	11	M	AF ...	11	K
YB ...	Mar. 7	M	ZG ...	10	L_2	AG ...	14	M
YC ...	7	M	ZH ...	15	L_2	AH ...	18	S
YD ...	9	M	ZJ ...	May 9	K	AJ ..	Oct. 4	K
YE ...	11	M	ZK ...	11	K	AK ...	4	K
YG ...	11	M	ZL ...	11	K	AL ...	4	K
YJ 631	21	K	ZM 633	12	K	AM ...	4	K
YM ...	Apr. 4	K	ZN 634	12	K	AN ...	4	K
YN ...	4	K	ZO ...	Apr. 5	L_2	AO ...	Nov. 1	K
YO ...	4	K	ZP ...	May 4	M	AP ...	3	K
YR ...	4	K	ZQ ...	5	M	AR ...	3	K
YS ...	4	K	ZR ...	11	M	AS ...	3	K
YT ...	4	K	ZS 635	June 9	L_1	AT ...	3	K

Letter and Number.	Date.		Discoverer.	Letter and Number.	Date.		Discoverer.	Letter and Number.	Date.		Discoverer.
AU ...	Nov.	4	P	BA ...	Nov.	7	S	BG ...	Nov.	10	K
AV ...		4	K	BB ...		8	W	BH ...		30	K
AW ...		4	S	BC ...		8	W	BJ ...	Dec.	4	K
AX ...		5	S	BD ...		8	W	BK ...	Nov.	27	M
AY ...		6	K	BE ...		3	M	BL ...	Dec.	11	M
AZ ...		6	K	BF ...		4	M				

A planet found on Greenwich plates exposed 1903 August 6, 31, September 1, has been designated LX[a], UW is identical with 66, UZ with 408, WO with 260, WQ with 167, XK with 129, XL with 462, XZ with 469, YH with 236, YK with 411, YL with 629, YP with 62, YQ with 450, ZA with 510, ZB with 31, 615 with KR and KV, WK probably with 34, 635 with TU, UT probably with GX, YA with YF, AQ with 528, AX (probably) with 518, BA (probably) with 566.

The following planets, not numbered at the date of the last report, have since received permanent numbers: TE 602, TJ 603, TK 604, UU 605, VB 606, VC 607, VD 608, VF 609, VK 610, VL 611, VN 612, VP 613, VQ 614, VR 615, VT 616, VY 617, VZ 618, WC 619, WE 620, WJ 621, WP 622.

The following have received names: 398 *Admete*, 410 *Chloris*, 411 *Xanthe*, 414 *Liriope*, 426 *Hippo*, 427 *Galene*, 429 *Lotis*, 430 *Hybris*, 431 *Nephele*, 437 *Rhodia*, 438 *Zeuxo*, 441 *Bathilde*, 453 *Tea*, 497 *Iva* 506, *Marion*, 508 *Princetonia*, 517 *Edith*, 518 *Halawe*, 519 *Sylvania*, 523 *Ada*, 533 *Sara*, 534 *Nassovia*, 535 *Montague*, 537 *Pauly*, 583 *Clotilda*, 588 *Achilles*, 602 *Marianna*, 617 *Patroclus*, 620 *Drakonia*, 624 *Hector*.

The initials represent the following: K = Herr Kopff, Heidelberg; L_1 = Herr Lohnert, Heidelberg; L_2 = Prof. Lowell, Flagstaff, Ariz.; L_3 = M. Liapin, Pulkova; M = Dr. Metcalf, Taunton, Mass.; P = Dr. Palisa, Vienna; S = Herr Scheifele, Heidelberg; W = Dr. Wolf, Heidelberg.

VY = 617 Patroclus and XM = 624 Hector prove to have, like TG = 588 Achilles, orbits with nearly or quite the same period as Jupiter, exemplifying more or less closely Lagrange's proposition of the stable motion of three bodies at the vertices of an equilateral triangle. The longitude of VY Patroclus is less than that of Jupiter, that of the other two is greater. Both Achilles and Patroclus have been well observed at their second opposition, so materials are available for determining their orbits accurately. The question of the stability of these orbits, both as regards the Jupiter perturbations and those of the remaining planets, is an interesting one, and has not yet been completely solved.

Dr. Ross has published new elements of Jupiter's satellites VI. and VII. Those of the former are based on the early places

secured from Harvard plates of 1894, 1899, as well as those since the discovery.

Elements referred to Earth's equator 1905·0:—

	Satellite VI.	Satellite VII.
ϖ	268°·82	114°·22
Ω	171·00	291·45
i	5·55	25·31
l	20·00	238·23
e	0·155	0·207
Sid. period	250^d·618	260^d·06
a for log $\Delta = 0{\cdot}71624$	3037″	3113″

	VI.	VII.
Annual increase of ϖ on Earth's equator,	1°·493	
Annual increase of ϖ on Jupiter's orbit,	1°·373	1°·44
Annual decrease of Ω on Earth's equator,	4°·358	
Annual decrease of Ω on Jupiter's orbit,	1°·207	1°·35
Inclination of orbit to Jupiter's orbit .	28°·747	27°·972

Dr. Ross also gives the principal perturbations, of some of which the approximate values are given here (Delaunay's notation).

$dL =$ VI.		VII.	$\frac{da}{a} =$ VI.		VII.	$di =$ VI.		VII.
+ °·3	sin (2D − 2l)	+ °·5	+ ·007	cos(2D)	+ ·014	− °·5	sin ($\mathrm{F}-l$)	− °·6
+ ·3	(2D − 2F)	+ ·3	+ ·003	(2D + l)	+ ·006	+ ·4	(2D − F − l)	+ ·6
+ ·2	(2F − 2l)	+ ·3	+ ·018	(2D − l)	+ ·024	− ·5	(F − 2l)	− ·9
− ·4	l^1	− ·4	+ ·002	(2D − l − l^1)	+ ·003	− ·6	(3F − l)	− ·7
+ ·5	2D	+ ·8	− ·007	2F	− ·010	− ·3	3F	− ·4
+ 2·3	(2D − l)	+ 3·2	− ·002	(2F + l)	− ·004	+ ·8	(2D − F)	+ ·8
+ ·3	(2D − l − l^1)	+ ·4	− ·024	(2F − l)	− ·030	+ ·5	(2D + F − l)	+ ·7
− ·5	2F	− ·8				+ ·3	(2D + F)	+ ·4
− 2·7	(2F − l)	− 3·4						

A. C. D. C.

The Comets of 1907.

Five comets have been discovered during the year, and the order of discovery has followed that of perihelion passage.

Comet 1907 (*a*), discovered on March 9 at Nice by M. Giacobini, was carried rapidly towards the Sun, and observations soon ceased. But late in the year the comet, though faint, was again in a favourable position for observation, and was photographed by Dr. Max Wolf on December 4, who estimated the brilliancy as that of the 14th mag. The elements, which are not yet definitely known, resemble those of Comet 1890 IV.; and though the identity

of the two objects is impossible, they may have had a common origin.

Comet 1907 (*b*) was first seen by Mr. Grigg, of Thames, New Zealand, on April 9, but intelligence of its discovery was first circulated by Mr. Mellish of the Washburn Observatory, who saw the comet on April 14. The trace of the comet was subsequently found on a plate taken on April 13 by Professor Barnard, while searching for Giacobini's comet. The object was faint and the observations were not numerous. The orbit is interesting, since the elements are similar to those of 1742, a bright comet first seen at the Cape of Good Hope, but also observed in Europe.

Comet 1907 (*c*), discovered on June 1 at Nice by M. Giacobini. This was also a faint comet and was observed for only a short time, but it is not impossible that it may be re-observed in the spring. There is no reason to think that the orbit differs from a parabola.

Comet 1907 (*d*), found by Mr. Daniel of the Princeton Observatory on June 9, was the brightest comet of the year, and in the morning sky of August was quite as conspicuous as a star of the second magnitude. Numerous observations and photographs were secured, the latter disclosing a tail of very complicated structure, extending at least 17° from the nucleus. There was also some evidence of a rotatory motion of the comet about a line joining the nucleus and Sun.

The perihelion passage took place on September 3, and after perihelion observations were possible, the comet being picked up on November 5 by Dr. Hartwig, who reported it as of the eighth magnitude.

Comet 1907 (*e*) was first observed by Mr. Mellish, of the Washburn Observatory, on October 13. For some weeks the comet increased in brilliancy, and was so favourably placed that it could be observed on the meridian. The brightness faded very quickly, however, after the middle of November, diminishing more rapidly than the ordinary accepted formula would explain.

It will be noticed that no periodical comet * has returned during the year, and no new elliptic comet of short period has been detected. Denning's Comet of 1881 V. unfortunately passed through perihelion without being perceived. A similar fate overtook the same comet in 1890 and 1899; and since the mean motion could not be determined with great accuracy in 1881, this object must be considered lost. Swift's Comet 1894 IV., an object of unusual interest on account of its supposed identity with the lost comet of De Vico, passed through its perihelion about the beginning of July, but was not seen. The chances of recovering this comet are very slender. Comet 1900 III. (Giacobini) also passed undetected.

On the other hand, as evidence of long continued visibility of

* Encke's Comet was photographed at Heidelberg by Dr. Max Wolf on December 25, but the cometary character was not recognised till January 3, 1908.

comets, attention may be drawn to the fact that observations of Comet 1905 IV. already extend over a period of more than three and a half years, and may be even further prolonged. Observations of Comet 1904 I. extend from 1903 May 14 to 1905 June 5, or 753 days.

Spectrum of Comet d 1907.—The position of this comet in the sky was not very favourable for spectrum observations, but MM. Deslandres and Bernard at Meudon, Dr. Rosenberg at Göttingen, and Mr. and Mrs. Evershed at Kodaikánal have obtained valuable results.

In addition to the details shown by the nucleus, which exhibit the ordinary characteristics, including the cyanogen band at λ 388, first seen by Huggins in the comet of 1881, the spectrum of the tail has been photographed for a distance of at least 45′ from the head. The spectra of the head and tail do not rigorously coincide. While some bright bands are entirely confined to the nucleus, fainter bands can be traced to a considerable distance in the tail; and further, some faint bands in the nucleus, whose chemical origin is not yet clearly ascertained, are not reproduced in the tail.

It has therefore been suggested that the constitution of the nucleus and that of the tail of the comet are not similar. In connection with the same subject, attention should be called to a paper by Mr. H. F. Newall, in which he has described the alteration produced in the spectrum of cyanogen under various conditions of pressure. In this same paper Mr. Newall, after pointing out that gases and vapours must exist in circumsolar space, a fact that is sufficiently indicated by the extension of the corona and the length of comets' tails, offers an explanation of the presence of those features of the spectrum common to all comets. He submits that it is more probable that the approximate similarity in the spectra of all comets is due to the fact that these objects come in contact with the same vapours, spread in their path as they approach the Sun, and by some process elicit the spectra of these vapours, than that comets starting from all parts of the universe should bring with them the cyanogen and hydrocarbons shown in their spectra.

The orbits of the following comets have been definitely determined during the year:—

Comet.	Character of Orbit.	Calculator.	Place of Reference.
1796	Parabolic	H. A. Peck	*Ast. Journal*, No. 596
1819 II.	,,	,,	,, ,, 593
1822	,,	,,	,, ,, 596
1826 II.	,,	Cowley and Whiteside	*Erganz. zu den Ast. Nach.* No. 13.
1905 V.	,,	G. Zappa	*Annali dall' Obs. Coll. Romano*, vol. iv.
1906 I.	,,	Terkan and Czuczy	*Ast. Nach.* No. 4218

The return of Halley's Comet in 1910 has attracted considerable attention. The discrepancy between the time of perihelion passage computed by Pontécoulant and the time indicated by Ångström's curve, founded on two supposed inequalities based on Hind's identification of ancient returns, suggested the necessity for further computations. Such a revision was the more necessary owing to the detection of a suspicious error in the value of the perihelion distance assigned by Pontécoulant.

Attention was first directed to this subject by Mr. Crommelin, but later Mr. Cowell has collaborated, with the result that the method of calculating the perturbations of long period comets has been examined and simplified, and the necessary formulæ have been put in a very practical form. The object of the inquiry has been twofold. First, to determine with the necessary accuracy the date of the comet's perihelion; secondly, to trace backwards the course of this famous comet, and to identify its several returns. With regard to the first part of the investigation, the result has been to confirm the general accuracy of Pontécoulant's prediction. He assigned 1910 May 23 as the date of next perihelion passage; and Messrs. Cowell and Crommelin show that the period he has assigned requires to be shortened by nine, and two days for the actions of Jupiter and Saturn respectively. Further, the large value for the perihelion distance given by Pontécoulant is clearly in error. The historical research, so far as it has been carried, proves that the comet returned in 1301, as suggested by Hind. The result is the more satisfactory, as very considerable doubt attached to that particular date owing to the misdescription of the comet's position by contemporary observers. The certainty of identification has been carried one revolution further back, and it is understood that more distant epochs will be investigated. W. H. P.

Solar Activity in 1907.

Sun-spots.—There has been a distinct increase in the spot-activity of the Sun during 1907 as compared with the preceding year, though the level of 1905 was not quite attained. The number of giant groups readily visible to the naked eye has been double that recorded in 1906, and equal to that of 1905, no fewer than 24 distinct groups large enough to be thus seen having been observed during the year. The chief groups of this rank were on the disc 1906 December 29–1907 January 9, January 28–February 7, February 6–19, June 13–25, July 11–23, September 24–30, October 15–25, November 9–22, and December 5–18. The tendency, so marked in 1906, to develop long processions of groups following each other at short intervals along a parallel of latitude has continued in 1907, and on more than one occasion the Sun, as seen merely through a dark glass, presented a very fair imitation of a Martian "canal," in the shape of a long, straight narrow scar.

The first two months of the year were the most disturbed, and it is not improbable that the crest of the wave may eventually be found to lie somewhere within their limits, unless the revival, evident in 1907, should be still further accentuated in 1908. Faculæ were most numerous in March, which was, so far, the most prolific month for these bright markings during the whole of the present maximum. Two days at the end of November showed the Sun free from spots. E. W. M.

Prominences.—The daily frequency of prominences deduced from spectroheliograms taken on 40 days during the year 1907 was somewhat greater than that deduced for the preceding year, also from photographs secured at the Solar Physics Observatory, South Kensington. The pictures were taken with the "K" line (calcium) of the solar spectrum, and the lower limit for the height of prominences accepted was about 20″. The photographs examined seemed to indicate that for the year 1907, unlike the previous year, the activity was fairly equally divided between the two hemispheres, the figures obtained for the last three years being as follows:—

	1905.	1906.	1907.
North hemisphere . . .	3·4	3·9	3·5
South hemisphere . . .	3·0	2·6	3·9
Total daily frequency	6·4	6·5	7·4

The above figures for the total daily frequency indicate a lag of the maximum prominence activity behind the maximum of spotted area which occurred late in 1905, though the spotted area of 1907 will probably show an increase over 1906.

Activity apparently in the neighbourhood of the solar poles was less marked than in 1906, while equatorial prominences were not infrequent.

From July 15th to the 20th the Sun appeared very active, showing many high and remarkable prominences; fortunately good weather allowed a number of excellent photographs to be obtained.

The visual observations of prominences made at Catania by A. Riccò* indicate for the first six months of 1907 a considerable increase over the mean frequency for 1906. The lower limit of height of observed prominences was 30″.

	1906.	1907 (first semestre).
North hemisphere . .	1·56	2·15
South hemisphere . .	1·06	1·66
Total daily frequency	2·62	3·81

The distribution in latitude shows for the activity in 1907 maxima at 36°–40° and 51°–55° in the north solar hemisphere, and 15°–19° and 40°–44° in the south. W. J. S. L.

* *Memorie della Società degli Spettroscopisti Italiani*, disp. 12, vol. xxxvi.

Solar Research.

The International Solar Union.—An important event in 1907 was the Third Conference of the International Union for Co-operation in Solar Research, which was held at Meudon in May, under the presidency of the late Dr. Janssen. Reports of the work of the committees on standard wave-lengths, sun-spot spectra, solar radiation, and spectroheliograph observations were presented, and new committees were appointed to deal with solar rotation problems, and the organisation of solar eclipse work.

The re-determination of the wave-length of the red line of cadmium has been completed by Messrs. Fabry and Perot, and is in such satisfactory agreement with the independent results obtained by Michelson and others that it was resolved to adopt it as the basis of the new wave-length tables. The work on the secondary standards was also reported as well advanced, and the wave-length committee was authorised to issue a definitive table without waiting for its formal adoption.

The next Conference will be held at Mount Wilson, California, in the autumn of 1910.

Sun-spot Spectra.—Investigations of the spectra of sun-spots have made great progress during the year. The visual observations which have been so laborious in the past have now been greatly simplified by the admirable photographic map of the spot spectrum which has been prepared at Mount Wilson, and, through the kindness of Professor Hale, distributed among the observers in this section of the work of the International Solar Union. Some very fine photographs have also been secured at Kodaikánal, and the photographic method is also being successfully employed by Mr. Newall at Cambridge, and by Father Cortie at Stonyhurst. The records of the phenomena are therefore now perfectly definite, and experimental work tending to their elucidation is enormously facilitated.

Hale and Adams have given an account of further investigations which they regard as supporting the view that the modifications of the metallic lines in spot spectra are to be explained by reduction of temperature, the flame lines being strengthened while the enhanced (spark) lines are weakened. Further evidence in the same direction is also afforded by their very satisfactory identification of the stronger flutings of titanium oxide in the spots, those having heads at 7054·60, 7087·90, and 7125·89 being very strongly marked (*Ap. J.*, xxv. 75–94). The spot spectrum thus shows an approximation to the third type stars, of which the titanium oxide flutings are characteristic. An interesting paper in this connection has been published by Father Cortie (*Ap. J.*, xxvi. 123), who points out that at the maximum of *Mira Ceti* in 1906 the flutings were much less developed than at the maximum of 1897, when the magnitude at maximum was lower. Assuming the greater

brightness to be associated with a higher temperature, the appearance of the flutings in sun-spots points to the conclusion that the temperature of sun-spots is lower than that of the photosphere.

Many of the numerous spot "band lines" in the region E to F have been identified by Fowler with components of the fluting of "magnesium hydride," the head of which is at 5211·0 (*M.N.*, lxvii. 530). It is probable that this will throw much light on the discontinuity of the dark background of the spot spectrum, which gives rise to the appearance of bright lines and bands. There is evidence that the absorption of this substance is also shown faintly in the normal solar spectrum.

The weakened lines in spots have received special attention from Mr. Nagaraja of Kodaikánal, who has catalogued 167 of this class between F and D. In accordance with the previous observations of Fowler, and Hale and Adams, it is found that enhanced metallic lines figure largely among the weakened lines, and that many of them occur as high-level lines in the chromosphere (*Ap. J.*, xxvi. 143).

A remarkable feature of the sun-spot spectrum is the tendency to disappearance of characteristic features in the violet and ultra-violet, the spectrum there gradually approximating to that of the ordinary disc. Hale and Adams have discussed this matter in the paper to which reference has been made above; and though they have reached no explanation which they consider final, it is suggested that the effect may be largely due to solar light diffused over the spot area. At Kodaikánal, "with improved apparatus and under exceptionally favourable atmospheric conditions," affected lines have been photographed in the region 3990 to 4350, but Mr. Evershed questions whether the admixture of skylight with the spot spectrum is a sufficient explanation of the poverty of detail. Titanium and vanadium account for many of the strengthened lines in this region as well as in the visible spectrum (*M.N.*, lxviii. 12).

Closely associated with the study of sun-spot spectra is the important investigation of the spectrum of the Sun's disc near the limb, which has been made at Mount Wilson (*Ap. J.*, xxv. 300). It is found that the "wings" of many lines disappear at the limb, and that the relative intensities of numerous lines are changed, generally in the same direction as in spots, though the weakening is relatively more marked than the strengthening of lines. The further discussion of these results, which is promised, will be awaited with great interest.

Useful notes on the conditions which accompany the appearance of bright D_3 over the umbræ, and of dark D_3 in the neighbourhood of spots, are given by Mr. Nagaraja and Capt. Daunt (*Observatory*, 214, 249).

The Sun's Rotation.—The spectrographic method of observing the Sun's rotation has been employed by Mr. Newall in order to determine whether the cyanogen band 3883 in the solar spectrum is wholly of solar origin (*M.N.*, lxviii. 2). In the observations made during August and September the band generally showed a double head at the east limb, as if there were two bands superposed, while

at the west limb there was only a slight intensification, without measurable duplication. It is suggested that some cyanogen having independent motion must have existed in interplanetary space between the Earth and Sun, and not far from the Sun, and the possible connection with the *debris* of comets is discussed.

The results of the first systematic investigation of the Sun's rotation by the spectrographic method have been announced by W. S. Adams (*Ap. J.*, xxvi. 203). Granted an apparatus as powerful as that used at Mount Wilson, the many advantages of the photographic method are obvious, particularly as regards the number of lines which may be utilised. In this investigation twenty lines of various origins were selected for measurement, and the general means give 24·46, 24·79, 25·48, 26·43, 27·63, 28·73, 29·63, 30·25, and 30·56 days as the rotation periods for heliographic latitudes 0, 10, 20, 30, 40, 50, 60, 70, and 80 degrees respectively. For the lower latitudes the values agree very closely with the results of Dunér and Halm, and for the higher latitudes are intermediate between the results of these observers. A fact of great interest is that different lines give slightly different rates of rotation, those of carbon and lanthanum showing 0".1 less than the daily rate indicated by the mean of all the lines, while two lines of manganese give systematically high results.

In connection with the Computing Bureau of the Solar Union, Professor Turner has made a preliminary investigation of the position of the Sun's axis, as deduced from the Greenwich sun-spot measures (*M.N.*, lxviii. 98). A small and possibly periodic departure from the axis deduced by Carrington is indicated.

The Spectroheliograph.—Records of the calcium flocculi are being regularly made at Mount Wilson, Kodaikánal and other observatories. In addition, Hale has made investigations in which lines affected in sun-spots have been adjusted to the position of the second slit. In the case of strengthened lines, the umbræ and penumbræ of spots are relatively darkened and the areas are considerably increased; and definite, though less marked, results have been obtained with enhanced lines. Successive pictures of the calcium flocculi have also been prepared for use in the stereoscope, showing the sphericity of the Sun and the apparent elevation of the flocculi. Features which escape notice in single photographs are brought into prominence by this method of observation (*Ap. J.*, xxv. 311). A. F.

The Moon's Physical Libration.

Between 1898 and 1900 Dr. F. Hayn conducted with the 30 cm. refractor of the Leipzig Observatory a series of measures in order to determine accurately the selenographic co-ordinates of a number of points on the Moon's surface. He intended to reduce these on the supposition that the rotation elements of the Moon and the position of Mösting A, as given by Franz's reduction of Schlüter's observations, were substantially accurate. In the course

of his work he found reason to doubt the truth of this assumption, and accordingly he undertook a complete revision of the analysis on which the reduction is based. The aim he set before himself was to include every term in Hansen's theory which could produce an effect amounting to 2″ in selenocentric or 0″·1 in geocentric arc, and to carry through the reductions to this degree of accuracy.

Since the first series of observations were planned for a different purpose, and were not suitable for determining the position of Mösting A, or the coefficients of the principal terms in the Physical Libration, a fresh series was undertaken towards the end of 1903, in which the distance and position-angle of Mösting A were determined on ten evenings, sometimes from as many as eighteen points on the limb. The measures were made with a filar micrometer, and the distances were so great that considerable reliance had to be placed on the driving-clock, but the results seem to justify the confidence placed in its accuracy.

When the observations were reduced it was felt that they did not justify a definitive determination of the rotation elements, and it was decided to make a fresh reduction of the similar measures made by Hartwig between 1877 and 1879 with the Strassburg heliometer. This work has now been accomplished, and the final results are published under the title of "Selenographische Koordinaten" in Bd. xxx. *der Abhand. der Math. Phys. Klasse der k. Sächs. Gesell. der Wiss.*, the previous work having been published in Bdn. xxvii. and xxix. of the same series. The principal points in the reduction are that greater care is taken in the integration of the equations not to neglect terms which would affect the result to the degree stated, and that account is taken of the departure of the surface of the Moon from that of a sphere, the amount of this departure being determined from the observations themselves.

The principal results may be stated as follows:—

If A, B, C are the Moon's principal moments of inertia, A being that about the equatorial diameter through the zero meridian, C about the axis of rotation,

I the mean inclination of the Moon's equator to the ecliptic,

λ, β the selenographic longitude and latitude of Mösting A,

h the radius to this point,

$h_{N.A.}$ the radius assumed in the *Nautical Almanac*, = 15′34″·21,

g the Moon's mean anomaly,

g' the Sun's mean anomaly,

ω the departure of the Moon's perigee from the ascending node of its orbit,

then Dr. Hayn finds—

$$f = \frac{C-B}{A}\,\frac{B}{C-A} = 0{\cdot}75 \pm 0{\cdot}04$$

$$I = 1^{\circ}\,32'\,6'' \pm 15''$$

$$h = h_{N.A.} + 0''{\cdot}50 \pm 0{\cdot}56$$

$$\lambda = -5^{\circ}\,10'\,13'' \pm 15''$$

$$\beta = -3^{\circ}\,10'\,58'' \pm 12'',$$

which correspond to the rectangular co-ordinates

$$\xi = -\cdot08998, \quad \eta = -\cdot05552,$$

whilst the physical librations are

in longitude $-12''\sin g + 59''\sin g' + 18''\sin 2\omega$
in latitude $-144''\sin\omega + 11''\sin(g+\omega)$.

The coefficients of sin g and sin g' are about half those found by Dr. Franz, whilst the libration in latitude takes a different form. Dr. Franz's value of f was 0·49.

The errors given are those of mean square.

Dr. Hayn also gives the co-ordinates of four points of the first order. Those of fifteen other points of the second order are promised in a fourth memoir.

The libration considered is that due to the forced oscillations whose periods are determined by the configuration of the system Sun, Earth, Moon. A search was made for evidence of free oscillations, but, like previous attempts, it showed only that the amplitudes must be very small.

The *Berliner Jahrbuch* for 1909 contains an ephemeris for Mösting A deduced from these elements.

But even those now published cannot be considered definitive: the magnitudes of some of the results depend upon an arbitrary system of weighting adopted for the conditional equations, and it is imperative that some of the other series of observations, of which there are several in existence, should be re-reduced. The work of reducing those made by Schlüter has already been commenced at Cambridge by Mr. Stratton.

Dr. Hayn also publishes a provisional table and contour map showing the difference from mean level at every point of the surface which can come upon the limb. In 1905 he published a table of the same kind in *Ast. Nach.* 4009; this he now withdraws in favour of the new table, depending on a more complete reduction of the same observations.

Dr. Hayn is to be congratulated on having brought to a successful termination a very laborious piece of work, in which he deals with quantities that are near the limit of what can be observed. He has considerably improved our knowledge of the rotation elements and the contour of the Moon, and has shown how a still further improvement may be effected. S. A. S.

Action of the Planets on the Moon.

In 1892 M. Radau published his researches on the planetary inequalities in the movement of the Moon. The past year has been marked by the publication of investigations by Professor Newcomb*

* Investigations of Inequalities in the motion of the Moon produced by the action of the planets.

and Professor Brown * dealing with the same subject. A comparison of the three memoirs is given by the latter in the *Monthly Notices* of last January. The differences between any two of the three theories only occasionally rise to quantities that could be detected by a discussion of the observations. The two later memoirs agree in increasing M. Radau's † coefficient of the Jupiter evection term by a quarter of a second, and there are instances when Professor Newcomb differs by a tenth of a second from the other two. Many of these discrepancies are traced to their source by Professor Brown in the paper referred to above.

No new light is thrown on the observed long period inequalities of the Moon. A fresh discovery could only have been hoped for on the supposition that M. Radau had made an oversight of considerable magnitude, and perhaps the deepest impression made by the two recent memoirs is the sense of the excellence of M. Radau's original research.‡ P. H. C.

Double Stars.

The abbreviations used are—

M.N. : *Monthly Notices.*
A.J. : *Astronomical Journal.*
A.N. : *Astronomische Nachrichten.*
L.O.B. : *Lick Observatory Bulletin.*

A number of new double stars have been added during the year:—Thus in *M.N.*, 1907 May and 1908 January, the Rev. T. E. Espin gives 148 new pairs, of which 78 are under 5″ separation and 70 over 5″. These make 551 discovered by Espin. In *L.O.B.* No. 109 are 250 new doubles by Aitken, of which 54% are under 1″ separation and 200 more in *L.O.B.* 125. Mr. Aitken's discoveries now number 1700. Mr. Hussey brings his discoveries up to 1337 by the new pairs in *L.O.B.* 127.

Measures of known pairs have been much the same as in previous years. The Greenwich measures made in 1906 are in *M.N.* 1907 November. They contain about 150 Hough stars and about 250 miscellaneous pairs. The measures of Southern doubles made with the 26-inch refractor of the McCormick Observatory are in *A.N.* 4166; and in Nos. 4180, 4193 are measures of 250 wide pairs made at the Urania Observatory, Copenhagen. Burnham's measures of 200 wide pairs with the 40-inch Yerkes refractor are in No. 4209. Less numerous sets will be found in

* The Inequalities in the motion of the Moon due to the direct action of the planets.

† Dr G. W. Hill obtained the same coefficient as M. Radau, but he has corrected his result in *A.J.* No. 600.

‡ Recherches concernant les Inégalités planétaires du mouvement de la Lune, *Paris Obs. Ann.* (*Mém.*), vol. xxi.

A.J. 591 of 55 pairs with the Kirkwood 12-inch; in *M.N.*, 1908 January, of 86 pairs by Rev. T. E. Espin; and of 44 pairs at the Morrison Observatory. The measures of 1100 pairs made at the Observatory of Chevreuse by M. Farman in the years 1904–1906 are worth special notice. The measures were made with a 9½-inch Cooke triplet.

The idea of M. Farman was to bring the Catalogues of Flammarion and Wilson-Gledhill up to date, and hence he supplements his measures by all those made of these particular stars since 1878, and it forms thus a fair-sized volume.

There have been two papers on the positions of the poles of Binary star orbits. One by Professor Turner and Mr. Lewis in *M.N.*, 1907 June, where 59 orbits are discussed and the evidence found to slightly favour the view that their poles lie near the Galaxy. The other paper is by Karl Bohlin (*Arkiv för Matematik, Astronomie och Fysik*, Stockholm, Band 3, No. 19), wherein, by making choice of the alternative poles, the opposite conclusion is deduced.

The orbits computed are — ξ Scorpii, Σ 2173, Σ 3121, μ^2 Herculis in *A.N.* 4169–70, and of α Centauri in No. 4189, all by Dr. Doberck, and of β 612 by Mr. L. Semenow in *Popular Astronomy*, 1907 December.

Herr Zeipal gives parallax of Σ 443 as $+''{\cdot}057$ in *A.N.* 4188, and Chase, in *A.J.* 593, gives $+''{\cdot}291$ for 61 Cygni.

The principal event of the year in double star astronomy is the publication of Burnham's *General Catalogue of Double Stars*. This title includes two large volumes: (i.) the Catalogue proper; (ii.) Notes on the Stars in volume i. The catalogue, consisting of 330 pages, contains 13,665 double stars, arranged in order of R.A. (epoch 1880), and with the usual data for a working catalogue. Volume ii., consisting of 830 closely printed pages, gives a few selected early measures of the great majority of the stars contained in volume i., and a number of recent measures of many pairs not published elsewhere, together with notes sufficient to enable anyone to make out a working catalogue suitable to his geographical position and equipment, as all stars from the N. Pole down to 31° South Declination are included. At the same time a person wishing to make investigations is supplied with complete references which will enable him to collect all the observations relating to each particular pair. Obviously their inclusion would have necessitated another volume. When various catalogues of discoveries are thus brought together and the pairs arranged in order of R.A. in one general catalogue the discovery numbers do not run in any particular order, and it becomes difficult to find a pair. Professor Burnham has, in vol. i., a compact table, very ingeniously arranged, which enables easy reference to be made to any pair, either in vol. i. or vol. ii. Thus, on the left hand are numbers from one upwards, and in the succeeding columns, under the heading of the discoverer, the reference number in the "General Catalogue." Suppose we wish to look up β 151—opposite 151, under Burnham,

is the reference. This table is not described in the introduction, and may well be overlooked. It also gives at once the number of pairs discovered by any observer. The catalogue has been published by the Carnegie Institute and freely distributed, and Professor Burnham is to be congratulated on the completion of so great and important a work. Reviews of the work are in various journals; and an appeal is made for measures of the wider pairs from the Astrographic plates. T. L.

Variable Stars.

The rate of discovery of new variable stars is well maintained in 1907, the last number allotted officially in the *A.N.* being 180, which represents the number of announcements during the year.

As usual, Harvard College Observatory shares very largely in this work, the principal notices being as follows:—

H.C.O. Circular.	Number of New Variables.	Remarks.
127	14	Discovered in pursuance of a scheme referred to below. All except one above 10^m at max. Maps 3 and 6.
129	15	In maps 31 and 62. All 10^m or above at max. except one.
130	71	In maps 9, 12, 21, 48, and 51. All above $10{\cdot}5^m$ at max.
131	...	Details of a group of red stars near *Nova Velorum*.
132	15	From an examination of stars with peculiar spectra. All above 10^m.
133	15	In maps 15, 18, and 27.

Independently of the above, *Harvard College Observatory Annals*, vol. lx. part 4, refers to 1777 variables discovered in the two Magellanic Clouds, some of which have been previously announced. Part 5 gives particulars of 10 new Algol type variables, all south of decl. $-15°$.

H.C.O. Circular 127 describes a practical method of search for variables of moderate brightness by superposing a negative upon a contact print of a second negative, covering the same region of the sky, but taken at a different date from the first. These photographs are large; that is, each covers a portion of the sky about 30° square, there being 55 maps for the whole sky. Already a considerable number of new variables have been brought to light, as indicated above; and a continuation of the scheme will probably result in approximately sweeping up all the known variables of a "moderate" degree of brightness.

Much work has recently been done at H.C.O. in continued photographic observations of known variable stars.

This observatory has issued a *Second Catalogue of Variable Stars* containing 1957 stars, including those in globular clusters. If we add the 1791 variables found in the Magellanic clouds, we get a total of 3748 known variables, of which considerably more than half have been found at Harvard. The full and accurate details of each star, together with a series of interesting remarks, critical and historical, render this undoubtedly the leading work of reference on the subject.

Vol. lvii. part 1, *Annals H.C.O.*, contains the observations of 75 classical variables of long period, made by the "method of Argelander," during the years 1902–1905 inclusive. The results of the observations are also given in the shape of dates of maxima and minima, as well as full details of the light curves, etc.

During the past year has been published series iv. of the *Atlas Stellarum Variabilium* by the Rev. Father J. G. Hagen, S.J. This fine work represents the practical completion of the atlas, which must be *the* handbook for all observers of variables. It is on the same general lines as the preceding series, and contains 100 variables of various types, whose minima can be followed in telescopes of moderate size. For each star there is a beautifully printed chart, quarto size, giving the telescopic vicinity of the variable, the lines of R.A. and Dec. being in red, and a separate list of comparison stars, the magnitudes of which have been determined with great care. It would take up too much space here to do full justice to this meritorious work, the publication of which marks an epoch in the history of variable star research. It may be added that series v. of the Atlas, which has not been referred to in these notes, and which contains 49 stars whose minimum is below 7^m, was published in 1906.

In addition to the foregoing, when account is taken of the work done in this country and on the Continent in 1907, it is plain that the department of Stellar Variation continues to occupy an important position in astronomical work. E. E. M.

Stellar Distribution.

A very noteworthy addition to our knowledge of the parallaxes of stars is contained in the *Yale Observatory Transactions*, vol. ii. pt. 1, published towards the end of 1906. An extensive list of parallaxes, all determined on a uniform plan and with approximately the same degree of trustworthiness, is provided. The list consists of 163 stars, the observations being made by Dr. Chase, Mr. Smith, and Dr. Elkin, using a heliometer. Some idea of the magnitude of the undertaking may be gathered from the fact that it had been in progress for thirteen years. With a very few exceptions the stars selected were those having proper motions exceeding 40″ per century, and not hitherto observed for parallax. It is noteworthy that not a single parallax exceeding 0″·20 was found; the mean parallax was about 0″·05. In general

the results appear to confirm Professor Kapteyn's tables of the mean parallax corresponding to given magnitude and proper motion. The deduced values of the linear velocities of the stars are on the average considerably higher than the mean velocities of stars deduced from line-of-sight determinations with the spectroscope; but it is pointed out that this is only to be expected, seeing that the stars were specially selected on account of their large apparent motions.

At the meeting of the British Association at Leicester, Sir David Gill devoted a large part of his presidential address to the subject of stellar distribution and the structure of the universe. Besides giving a general survey of the subject, his address included some unpublished results by Professor Kapteyn as to the distribution of stars in space and their luminosities, revising his earlier estimates, though not changing their general character.

In Groningen *Publications*, No. 18, Professor Kapteyn collects and compares all the available data as to the numbers of stars of various magnitudes in different parts of the sky. The difficulty of reducing the estimates of magnitude made by the different observers to a common standard, so as to render the enumerations of stars strictly comparable, is naturally very great, but seems to have been successfully overcome. The principal result of the paper is an empirical formula exhibiting the number of stars per square degree as a function of the magnitude and galactic latitude. Professor Kapteyn concludes, moreover, that, except in the immediate neighbourhood of the Milky Way, the distribution of the stars does not vary to any great extent with the galactic longitude. This must be taken to indicate that there is little important clustering of the stars except in the Milky Way.

In *Monthly Notices*, 1907 December, the mean distances of the stars of Groombridge's Catalogue are examined from the standpoint of the hypothesis of two star-drifts. Further evidence is given to show that the two drifts must be at sensibly the same mean distance from us, and that accordingly they must permeate one another. It also appears that the mean distance of these stars increases continually as we proceed from the galactic pole to the galactic equator. This is in accordance with the view that the apparently closer aggregation of stars in the galactic plane is not due to a clustering of the nearer stars, but to the fact that additional more distant stars are visible in those directions.

Professor Comstock continues his investigations on the solar motion relative to the very faint stars in *Astronomical Journal*, No. 591. He has now available proper motions of 149 stars, for the most part between the ninth and twelfth magnitudes, derived from double-star measures. From these he derives a position of the solar apex, R.A. 20^h, Dec. $+54^\circ$. The most noteworthy feature is the high declination of the apex. Other investigators have noticed a progressive increase of the declination as fainter stars are used, so that this result, derived from stars fainter than had hitherto been considered, is not unexpected. The same author contributes a paper

on the "Luminosity of the Fixed Stars" to *Astronomical Journal*, No. 597.

Dr. K. Schwarzschild (*Nachrichten der K. Gesell. der Wissen. zu Göttingen*, 1907) has discussed the law of distribution of the motions of the stars. He accepts the recent conclusion that there are two strongly favoured directions of motion, which, when referred to the centre of gravity of the stars, must be directly opposite to one another. On this basis he puts forward a frequency law, which, while differing only slightly from that embodied in the two-drift hypothesis, is remarkably well adapted for mathematical treatment and calculation, and, moreover, regards the universe as a single instead of a dual system. According to Dr. Schwarzschild's law, the frequency of a velocity (u, v, w) is proportional to $e^{-Au^2-B(v^2+w^2)}$, where $A<B$. This stands in much the same kind of relation to the Maxwellian law, $e^{-A(u^2+v^2+w^2)}$, that a prolate spheroid does to a sphere; this distribution may in fact be derived from the random distribution by increasing all the component velocities parallel to the axis of x in a certain ratio. This axis corresponds to the direction of relative motion of the two drifts on the other theory. A careful analysis of the Greenwich-Groombridge proper motions on the new theory shows a very satisfactory accordance between theory and observation. No suggestion is made in the paper as to a theoretical basis for the law proposed; presumably some such distribution might arise if the universe (or that part of it which is here concerned) had originally been in the form of a very elongated ellipsoid. A. S. E.

Astrographic Chart and Catalogue.

Additions to the published *Astrographic Catalogue* have been made during the year by the following observatories:—Catania, Zone 51° N., 0^h to 3^h R.A.; Potsdam, volume iv., Zone 32° N.; Oxford University, volume ii., containing the whole of Zone 30°, and volume iii., Zone 29°; Paris, volume ii., Zone 23° N.; and Bordeaux, volume ii., Zone 16° N.,—the word "Zone" implying in all cases the plates whose centres are at the declinations named, and the stars included therefore lie between declinations approximately 1° less and greater. The second volume of the *Greenwich Astrographic Catalogue*, which completes the northern cap 65° to the pole, allotted to the Royal Observatory, is on the eve of publication, so that substantial progress is being made.

The above-named observatories have all published previous volumes except Catania, whose contribution, though entitled vol. v. part 1, is the first issued from that observatory. The scheme of this work follows that of the Helsingfors Catalogue, each plate being taken as a unit, and all the stars it contains measured. In the columns of the catalogue these measures are given, and with these, in parallel columns, their values when corrected by application of the plate constants. These corrected measures are con-

verted into Right Ascension and Declination, and are given for each star on every plate to thousandths of a second of R.A. and hundredths of a second in Declination. This is a very complete programme, since it makes the results at once available for comparison with other catalogues, but entails much work, and publication is therefore not rapid. Since the original resolution of the organising committee provided only for the publication of rectilinear co-ordinates, most of the co-operating observatories prefer to first publish simply the measures, and to leave the equatorial co-ordinates for later volumes.

It appears from the reports of the Colonial observatories that the measurement of the plates of Zones 41° S. to the South Pole is proceeding. The plates of Zones 24° to 40° S., divided between Cordoba (S. America) and Perth (W. Australia), are being taken, and there is prospect that these will be measured soon. The work both on the Chart and Catalogue is progressing actively at Tacubaya (10° S. to 16° S.).

Enlargements of the chart plates have been distributed during the year by the Directors of the Paris, Algiers, Toulouse, Bordeaux, San Fernando, Tacubaya, and Greenwich Observatories. The total number issued from these observatories is now—

Paris	296	San Fernando	145
Algiers	281	Tacubaya	23
Toulouse	147	Greenwich	736
Bordeaux	65		

It is understood that the Potsdam Observatory does not propose to take the chart plates 32° to 39° N., but that these zones will be undertaken by the Royal Observatory of Belgium.

It would be unfitting to close this note without making reference to the loss that the work has suffered by the death of M. Loewy, Director of the National Observatory, Paris, who was so much associated with the International Chart of the Heavens. Four French observatories are taking part in this undertaking, and within the last few years three of them—Algiers, Bordeaux, and Paris—have lost their Director by death. M. Baillaud, who succeeds M. Loewy at Paris, is the only survivor of the four French astronomers who superintended the work when it was begun. H. P. H.

Stellar Spectroscopy in 1907.

New Star.—In last year's report, reference was made to the discovery of a star exhibiting the photometric peculiarities of a Nova. Professor E. C. Pickering announced it (*H.C.O. Circular* 121) as Nova *Velorum*. The spectrum had not then been observed, but in *H.C.O. Circular* 131 (*Ast. Nach.*, **176**, 255) Professor Pickering announces that Mrs. Fleming has recently found that its

spectrum exhibits bright lines which appear to coincide with bright lines in the later spectrum of Nova *Persei* No. 2 1901. Many other interesting spectra of red stars have been found in its neighbourhood.

Professor Hartmann has succeeded in photographing the spectrum of Nova *Persei* No. 2 (1901) with a specially constructed spectrograph. The star was fainter than the eleventh magnitude. The spectrum was obtained with 8¼ hours' exposure, and was found to resemble the spectrum of the *Wolf Rayet* star (*Ast. Nach.*, 177, p. 113).

Studies of Special Stars.—*o Ceti.*—The bright maximum of this star in December 1906 and January 1907 was well observed by spectroscopists. Father Sidgreaves (*M.N.*, lxvii. 534) ascribes the unusual intensity of the maximum to weakness of the absorption bands, which usually form the main feature of the spectrum. He also records peculiar behaviour of the hydrogen lines. Father Cortie (*Astroph. Jour.*, xxvi. 123) discusses the bearing of recent observations of *o Ceti* on the question of the temperature of sun-spots. Mr. Slipher contributes two notes (*Astroph. Jour.*, xxv. 66 and 235) on the spectrum of this star. He points out that the hydrogen line Hα (C), as well as Hβ, Hγ, and Hδ, appears bright on his photographs, and that the absorption lines of *Vanadium* are very strong, a fact which is well brought out by a beautiful plate illustrating the spectrum from λ 4100 to λ 7000 with comparison spectra of Va, Fe, and Na. Mr. Plaskett (*Jour. R.A.S.*, Canada, i. 45, with plate) gives an account of his studies of the spectrum; his measurements of the velocity in the line of sight are in good agreement with those made by Professor Campbell in 1898.

α Orionis.—Mr. Newall and Mr. Cookson (*M.N.*, lxvii. 482) have discovered three absorption flutings in the extreme red end of the spectrum of this star. They also give results of a comparison of sun-spot lines with marked lines in the green region of the spectrum of the star.

β Orionis.—Sir N. Lockyer (*Proc. R.S.*, **80**, 50) has detected lines attributable to sulphur in the spectrum of *β Orionis*.

ε Capricorni.—Mr. Slipher (*Astroph. Jour.*, xxv. 285) finds bright lines in the spectrum, and regards the star as likely to prove to be a binary.

α Scorpii.—Mr. Lunt (*M.N.*, lxvii. 487) gives reason for suspecting the presence of tin in the atmosphere of *α Scorpii*.

α Boötis.—Mr. Lunt (*Proc. R.S.*, **79**, 118) has found several lines attributable to *Europium* in the spectrum of *α Boötis*, and discusses the evidence afforded by the spectrum of the solar chromosphere for the presence of this element in the Sun.

These careful researches of Mr. Lunt, originating in the detection of small abnormalities in velocity deduced for special stellar lines, afford a good instance of the way in which our knowledge of terrestrial spectra may be helped by studies of stellar spectra.

Two lists of stars, having peculiar spectra, have been issued by Professor Pickering. The first (*H.C.O. Circular* 124, and *Ast.*

Nach., **174**, 101) relates to 33 stars, of which 18 have been found to be variable stars; the second (*H.C.O. Circular* 132, and *Ast. Nach.*, **176**, 258) relates to 23 stars, of which 15 are found to be variable stars. The long-continued success of the Harvard College observers in connecting peculiar spectra with variable stars serves to emphasise the view that typical spectra are evidences of special states of equilibrium in stellar conditions.

Variable Radial Velocity.—Notes on variability of velocity are given for the following stars :—

RZ Cassiopeiæ	Yerkes Observatory	*Astroph. Jour.*, xxv. 59
X Cygni	,,	,, ,,
13 Ceti	,,	,, ,,
ω Leonis	,,	,, ,,
85 Pegasi	,,	,, ,
19 τ_5 Eridani	,,	,, ,,
33 τ_8 Eridani	,,	,, ,,
20 τ Orionis	,,	,, ,,
4 ξ_1 Canis Majoris	,,	,, ,,
ε Cygni	Küstner	*Ast. Nach.*, 175, 87
ζ Cygni	,,	,, ,,
o Leonis	Zurhellen	,, 173, 353
α Draconis	Harper (Ottawa)	*Jour. R.A.S.*, Canada, i. 237.
U Cephei	Slipher	*Astroph. Jour.*, xxv. 284
X Carinæ	Wright	*L.O.B.*, No. 123
ι Gruis	,,	,, ,,
o Tauri	Campbell and Moore	,, ,,
f Tauri	,, ,,	,, ,,
7 Camelopardalis	,, ,,	,, ,,
A Boötis	,, ,,	,, ,,
β Coronæ	,, ,,	,, ,,
ξ Cygni	,, ,,	,, ,,
d Tauri	,, ,,	,, ,,
ζ Cephei	,, ,,	,, ,,

Orbits of Spectroscopic Binaries.—Orbits have been calculated for the following spectroscopic binaries :—

			e
α Andromedæ	Ludendorff	*Ast. Nach.*, 176, 327	0·4
κ Cancri	Ichinohe	*Astroph. Jour.*, xxv. 315	0·15
β Arietis	Ludendorff	,, xxv. 320	0·88
μ Sagittarii	Ichinohe	,, xxvi. 157	0·44
α Carinæ	H. D. Curtis	*L.O.B.*, No. 122	0·18
α Pavonis	,,	,, ,,	0·01
κ Velorum	,,	,, ,,	0·19
θ Draconis	,,	,, ,,	0·01
ω Draconis	A. B. Turner	,, No. 123	0·01
η Virginis	Ichinohe	*Astroph. Jour.*, xxvi. 282	0·25
		Mean eccentricity	0·25

Dr. See (*M.N.*, lxviii., current volume, 201) states that the average eccentricity among visual binaries is considerably more than twice that among spectroscopic binaries, the average value for 30 of the latter systems being 0·22.

Dr. Zurhellen (*Ast. Nach.*, 175, 246) contributes remarks on the analytical determination of orbits of spectroscopic binaries.

Dr. Knut Laves (*Astroph. Jour.*, xxvi. 164) describes a simple graphical method based on the use of the hodograph.

Standards of Wave-length.—Messrs. Benoit, Fabry, and Perot (*C.R.*, 144, 1082) have redetermined the number of wave-lengths of the red cadmium ray in a metre. The resulting wave-length of the cadmium ray, viz. 6438·4702, has been adopted by the International Union for Solar Research as the single primary standard for spectroscopic measurements. This value agrees with the previous determination by Michelson and Benoit within one ten-millionth part.

Messrs. Fabry and Buisson (*C.R.*, 143, 165, and 144, 782) have published wave-lengths of 115 standard lines in the spectrum of the iron arc, between the wave-lengths λ 2373 and λ 6495. This work is a contribution to the plan of co-operation in the determination of wave-lengths of lines, which will be adopted as secondary standards, lines at intervals of 50 Ångström units throughout the spectrum: the wave-lengths being determined by the interferometer method, as far as possible with an accuracy of 0·001 unit.

Herr Eversheim (*Astroph. Jour.*, xxvi. 172) describes his investigations in determinations of secondary standards. He points out that his wave-lengths seem to be all slightly larger than those of Fabry and Buisson, and states that he is looking for the origin of this systematic difference.

The system of tertiary standards at intervals of from 5 to 10 units will be chosen and determined when the secondary system is complete. The wave-lengths of the tertiary standards will be obtained by interpolation between the secondary standards with the help of large gratings.

Dr. Kayser (*Astroph. Jour.*, xxvi. 190) regards the question of the constancy of wave-lengths as decisively settled. Presumbly his statements must be taken as referring to suspected displacements much larger than those attributable to small changes in pressure.

New Spectrographic Installations.—In the publications of the Lick Observatory, vol. ix., parts 1–3, Mr. Wright gives an introductory account of the Mills Expedition to the Southern Hemisphere. A 36-inch pierced mirror is mounted at Cerro San Cristóbal, near Santiago. The Cassegrain mounting has been adopted, and a three-prism spectrograph is rigidly fixed at the back of the large mirror. Mr. H. D. Curtis, who, on Mr. Wright's return to the United States in March 1906, was appointed to take charge of the work, gives an account of recent changes (*Pub. A.S.P.*, xix. 227), and describes (*Astroph. Jour.*, xxvi. 256) the method

adopted for minimising the change of focus of the silvered mirror in the course of the night's work. A refrigerator is applied two hours before sunset to reduce the temperature of the mirror by 5° or 6° C. By this plan the changes of focus have been reduced from about 25 mm. to about 5 mm. in the course of the night.

At the Dominion Observatory, Ottawa, Canada, Dr. King has put the spectroscopic work in the charge of Mr. Plaskett. Great activity is shown both in the careful preparation of the instrumental appliances for giving the best results (*Astroph. Jour.*, xxv. 195), and in measurement and reduction of photographed stellar spectra (*Jour. R.A.S.*, Canada, vol. i.). The spectrograph is attached to an equatorial of 15 inches aperture.

Measurement and Reduction of Observations.—Papers dealing with determinations of orbits of spectroscopic binaries are referred to above in the paragraph on binaries.

Mr. Moore (*Pub. A.S. Pac.*, xix. 13) gives a résumé of methods of measurement and reduction of spectrograms.

Mr. Schlesinger (*Pub. Allegheny Obs.*, vol. i.) describes a method which he has found useful for simplifying the reduction of measurements.

Colours of Telescopic Binary Stars.—Sir William and Lady Huggins (*Astroph. Jour.*, xxv. 65) call attention to the corroboration which their view, expressed in 1897, receives from Mr. Lewis's discussion of the relative masses of the components of 18 binary stars: "the apparent satellite is in fact the primary of the system" (*Mem. R.A.S.*, 56, xx). Sir W. Huggins had suggested in 1897 that the bluish component of a pair might have the greater mass, and for this reason be still at the earlier evolutionary stage.

Theoretical Investigations.—Professor Schwarzschild (*Nach. K. Gesellsch. Gottingen*, 1906, Heft 1) has suggested a third alternative to isothermal and adiabatic equilibrium in radiating stellar atmospheres. He calls it radiative equilibrium. The problem to be solved is as follows: If in the Sun's atmosphere the various strata are regarded as simultaneously radiating and absorbing, and if convective motions were to cease, what distribution of temperature would have to be reached in the various strata in order that the observed steady flow of energy should be transmitted without further change of temperature? He finds a solution, proves that the equilibrium is stable, and proceeds to show that radiative equilibrium under certain simplified assumptions gives a good account of the observed darkening of the Sun's limb, whereas adiabatic equilibrium fails. H. F. N.

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Bologna, Royal Academy of Sciences.
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Bombay, Government Observatory.
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Bordeaux, Society of Physical and Natural Sciences.
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Brazil, Sociedade Scientifica de São Paulo.
Brussels, Belgian Astronomical Society.
Brussels, Royal Academy of Sciences of Belgium.
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Buda-Pesth, Royal Hungarian Institute for Meteorology and Terrestrial Magnetism.
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Leiden Observatory.
Leipzig, Astronomical Society.
Leipzig, Prince Jablonowski Society.
Leipzig, Royal Society of Sciences of Saxony.
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Paris, École Polytechnique.
Paris, Mathematical Society of France.
Paris Observatory.
Paris, Philomathic Society.
Pennsylvania, Lehigh University.
Perth Observatory, Western Australia.
Philadelphia, American Philosophical Society.
Philadelphia, Franklin Institute.
Pola, Imperial Hydrographic Office.
Potsdam, Astrophysical Observatory.
Potsdam, Central International Geodetic Bureau.
Potsdam, Royal Prussian Geodetic Institute.
Prague, Imperial Observatory.
Pulkowa Observatory.
Pulkowa, Physical Observatory.
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ADDRESS

Delivered by the President, Mr. H. F. Newall, on presenting the Gold Medal of the Society to Sir David Gill, K.C.B., F.R.S.

For the second time in the history of our Society, your President is called upon to present the Gold Medal to Sir David Gill. By their award in 1882 the Council desired to recognise the value of his work in determining the solar parallax from observations of Mars in Ascension; and now, after a quarter of a century, they have awarded the Gold Medal to him in recognition of his contributions to the Astronomy of the Southern Hemisphere, and his other astronomical work.

In fulfilling the task which it is my privilege to attempt this afternoon, I would begin by assuring Sir David Gill of the pleasure which it is to us to have him at home again among us with his vigour unabated. We look forward to the prospect of our having his help in our counsels for many years to come.

The terms of the Council's award are wide. They cover the large range of work which Sir David Gill has accomplished during the twenty-eight years of strenuous activity which he has spent in his capacity as His Majesty's Astronomer at the Observatory of the Cape of Good Hope. It would be a difficult matter to decide whether Sir David Gill has contributed to the advance of astronomy in those years more by his own observational work or by the active performance of his administrative duties. Fortunately it is not part of my task to try to disentangle these two aspects of his work.

Broadly speaking, by his personal labours we have such achievements as his two sets of heliometer determinations of the parallaxes of certain southern stars, and his three determinations of the solar parallax. By his administrative activity the Cape Observatory has been equipped entirely anew with modern first-class instruments; a vast store of astronomical material has been rendered available for the use of astronomers; not only are the last remaining observations of his predecessors reduced and published, but also the observational materials collected in Sir David Gill's régime are reduced and published up to date.

And, as if these duties were not enough to occupy him, Sir David Gill devoted himself to promoting a unified scheme of geodetic survey of South Africa. In his latest utterance about this work in his Presidential Address to the British Association, Sir David Gill speaks of the great African arc on the 30th meridian.

He says of it that it is the dream of his life to see it completed. When I heard these words at Leicester, it seemed to me that they represented Gill's general frame of mind. Whenever he set his hand to any bit of work, it became the dream of his life to see that particular bit of work completed in the most comprehensive way that he could attain. This frame of mind does not always meet with sympathy; it is liable at times to rouse keen opposition. But Gill has seldom failed to infect others with something of his own enthusiasm.

In doing honour to Gill's work, we shall not in the least detract from it if we pay the sincerest tribute to the co-operation which he has been able to elicit from many workers in many lands. We must not forget the devotion of his staff at the Cape Observatory. We must remember the liberal support he has received from the Lords Commissioners of the Admiralty, both for increased instrumental equipment and for increased staff to deal with the expanded scope of the work. We must pay a tribute to the memory of our late Fellow, Frank McClean, whose generous munificence provided the astrophysical department at the observatory. We shall not forget the services rendered by Professor Kapteyn in the *Cape Photographic Durchmusterung*, nor those of Dr. Elkin and Dr. Auwers in their allotted tasks in the solar parallax work, nor yet those of Sir W. Morris and many other officers in the geodetic work.

Paying tributes in this wise is only a way of recognising Gill's power of inspiring others to take part in advancing the subject which it is the object of our Society to encourage.

History and Traditions at the Cape.—It has been of great interest, in preparing to write this address and in trying to attune myself to the nature of the surroundings in which Sir David Gill has worked, to trace back through the prefaces of early volumes of observations and through the annual reports something of the history and traditions of the Royal Observatory at the Cape of Good Hope. What an enthralling history it would make for us astronomers if it were written by someone who had the power of picturesque presentation.

We should see Fallows, the first astronomer at the Cape (1821–1831), awaiting for three and a half years the delayed plans of his observatory, and filling up his spare moments by opening a school and teaching the children of neighbouring farmers—his fee for each lesson a load of earth, which helped to make the foundation of a garden on the rocky hill chosen for the site of the observatory. We should hear of his applying in vain for permission to measure an arc of meridian.

We should read how he struggled on, making the best of his portable instruments, till the observatory buildings were finished and the permanent instruments were installed—seven years after his arrival at the Cape. Then, after a couple of years of vigorous observations, his able assistant Captain Ronald fell sick, and Fallows would have been left alone at the work but for the

assistance which his wife learned to render by taking the circle readings whilst he was engaged with the transit. It was generous official help that he needed, but before it arrived fever had attacked him and he died in the forty-third year of his age. It is a sad story of a brave man struggling to implant new work in a distant land, and it should make us remember the debt of help that we at home owe to those who are working at a distance from headquarters.

Next we should read of Henderson, who went to the Cape in 1832 and worked there assiduously for thirteen months before he uttered that wail of despair about the situation of the observatory on Snake Hill. He describes it as "on the verge of an extensive sandy desert, exposed to the utmost violence of the gales which frequently blow, without the least protection from trees or other objects to shelter from the wind or sun, some miles from markets, shops, or the habitation of persons with whom those belonging to the observatory can associate." It was in such surroundings that Henderson, with the assistance of Lieut. Meadows, achieved those observations which have made his name famous. He took most of his material back to Edinburgh, and year by year he found time, in the midst of his duties as Astronomer Royal for Scotland, to reduce it all. Our Society had the honour of publishing in its early memoirs fifteen contributions from him dealing with his Cape observations, including his memorable determination of the parallax of α Centauri.

Next we should find Maclear (1834–1870) boldly coming out, at heart a true colonist, making roads, harnessing the wind to draw a water-supply, carting earth, planting trees and rows of pine and wattle to form shelter. By exchange and sale and purchase of land he consolidated the observatory property, and succeeded in getting convenient communication with the main road to Cape Town. Of his scientific labours, it is enough to recall his measurement of an arc of meridian, his multitudinous meridian observations, partly reduced under his own régime, and revised and published by his successors Stone and Gill in the form of four Cape Catalogues for the equinoxes 1840, 1850, 1860, and 1865, which deal with a total of 10,766 stars. In his régime, too, two equatorials were added to the equipment, and with these his observations of comets, nebulæ, double stars, and occultations were made. A new transit-circle was installed.

So, at the end of Sir Thomas Maclear's directorate in 1870, when he withdrew from the observatory to live for the remaining nine years of his life at Mowbray, within a mile of the scene of his former labours, we should see Stone (1870–79) coming out to the Cape to find an established home and an observatory containing a transit-circle the facsimile of that at Greenwich. Stone was at once engrossed in completing two great objects, (i) the preparation of Maclear's meridian observations for press, and (ii) the re-observation of the stars which had been observed by Lacaille more than a century before. He completed the latter undertaking by the formation of the Cape Catalogue for 1880, containing the places of 12,441 southern stars to the seventh magnitude. During his directorate

he passed through the press two Cape Catalogues for 1840 and 1860, based on Maclear's observations made during thirteen years.

This is in briefest outline the history of the observatory at the Cape of Good Hope up to the year 1879, when Stone returned to England to take up his duties as Radcliffe Observer at Oxford. We have seen the traditions forming—

(1) The stellar parallax tradition, implanted by Henderson in his studies of α Centauri, and fostered by Maclear.

(2) The geodesy tradition, implanted first in African soil by the Abbé Lacaille, in vain desired by Fallows, and firmly rooted by Maclear.

(3) The tradition of the Cape Catalogue, implanted by Fallows in his first catalogue of 273 stars, fostered by Airy in his reduction of Fallows' last observations, and firmly fixed by Maclear and Stone,—in fact, the traditions of general astronomy.

Into all this goodly heritage of history and tradition Sir David Gill entered in 1879; and looking back upon the records of his directorate, we find he has fostered and extended each one of the traditions he found at the Cape, and has implanted as many more as he found.

Thus by the accumulated labours of its Directors the Cape Observatory has grown to be an observatory of the first rank, of which the nation may justly be proud.

Stellar parallax.—First and foremost among the new traditions we must put the heliometer tradition. Gill's masterly use of this instrument has given us those refined determinations of stellar parallax and of solar parallax, those observations of planets and satellites, and triangulations of stars, that are so honourably connected with his name.

Let me first refer to the determinations of stellar parallaxes, one of his most notable contributions to the astronomy of the southern hemisphere. He has made two sets of determinations; the first were made with a 4-inch heliometer. This instrument was an old friend, for he had used it in Lord Crawford's expedition to Mauritius in 1874, and had also made his observations of Mars with it at Ascension in 1877. Gill knew he could use it effectively in stellar parallax determinations, and he had acquired it from Lord Crawford for this purpose. Just before he left Europe to take up his duties at the Cape, Gill was fortunate enough to find a young and able astronomer eager to join him in the parallax work. He invited him to come to the Cape as soon as he should have finished his studies at Strassburg. Thus it was that Dr. Elkin went out to the Cape in 1881. For more than two years, a guest in Gill's house, he took his share in Gill's labours, and made it a labour of love.

Their programme of parallax observations was an interesting one. First on the list of chosen stars was, of course, α Centauri, Henderson's star, which still retains its place as the nearest star in the whole sky. Altogether nine of the most interesting stars in the southern heavens were included on the observing list, four

stars, including α Centauri, being chosen for historical associations or for conspicuous brightness, and five others for large proper motion. These stars were judiciously divided between the two observers, so that each had six stars on his list. The comparison stars were chosen with great care, and in the case of the three stars common to the two lists each observer had his special comparison stars, so that we must regard the determinations as entirely independent. Each observer discussed his own observations, and the results were combined in the final account of the research published in the *Memoirs* of our Society, vol. xlviii. (1884).

The results were valuable, both in their bearing on the exact solution of the problem undertaken and by the effects produced by their attainment, for immediately after the completion of the work Sir David Gill represented to the Admiralty that continued research on stellar parallax and a new determination of the solar parallax by observations of minor planets were much to be desired, if only they could be carried out with a larger instrument provided with certain structural improvements which experience with the small heliometer had suggested. The Lords Commissioners promptly sanctioned the addition of a 7-inch heliometer to the equipment of the Cape Observatory.

Regular observing was begun with the new heliometer in January 1888, and in the next four years an astonishing programme of observations was successfully completed. It included—

(1) The complete determination of the constants and errors of the heliometer (scale value, errors of the micrometer screw and of the scale divisions, etc., involving nine months' labour).

(2) Observations for parallax of twenty-two stars.

(3) Observations of the three minor planets, Iris, Victoria, and Sappho, for the determination of solar parallax.

(4) Observations of Jupiter's satellites.

Gill's expectations of the new heliometer were fully realised; he satisfied himself that one observation made with it was of the same weight as three observations with the old instrument, and that one set of observations could be made in half the time that had been previously required.

The tables which summarise (in vol. vii. of the *Annals of the Cape Observatory*) the results for stellar parallax contain details relating to 12 *bright* stars and 10 stars of *large proper motion*; and among the 32 distinct determinations the name of our Medallist is attached to 18, Elkin's to 7, Finlay's to 3, and de Sitter's to 4. In several instances, by way of test, the parallax of the same star was investigated with both instruments, with substantially the same results. Gill completes his exposition of the observations by a discussion of the conclusions to be drawn from the results obtained. So far as so small a number of parallaxes as 22 can be utilised for such a generalisation, the Cape results seem to support the view which has gradually formed in the last twenty years, namely, that large proper motion is a safer criterion of proximity of a star to our system than brightness. We have learnt from the work of

Stumpe, Kapteyn, and Newcomb and others to realise how much emphasis is to be laid on the probability that stars differ enormously in actual luminosity.

The point is beautifully illustrated by the tables which Gill has drawn up. It must be remembered, as he points out, that the derived parallaxes are differential or apparent, not absolute; for we are not at liberty to assume that the comparison stars are infinitely remote. The only legitimate procedure is to deal with limiting values, which can be obtained by adopting different hypotheses as to the absolute parallax of the comparison stars. Gill accordingly gives tables exhibiting the variation of the total light radiated by each of his parallax stars (in terms of the Sun's light), with different hypothetical parallaxes of the comparison stars, ranging from $0''{\cdot}00$ to $0''{\cdot}05$. It is thus made clear that the southern stars whose apparent parallaxes have been determined would differ in brightness by about 12–15 magnitudes if they were all removed to the same distance from the Sun.

The further tables which Gill gives to convert the observed proper motions of his parallax stars into absolute velocities in miles per second afford strong confirmation that the deductions about actual luminosity are legitimate. For the velocities deduced are nearly all within the limits which spectroscopic determinations have led us to expect among stars.

The beautiful photographic reproductions, given in the volume of results, of the spectra of α_2 Centauri and the Sun show what may be expected of the astrophysical department which Sir David Gill has laboured to develope at the Cape. I will not attempt to refer in detail to this spectroscopic work; for though the papers published by the Cape Observatory hitherto have dealt with important points, we are able to gather from references in the annual reports that the main forces are concentrated on yet another determination of the solar parallax by the spectroscopic method. We can well imagine the interest with which Sir David Gill is looking forward to the results of the work.

Cape Catalogues.—Before passing on to refer to Gill's work on solar parallax, I would advert to another of his large contributions to the astronomy of the southern hemisphere, namely, his faithful fostering of the Cape Catalogue tradition. Meridian work must necessarily form an important part of the work of a large national observatory. At the Cape much of the energy of the staff has always been devoted to this branch, and the result is a series of catalogues of star places which have a special value.

When Gill took up his duties at the Cape he found that, in spite of the indefatigable zeal of Mr. Stone, there were two periods (1849–52 and 1861–70) of valuable observations of Sir Thomas Maclear which still remained to be reduced and published. Stone had already begun the reduction of the observations of the first period, and Gill made it one of his first duties to complete and revise the reduction, and in 1884 the Cape Catalogue of 4810 stars for 1850 was published. The last remaining period (1861–70) of

Maclear's observations was dealt with by degrees, and in 1900 the Cape Catalogue of 1905 stars for 1865·0 was published. We can well understand with what satisfaction the completion of this work was hailed. The preface contains the following remark:—"The publication of this catalogue marks an epoch in the history of the observatory. For the first time in that history the Director can feel that the accumulated labours of his predecessors are available for the use of astronomers, and that the work being done under his own direction is in a healthy and forward state of reduction and publication."

The first catalogue for which Sir David Gill is solely responsible is a catalogue of 1713 stars for 1885·0 published in 1894; the next is a catalogue of 3007 stars for 1890·0 published in 1898. In 1906 two more catalogues were published, an achievement of which the meridian circle department might well be proud. One is a catalogue of 8560 astrographic standard stars for the equinox 1900, to serve for the Cape section of the Astrographic Catalogue. The other is a catalogue of 4464 stars for the equinox 1900, based on observations made between 1900 and 1904, and containing, *inter alia*, the results of Cape observations of 2798 zodiacal stars which were entered on the list circulated by Sir David Gill, and adopted at the Paris Conference in 1896 as a list of stars suitable for reduction of heliometer observations of planets, etc. In this connection I may refer to another large undertaking. Since 1897 all the oppositions of major planets have been observed with the heliometer at the Cape.

The transit-circle, which has been in use since 1856, was carefully renovated in 1885. The instrument is similar to that at Greenwich. I wish I could have included in these remarks reference to a fifth catalogue, which will deal with other observations, already on the way to completion at the time of Sir David Gill's leaving the Cape. We shall look forward to it with great interest, for it will contain the results of comparison of the old and the new transit-circles at the Cape.

Gill has laboured to embody in the new transit-circle, which was erected in 1901, many new features which were designed with the object of making it possible to detect or eliminate all ascertainable sources of error in observations of fundamental stars.

In connection with his efforts to probe into these refinements of observational work, I would recall to your memory his careful discussion of the meridian observations made for the reduction of his heliometer measures of Mars at Ascension. He then definitely discovered that curious variation of personal equation known as the magnitude equation. In virtue of this troublesome peculiarity, a transit observer watching the passage of bright and faint stars over a wire records the transit a little later for a faint star than for a bright star. The result is that the right ascension interval between a faint and a bright star is a little too large when the bright star leads, and is a little too small when the faint star leads. The effect is a small one, but is very troublesome in cases where

great refinement is needed in star places; the trouble is all the greater because the magnitude of the effect varies appreciably among different observers.

Sir David Gill has introduced in the Cape Catalogues, *e.g.* that of Astrographic Standard Stars for 1900·0, corrections for magnitude equation applied separately for each observer. The latest contribution to the experimental study of the effect is one made at the Cape Observatory, and communicated to our Society last April. The authors, Sir David Gill and Mr. Hough, discuss therein the results of a comparison of right ascension differences measured in three independent ways—(1) with the old transit method, (2) with the new transit and travelling wire, and (3) with the heliometer. The results show that within the range of magnitudes ($3\frac{1}{2}$–$8\frac{1}{2}$) of the stars observed, the magnitude equation is barely appreciable with the new transit-circle fitted with the travelling wire.

Cape Photographic Durchmusterung.—I turn now to remind you of another great undertaking which we owe to the initiative of our Medallist, viz. the *Cape Photographic Durchmusterung*. There is scarcely need for me to refer to it in any detail, for the history of it must be vivid in your memory from the account which Dr. Glaisher gave of it in presenting the Medal to Professor Kapteyn in 1902. The *Cape Photographic Durchmusterung* arose out of the success which Gill achieved in photographing the comet of 1882; he saw the beauty of the many star-images that appeared on the photograph down to the 9th magnitude, and soon afterwards he proved that large photographic cameras could be effectively employed for constructing photographic star maps. He proceeded forthwith to secure the systematic photographical survey of the southern skies, and gave us what may be briefly described as an extension of the *Bonn Durchmusterung* to the South Pole. Kapteyn's measurement and reduction of the plates remains a splendid instance of scientific devotion. It gives us, under the title of the *Cape Photographic Durchmusterung*, in three large volumes of the *Annals of the Cape Observatory*, the approximate places and magnitudes of 454,875 stars,—in fact, a catalogue of all the stars down to the $9\frac{1}{4}$ magnitude, and most of those to the 10th magnitude between declination 19° S. and the South Pole. This survey has not only served admirably for a study of stellar distribution in the sky, but by the industrious co-operation of Professor Kapteyn at Groningen and Mr. Innes at the Cape it has also led to the discovery and study of many variable stars, a fact which is attested by observations recorded in the fourth volume, containing the details of the work of revision of the *C.P.D.*

Astrographic Chart and Catalogue.—The present year is the twenty-first anniversary of the inception of the *Astrographic Chart* and *Catalogue*. In the original initiation of this scheme of international co-operation Sir David Gill took a deep interest and also a large share. Admiral Mouchez, by whose tact and energy the initial steps of promoting the harmonious co-operation of

eighteen observatories in different parts of the world were successfully conducted, has testified to the fact that Gill's success in photographing the comet of 1882 opened the way to the Chart, for it proved that we had reached the point of being able to construct the chart of the heavens by photography. The skill of the Brothers Henry, not only in developing the construction of object-glasses large enough and suitably achromatised for stellar photography, but also in producing their superb photographic star charts, was probably the decisive factor in the acceptance of the plan agreed upon in 1887.

The Cape Observatory at once became one of the eighteen contributories to the scheme; the zone $-40°$ to $-52°$ was allotted as its portion of the sky. The astrographic telescope made by Sir H. Grubb arrived at the Cape in 1890, and after some alterations and experimental researches the work was begun in 1893 and has gone steadily forward. By the end of 1897 the required 1512 catalogue plates had all been obtained and nearly half of the chart plates. In the following year it was decided to re-photograph the whole zone for the catalogue, in order to bring the epoch at which the plates were taken nearer to that at which the comparison stars were observed on the meridian. This plan also rendered it possible to determine with sufficient accuracy the proper motions of the reference stars in the numerous instances where no meridian observations existed previous to the epoch of the latest Cape observations.

In 1900 Sir David Gill undertook, in addition to the zone, to photograph the area within $2°$ of the South Pole, originally allotted to the Melbourne Observatory; it will thereby be possible to combine the photographic work with the Cape heliometer triangulation of this region, and with the results of the Cape discussions of all the meridian observations of southern close circumpolar stars.

The measurement of the plates was well started in 1898, with the aid of a new type of measuring apparatus designed by Gill (*M.N.*, lix. 61); a second measuring instrument of the same type was afterwards added; and at the time of Sir David Gill's retirement from the directorate in February 1907 he was able to announce that 1086 out of the 1512 catalogue plates had been measured. It is thus seen that the zone undertaken by the Cape Observatory is well on the road to completion.

Solar Parallax.—In these days, since the discovery of minor planets with peculiar orbits like that of Eros, it is not easy to speak of the older methods of determining the solar parallax in such a way as not to appear antiquated. Yet, in a retrospect over the gradual development of the methods, there are points of great interest.

It is just half a century since Airy summarised his views of the relative values of available methods of determining the solar parallax from observations of Venus and Mars. His summary was given by way of calling attention to approaching favourable oppositions of Mars in 1860, 1862, and 1877, and the two transits of

Venus in 1874 and 1882. He pressed on the attention of astronomers the importance of observing Mars; in fact, he summed up strongly in favour of the Mars method, but at the same time he urged that the future astronomical public would not be satisfied unless all practical use were made of the transits of Venus.

It is easy to criticise after events, but looking back, with our present knowledge as to where success lies, we cannot fail to be struck with the statesmanship which Airy showed in looking far ahead and calmly reviewing beforehand the broad chances of success in attempts to utilise special opportunities for the settlement of astronomical problems.

Airy's summary reads as if he had been truly a seer of what was to be, but also as if he hardly had the courage to let so uncommon an occurrence as a transit of Venus pass by without making observations. His view was that Mars, approaching within 0·4 unit of the Earth in favourable opposition, would give better values of the Sun's parallax than Venus.

Galle of Breslau proposed in 1872 that instead of Mars one of the minor planets would be more suitable, on the ground that the smaller disc of such a planet would more than make up for the greater distance from the Earth. Galle favoured observations in both hemispheres; Airy seems rather to have thought of observations to be made at a single observatory.

It was Gill's fortune to have learned at Dunecht Observatory to appreciate the worth of the heliometer as an instrument of precision; he saw that to apply the heliometer to observations for solar parallax was the great opening. Through many of the years in the '70's he worked at this problem. At Mauritius, whither he went in 1874 with Lord Lindsay to observe the transit of Venus, he made his heliometer measures of the minor planet Juno. At Ascension, in 1877, he used the heliometer to observe Mars, and his discussion of the observations gave results of high importance. Still Gill was unsatisfied: the size, colour, and phase of Mars made observations and discussion not perfectly satisfactory. The dream of his life would not be accomplished till he could determine the solar parallax afresh from observations of minor planets with a large heliometer.

He contributed a broad discussion of the other available methods in a series of papers communicated to *The Observatory* in 1877. He summed up in the sense that the minor planet method promised a completely satisfactory solution of the problem. This conviction was only deepened by the results of the heliometer observations of stellar parallax to which I have already referred.

Looking forward to available opportunities, Gill found that Iris in 1888 and Victoria and Sappho in 1889 would be exceptionally favourably placed; and being assured of the co-operation of Elkin at Yale, the work was undertaken. I have already alluded to the prompt response of the Admiralty to Gill's request for a 7-inch heliometer for the execution of the project.

The employment of the diurnal method at the Cape was not

completely satisfactory, nor was the transport of the observer and instrument to some equatorial station compatible with Gill's other duties. The only course open was to combine the southern heliometer observations with corresponding observations in the northern hemisphere. Hence the co-operation of the directors of the observatories that possessed heliometers—viz. Yale, Leipzig, Göttingen, Bamberg, and Oxford (Radcliffe)—was invited and promptly given.

Methods were devised to meet the case of the employment of heliometers in different hemispheres. To this end the comparison stars were as far as possible selected so that when the planet was situated at the greatest zenith distance at which good observations can be made (that is, when the parallax factor is the greatest attainable), the two comparison stars should be situated one above and one below the planet. In this way the determination of the parallax is made to rest upon differences of two nearly equal distances measured by means of the heliometer.

Special attention was directed to the important point of arranging matters so that the observations themselves furnish the material for the determination and elimination of the errors. The method was to be, so far as practicable, "self-correcting." Errors of scale were eliminated by selecting stars nearly equidistant from the planet; errors due to personal habit of the observer were minimised by the use of a reversing prism inserted between the eyepiece and the observer's eye, so as to bring the apparent direction of separation of the images always into the same position with reference to the vertical. The same comparison stars were used in both hemispheres, and thus errors in the adopted positions of the stars were eliminated. The cunning choice of the comparison stars along the course of the three planets is well shown in the charts given in the volumes vi. and vii. of the *Annals of the Cape Observatory*, which contain the full account of the observations.

For the meridian observations of the stars the co-operation of many different observatories was asked for, and Sir David Gill summed up the situation as follows:—

"The wide-felt scientific need of such an undertaking may be assumed to be demonstrated by the fact that almost every astronomer who was appealed to entered with heart and soul into his allotted share of the work, and probably no similar astronomical undertaking has ever before received such universal and powerful co-operation."

Twenty-two observatories co-operated. Altogether 9620 observations of 115 stars were utilised; they were discussed and combined by Dr. Auwers, who also visited the Cape and took part in the observations of Victoria. From this able and thorough discussion of the meridian observations, coupled with the heliometer triangulations, the places of the comparison stars were derived with unusual accuracy. Dr. Auwers also reduced the 760 meridian observations of the planets and derived a value of the solar parallax, viz. $8''{\cdot}806 \pm 0''{\cdot}03$, a value which agrees well with

that derived from the heliometer observations, though it is of much smaller weight.

The resulting heliometer determinations of the solar parallax were—

From Victoria,	. .	$8''\cdot8013 \pm 0''\cdot0061$
„ Sappho,	. .	$8\cdot7981 \pm 0\cdot0114$
„ Iris,	. . .	$8\cdot8120 \pm 0\cdot0090$
Mean,	.	$8\cdot8036 \pm 0\cdot0046$

—three independent determinations which agree within the limits of their probable errors.

Geodesy.—At a meeting of the British Association in South Africa (Report B.A., 1905) Sir David Gill gave an account of yet another enormous piece of work which has been carried out under the influence of his far-seeing initiative. He records how he felt that one of the duties laid on him by the traditions of the observatory and the labours of his predecessors was to interest himself in the geodetic survey of the colony.

Lacaille had measured an arc of $1\frac{1}{4}°$ northwards from Cape Town in 1752. The result showed that this southern arc indicated a figure of the Earth unsymmetrical about the equator.

Nearly a century later (1841–48) Maclear revised and extended Lacaille's arc to $4\frac{1}{2}°$. His result showed that the arc measured was, within narrow limits, consistent with a figure of the Earth that exhibited no difference in the northern and southern hemispheres. Lacaille's contrary deduction was found to be mainly attributable to a large local disturbance of the direction of gravity at one of the terminal stations.

Soon after his appointment as Astronomer at the Cape, Sir David Gill laid before Sir Bartle Frere, then Governor of Cape Colony, a proposal to create a system of geodetic triangles as a basis for the future accurate survey of the colony. It was a fortunate concurrence for the colonies and for geodesy that these two men—the one with his experience of Indian administration and his knowledge of the requirements of the colony, and the other with his scientific weight and practical enthusiasm—were able to deal with this question. After many administrative difficulties had been overcome, the actual work was begun in 1883. Thus was inaugurated a survey which must be an enduring benefit to the colonies, for already troubles and litigation were beginning to arise from incompatible surveys over small areas. The survey is founded upon a complete principal triangulation laid down with scientific accuracy. This could hardly have been achieved but for the initiation of Sir David Gill and the splendid devotion of Colonel Morris, R.E. (now Sir William Morris), who was in charge of the field work, ably supported by Lieut. Laffan, R.E., and other officers.

Step by step the geodetic survey has been extended, always under the scientific direction of Sir David Gill. As astronomers we may perhaps be allowed for the moment to lose sight of the

practical advantages accruing to the colonies, and to concentrate attention on the geodetic aspect of this great work.

The surveys of Cape Colony and Natal were completed in 1896, including a rediscussion of Maclear's triangulation (*Geodetic Survey of South Africa*, vol. i.), and a new reduction of Bailey's survey 1859–62 (vol. ii.). Rhodesia was then begun in 1897, and the triangulation was carried from near Bulawayo (lat. 22° S.) to within 75 miles of the southern end of Tanganyika (8°40′ S.).

Next, in 1902, the principal triangulation of the Transvaal and the Orange River Colony was undertaken, under the superintendence of Sir W. Morris, Gill being responsible for the initiation of the work, and acting as scientific adviser. The field work was nearly completed in 1905, and the trained parties were just about to be disbanded. Sir David Gill realised that if this occurred there would be a gap left between the Limpopo River and Bulawayo in South Rhodesia, with little likelihood of its being completed. But his energetic persistence succeeded in securing support by the intervention of Sir George Darwin, whereby the relatively small link of 120 miles in the chain was made good. Thus the triangulation is complete from the southern extremity of Africa nearly to the southern end of the German Protectorate, an arc of 25° in latitude.

It is to be hoped that the German Government will be able to continue the measurement of the arc along the eastern shore of Lake Tanganyika on German territory; all the more so because, by yet another instance of Sir David Gill's energy, the measurement of an arc of 2½° to the north of the German territory is now being carried out.

The delimitation of the Anglo-German boundary between British Bechuanaland and German South-West Africa along the 20th meridian gave rise to some temporary difficulties in 1896. Sir David Gill was in England at the time, and was consulted by the Colonial Office. His knowledge of local conditions was invaluable; he received instructions to proceed to Berlin, and there he was able to arrange an agreement satisfactory to both of the Governments concerned, and the direction of the work was placed in Sir David Gill's hands. Bosman's longitude arc across Bechuanaland to the 20th meridian was connected at both east and west ends with the geodetic circuit in Cape Colony.

This great system of triangulation pervading South Africa will eventually give geodetic results relating to three considerable arcs of meridian:—

First, the arc along the meridian of 19° E. longitude, with an amplitude of 12½°.

Second, the arc along the meridian of 26° E. longitude, with an amplitude of 8½°.

Third, the great arc along the 30th meridian, with an amplitude of 25°.

As far as the reductions are completed, it would appear that there is but slight deviation from Clarke's elements of the figure of

the Earth on any of these meridians. But in the second and third there seems to be evidence that the astronomical amplitude exceeds the geodetic by a small fraction of a second of arc per degree.

From the north, preliminary operations have already been begun by Captain Lyons of the Egyptian Survey to connect Cairo with the southern triangulation.

Sir David Gill's dream will be realised when, by the junction of Cairo round the east shores of the Mediterranean with Greece, and so with Struve's arc, the North Cape in lat. 70° N. is connected by triangulation with the southernmost point of Africa, lat. 35° S., an arc of meridian 105° in length.

My task has been a difficult one. Wisely or unwisely, I have attempted to set before you the broad lines along which Sir David Gill's achievements lie. In doing so, I have had, perforce, to leave many points of interest and importance untouched, even at the risk of seeming to fail in doing justice to his work.

I hope I have been able, however inadequately, to make clear to you that the recognition of the Council has fallen upon work of extraordinary scope, not only in administrative activity, but also in investigations of high refinement and permanent value.

Sir David Gill,—In presenting this medal to you, let me say that we know that for yourself the successful achievement of your tasks is the highest reward. But we could not deny ourselves the satisfaction of recording our appreciation of the mark which your labours in a distant land have made in the advance of astronomy. Let me convey to you the hope that your well-earned leisure may be filled with continued study of the science which you love.

The Meeting then proceeded to the election of the Officers and Council for the ensuing year, when the following Fellows were elected:—

President.

H. F. Newall, Esq., M.A., F.R.S.

Vice-Presidents.

Sir David Gill, K.C.B., LL.D., D.Sc., F.R.S.
Major P. A. MacMahon, D.Sc., F.R.S.
W. H. Maw, Esq.
H. H. Turner, Esq., D.Sc., F.R.S., Savilian Professor of Astronomy, Oxford.

Treasurer.

Major E. H. Hills, C.M.G.

Secretaries.

Thomas Lewis, Esq.
S. A. Saunder, Esq., M.A.

Foreign Secretary.

Sir William Huggins, K.C.B., O.M., LL.D., D.C.L., F.R.S.

Council.

Sir W. H. M. Christie, K.C.B., M.A., D.Sc., F.R.S., Astronomer Royal.
Bryan Cookson, Esq., M.A.
Rev. A. L. Cortie, S.J.
P. H. Cowell, Esq., M.A., F.R.S.
A. C. D. Crommelin, Esq., B.A.
A. S. Eddington, Esq., B.A., M.Sc.
Alfred Fowler, Esq., Assistant Professor of Physics, South Kensington.
J. W. L. Glaisher, Esq., M.A., Sc.D., F.R.S.
J. A. Hardcastle, Esq.
A. R. Hinks, Esq., M.A.
Richard Inwards, Esq.
E. B. Knobel, Esq.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVIII. MARCH 13, 1908. No. 5

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Frederic Hermann Albert Alfred Buss, 2 Lansdowne Terrace, Grosvenor Square, Ashton-on-Mersey, near Manchester;
Arthur du Pré Denning, M.Sc., Ph.D., Birmingham University, and 18 Lightwoods Hill, Birmingham;
Herbert Shaw, A.R.C.S., Royal College of Science, South Kensington, and 6 Gowan Avenue, Fulham, S.W.; and
James Henry Worthington, Student in the University of Oxford, Bindon, Wellington, Somerset,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Captain Richard Algernon Craigie Daunt, D.S.O., Lynalta, Newtownards, Co. Down, Ireland (proposed by Rev. A. L. Cortie); and
Edgar Odell Lovett, Ph.D., Professor of Astronomy, Princeton University, New Jersey, U.S.A. (proposed by J. W. L. Glaisher).

One hundred and four presents were announced as having been received since the last meeting, including, amongst others:—

M. Jarry-Desloges, Observations des surfaces planétaires, fasc. 1, presented by the author; E. W. Maunder, The Astronomy of the Bible, presented by the author; C. H. F. Peters, Heliographic positions of Sun-spots, 1860–1870, edited by Professor Frost, pre-

sented by the Carnegie Institution; J. A. Repsold, Geschichte der Astronomischen Messwerkzeuge, presented by Mr. Franklin-Adams; E. B. H. Wade, Field method of determining longitudes by observations of the Moon, presented by the Egyptian Survey Department.

Astrographic Chart; 32 charts, presented by the Royal Observatory, Greenwich; 20 charts, from Algiers and Paris Observatories, presented by the French Government; and 20 charts, presented by the San Fernando Observatory.

Series of 36 collotype reproductions of photographs of the Milky Way, etc., presented by Professor E. E. Barnard; photograph of the Nebula in Orion (transparency) from negative taken by Professor Perrine with the Crossley reflector, presented by the Lick Observatory.

A suggested explanation of the ancient Jewish Calendar Dates in the Aramaic Papyri translated by Professor A. H. Sayce and Mr. A. E. Cowley. By E. B. Knobel.

The Aramaic papyri discovered at Assuan, on the site of the ancient Syene, which have been recently translated and published by Professor Sayce and Mr. Cowley, are of unique interest and importance owing to the duplicate dates given to each document. These documents cover a large part of the fifth century B.C., extending from B.C. 471, nine years only after the battle of Salamis, to B.C. 410. The papyri all relate to a Hebrew colony established at that period at Syene, and deal with rights of property, conveyance of land and buildings, marriage portions, and legal processes. They are all deeds most carefully drawn, signed, sealed, and witnessed, and they are dated according to both the Egyptian and Hebrew calendars, in the regnal years of the kings of Persia.

The Egyptian year and calendar are well understood. The year was a vague solar year, and consisted of 365 days without intercalation or correction, consequently the Julian date of the commencement of the Egyptian year recedes one day every four years. The year consisted of twelve months, each of thirty days, and five additional days, called *epagomenæ*, were added after the last month. There is consequently no difficulty with this calendar in determining the corresponding Julian date.

Very little, however, is known of the Jewish calendar in use at the period under consideration. The present reformed calendar dates only from the time of Hillel in the fourth century A.D., though it was probably not finally settled until after the fifth century. It is known that in olden times the year was a lunar year, and certain months, and ordinances connected with the months and seasons, are mentioned in the Old Testament. There is no mention of an intercalary month in the Bible, and it is not

known whether the correction to the solar year was applied in ancient times by the addition of one month in three years, or by the adding of ten or eleven days at the end of each year. No information appears to exist that there was anything like a settled Jewish calendar so far back as the fifth century B.C.

It is very generally stated that prior to the adoption of the reformed calendar the Jews employed the era of the Seleucidæ, the years of which were Julian of 365 days, but this could not have been the case at the period under discussion. Burnaby's work on the Jewish calendar gives little assistance in the present investigation.

Mr. Margoliouth—a high authority—writes: "No lists of pre-Christian Jewish dates reconciled with Egyptian or other dates are so far available to throw light on the exact form of the calendar used for the dating of the Aramaic documents published by Professor Sayce and Mr. Cowley. In the fifth century B.C. the Jewish calendar depended entirely on the observation of the Sun and the Moon, particularly the latter. The decisions must have been made by a central court, as was practically the case down to 359 A.D., so that great uncertainty would be caused in distant parts (such as Syene in Upper Egypt, to which the papyri belong) by the delay in transmitting the announcements.

"It is also uncertain whether the Jewish lunar year was in ancient times harmonised with the solar year by the addition of one month in three years, or by lengthening the last month in each year. The difficulties connected with the dates given in the recently published papyri may possibly have to be ascribed to the uncertainties mentioned."

Professor Schürer has discussed the subject in the *Theologische Literaturzeitung* for February 1907, in which he claims that the papyri confirm the fact that the Jews began their months with the *appearance* of the new moon, and further that they show that "it was far from the case that any definite system had been adopted."

Dr. Lidzbarski has also reviewed these papyri in the *Deutsche Literaturzeitung* for 1906, but his discussion is more particularly philological, and contributes little towards the question of the ancient calendar of the Jews.

The object of the present paper is to inquire whether more definite information on the subject cannot be derived from the Aramaic papyri themselves.

The dates of each papyrus, as given by the translators, are as follows. The figures in brackets indicate possible alternative dates according as a certain slanting mark in the writing is considered as forming part of the numeral or not. The present opinion is that it should do so, and that the higher number is the correct one, which I have accordingly adopted.*

A. On the 17th (18th?) of Elul, that is the 27th (28th?) day of Pachons, the 14th (15th?) year of Xerxes the king . . . .

* An exception may probably be made in the day of Thoth in B.

B. On the 18th (?) of Chisleu, that is the 6th (7th?) day of Thoth, the 20th (21st?) year (of Xerxes), the beginning of the reign when Artaxerxes the king ascended his throne . . . .

C. Mutilated as to the dates.

D. On the 21st Chisleu, that is the 1st of Mesore, the 6th year of Artaxerxes the king . . . .

E. On the 3rd of Chisleu, that is the 10th day of the month Mesore, the 19th year of Artaxerxes the king . . . .

F. On the 13th (14th?) of Ab, that is the 19th day of Pachons, the 25th year of Artaxerxes the king . . . .

G. On the 26th (?) of Tishri . . . . the 6th (day) of the month Epiphi [the 25th year of Artaxerxes the king] . . . .

H. In the month Elul, that is Payni, the 3rd (4th?) year of Darius the king.

J. On the 3rd of Chisleu, the 7th (8th?) year, that is the 11th (12th?) day of Thoth, the 7th (8th?) year of Darius the king . . . .

K. On the 23rd (24th?) of Shebat, the 13th year, that is the 8th (9th?) day of Athyr, the 13th (14th?) year of Darius the king . . . .

The dates definitely adopted from the translation are as follows:—

A.	15th	year of	Xerxes,	28th Pachons	= 18th Elul.
B.	1st	,,	Artaxerxes,	6th Thoth	= 18th Chisleu.
E.	19th	,,	,,	10th Mesore	= 3rd Chisleu.
F.	25th	,,	,,	19th Pachons	= 14th Ab.
J.	8th	,,	Darius,	12th Thoth	= 3rd Chisleu.
K.	14th	,,	,,	9th Athyr	= 24th Shebat.

For the regnal years of the kings I have adopted the dates given by Ricard in his edition of Plutarch, thus:—

Cambyses,	1st year	B.C.	529
Smerdis (7 months),*	,,	,,	522
Darius Hystaspes,	,,	,,	521
Xerxes the Great,	,,	,,	485
Artabanus (7 months),	,,	,,	464
Artaxerxes Longimanus,	,,	,,	464
Xerxes II. (a month),	,,	,,	425
Sodgianus (7 months),	,,	,,	424
Darius II. (nothus)	,,	,,	423

The order of the Egyptian and Hebrew months is as follows:—

Egyptian Months.	Days.	Hebrew Months.	Days.
Thoth.	30	Tishri.	30
Phaophi.	30	Marheshvan.	29 or 30
Athyr.	30	Chisleu.	30 or 29
Choiak.	30	Tebeth.	29
Tybi.	30	Shebat.	30
Mechir.	30	Adar.	29

* Oppert.

Egyptian Months.	Days.	Hebrew Months.	Days.
Phamenoth.	30	Ve-Adar.	30
Pharmuthi	30	Nisan.	30
Pachons.	30	Iyyar.	29
Payni.	30	Sivan.	30
Epiphi.	30	Tammuz.	29
Mesore.	30	Ab.	30
5 Epagomenæ.		Elul.	29

In the papyri the Julian dates corresponding to the Egyptian dates are all known, and the problem, in the absence of all information on the subject, is to construct a reasonable and probable Jewish calendar which shall satisfy all the Jewish dates.

Fortunately the papyri E. and J. offer some assistance towards the solution of this difficult question. The Egyptian dates in Julian reckoning are as follows:—

E. B.C. 446 ... 10th Mesore = November 17,
J. B.C. 416 ... 12th Thoth = December 16,

but the Jewish date of both documents is the same, viz. 3rd Chisleu; consequently the period B.C. 446 November 17 to B.C. 416 December 16 should be an *exact number* of Jewish years.

It has been assumed by writers generally that the commencement of each month was determined by observation and announcement, and this was no doubt the common practice in the ecclesiastical year, which began with the 1st Nisan. The Jewish civil year, however, began unquestionably with the 1st Tishri; and with such a practical business people as the Jews, who, as we should infer from the papyri under consideration, enjoyed at this period a high state of civilisation, it is almost inconceivable that they should not have had in current use some calendar upon which they could base their business negotiations.

The reformed Jewish calendar is based upon the Lunar cycle of nineteen years—the so-called Metonic cycle—and it is not unreasonable to assume that this cycle was in use with the Jews long before the time of Hillel. With one exception, that of the French Revolution calendar, history does not record the *creation* of any calendar, but only the correction, reformation, or amendment of pre-existing calendars. Dr. Mahler pointed out in a paper read to the Oriental Congress of 1892 ("Das Kalenderwesen der Babylonier") that the Lunar cycle was in use by the Jews at Babylon before it was adopted by the Greeks, and that it was really of Babylonian origin. In discussing the order of the intercalary months, Al Biruni (A.D. 973–1048) (*The Chronology of Ancient Nations*) mentions one particular order which he says is preferred by the Jews, because they attribute its invention to the Babylonians.*

In this attempt to explain the Jewish calendar dates in the

* The period we are dealing with was only about sixty years after the Captivity, and it is reasonable to suppose that some of the colonists at Syene may have migrated from Babylon, as Professor Sayce particularly indicates Babylonish names among those mentioned in the documents.

papyri, it may therefore be justifiable to assume that the nineteen-year Lunar cycle was in current use. I have accordingly adopted the cycle with the same intercalations as are to be found in the present Jewish calendar, which is unchanged since the fourth century A.D., and upon this basis I have constructed a table for the whole period covered by the MSS., the intercalary months disposed according to Scaliger's rule, "ter, ter, bis, ter, ter, ter, bis." *

Normal Lunar Cycle.

No of Year.	Days.
1	354
2	354
3 Emb.	384
4	354
5	355
6 Emb.	384
7	354
8 Emb.	384
9	354
10	355
11 Emb.	384
12	354.
13	354
14 Emb.	384
15	355
16	354
17 Emb.	384
18	354
19 Emb.	384

Applying this tentative calendar to the cases of papyri E., B.C. 446, and J., B.C. 416, it will be seen that there is *only one possible* position for those years in this Lunar cycle, and that B.C. 446 was the 17th and B.C. 416 the 9th year of that cycle, for this is the only position in which twelve intercalary years can be brought into a period of thirty years.

This gives coincidence between the number of days from B.C. 446 November 17 to B.C. 416 December 16, and the number of days in thirty Jewish years beginning with cycle No. 17 and ending with cycle No. 8 inclusive. On any other calculation there would be a difference of a month, and both deeds could not be dated in the same month Chisleu.

B.C. 446 Nov. 17 to B.C. 416 Dec. 15 inclusive = 10,987 days
30 Jewish years, cycle No. 17 to cycle No. 8 inclusive = 10,986 „

* In the old Chinese and Japanese calendar the intercalary months are disposed in this order.

It should be mentioned in explanation that were Dr. Mahler's Babylonian cycle employed, then B.C. 446 would be the 6th and B.C. 416 the 17th year of that particular cycle. So again in the cycle which Al Biruni says was preferred by the Jews, B.C. 446 would be the 14th and B.C. 416 the 6th year.

The table appended to this paper of the 1st day of Tishri from B.C. 523 to B.C. 406 has been constructed in the following manner:—The Lunar cycle numbers are laid down for the whole period from the numbers fixed for B.C. 446 and B.C. 416, and the days of each Jewish year appended. The Jewish astronomical computation of the length of a Lunar cycle is 6939 days 16 hours and 595 chalakim.* As the table extends over six cycles, an empirical correction had to be made making some cycles 6940 days, so that the mean length of the six cycles is 6939 days 16 hours.†

It was then necessary to find reliable data for determining the 1st day of Tishri for any year, so that a calendar could be constructed so far on a sound basis. Fortunately this was afforded by the most interesting discovery a few years ago by Father Strassmeier of a Babylonian tablet recording a partial lunar eclipse at Babylon in the 7th year of Cambyses. This cuneiform tablet has been fully translated and discussed by Oppert (*Zeitschrift für Assyriologie*, vol. vi.). It has an entirely unique interest, as it is an account of one of the eclipses recorded by Ptolemy in the Almagest.

Ptolemy states that the eclipse occurred in the 7th year of Cambyses, in the 225th year of Nabonassar, on the night of the 17th and 18th of the Egyptian month Phamenoth. Strassmeier's Babylonian tablet gives the date as the 7th year of Cambyses, on the 14th day of the Jewish month Tammuz. The Julian date of the eclipse is determined by Pingré and Oppolzer as B.C. 523 July 16.

From this it is easy to calculate the date of the 1st Tishri as September 29; and as the 7th year of Cambyses is well identified as B.C. 523, the table appended is calculated entirely from this date—from B.C. 523 to B.C. 406. It gives the Year B.C.—Julian period—No. in Lunar cycle—Days in each year—Julian date of 1st Thoth—Julian date of the 1st Tishri; and Greenwich Mean Time of New Moon nearest to the 1st Tishri taken from Ginzel's *Handbuch der Mathematischen und Technischen Chronologie*.

In considering the coincidence of Julian and Jewish dates, it should be remembered that the Jewish day is defined in Genesis: "And there was evening and there was morning, one day,"—that is to say, the day begins at 6 o'clock in the evening and goes on to 6 o'clock the next evening, consequently one Jewish date extends over part of two Julian days.

* 1080 chalakim equal 1 hour.

† I have avoided complicating the question by reference to the "regular," "deficient," and "abundant" years, as exactitude is impossible, and it seemed sufficient to secure the correctness of the mean Lunar cycle.

Discussion of Dates.

A.

15th year of Xerxes	... B.C. 471 ...	1st Thoth	... Dec. 19
		28th Pachons	... Sept. 12
		1st Tishri	... Sept. 24
		18th Elul	... Sept. 12

B.

1st year of Artaxerxes	... B.C. 464 ...	1st Thoth	... Dec. 17
		6th Thoth	... Dec. 22
		1st Tishri	... Oct. 6
		18th Chisleu	... Dec. 21

C.

This papyrus is too much injured for the dates to be deciphered. The authors state that it is written by the same scribe as D., and that there is strong evidence for considering both C. and D. as of the same date.

D.

The MS. states: "On the 21st Chisleu, that is the 1st Mesore, the 6th year of Artaxerxes the king." By no possibility can these dates—21st Chisleu and 1st Mesore—be harmonised. But there is a crease in the papyrus just before the words "1 Mesore," and in this crease there is an indication of a character which cannot be deciphered until the crease is flattened out. It is probable that the Egyptian date has not been correctly deciphered. Mesore is the last month of the Egyptian year, and it is followed by the five Epagomenæ, which were kept as feast days. The question may be asked, whether in dating deeds such as those under consideration the five Epagomenæ were not treated as continuous dates of the previous month, Mesore? Dr. Budge informs me that he has no experience of such a case, but he sees no reason why it should not be suggested. I venture to hazard the suggestion that the first Epagomene was designated as the 31st Mesore. Upon this pure assumption we should have, as the best that can be done for D.,—

B.C. 460 ...	1st Thoth	... Dec. 16	
	31st Mesore	... Dec. 11	
	1st Tishri	... Sept. 21	
	21st Chisleu	... Dec. 9	

E

19th year of Artaxerxes	... B.C. 446 ...	1st Thoth	... Dec. 13
		10th Mesore	... Nov. 17
		1st Tishri	... Sept. 17
		3rd Chisleu	... Nov. 17

F.

25th year of Artaxerxes ... B.C. 440 ...	1st Thoth	... Dec.	11
	19th Pachons	... Aug.	26
	1st Tishri	... Oct.	10
	14th Ab	... Aug.	25

G.

The papyrus is very mutilated. The dates 26th Tishri and 6th Epiphi are fairly certain, but the regnal year of Artaxerxes is conjecture. The authors state that the date of this deed cannot be earlier than 446, and hardly later than 440. We have to find coincidence between 6th Epiphi and 26th Tishri. The table gives the following dates:—

B.C. 446	6th Epiphi ...	Oct.	14	26th Tishri ...	Oct. 12
445	,,		13	,,	30
444	,,		13	,,	19
443	,,		13		Nov. 7
442	,,		13		Oct. 28
441	,,		12	,,	16
440	,,		12		Nov. 4

From this it is probable that the year is B.C. 446, and this conclusion is supported by the fact that the scribe of G. is also the scribe of E., which is clearly B.C. 446. The regnal year would thus be the 19th of Artaxerxes.

H.

The papyrus states, "in the month Elul, that is Payni, the 3rd (4th ?) year of Darius."

3rd year of Darius	Payni began	Sept.	2	ended	Oct.	1
B.C. 421	Elul ,,	Sept.	11	,,	Oct.	9
4th year of Darius	Payni began	Sept.	2	ended	Oct.	1
B.C. 420	Elul ,,	Aug.	31	,,	Sept.	28

Clearly the 4th year of Darius, B.C. 420, suits the case best.

J.

8th year of Darius ... B.C. 416 ...	1st Thoth	... Dec.	5
	12th Thoth	... Dec.	16
	1st Tishri	... Oct.	15
	3rd Chisleu	... Dec.	15

K.

14th year of Darius ... B.C. 410 ...	1st Thoth	... Dec.	4	... B.C.	411
	9th Athyr	... Feb.	10	... B.C.	410
	1st Tishri	... Sept.	20	... B.C.	411
	24th Shebat	... Feb.	8	... B.C.	410

The final results are as follows:—

		Julian Date from Egyptian.	Computed Date from Table.
A.		Sept. 12	Sept. 12
B.		Dec. 22	Dec. 21
C.	Mutilated.		
D.	Uncertain.	Dec. 11?	Dec. 9?
E.		Nov. 17	Nov. 17
F.		Aug. 26	Aug. 25
G.		Oct. 14	Oct. 12
H.	B.C. 420		
J.		Dec. 16	Dec. 15
K.		Feb. 10	Feb. 8

The above results are too near coincidence to be fortuitous, and, so far as the civil year is concerned, they refute the opinion that the commencement of the month was determined by the appearance of the new moon.

Two conclusions from the foregoing investigation may be safely hazarded: first, that the Lunar cycle of 19 years was in use in the Jewish calendar at this remote period, which, as Professor Sayce says, was little more than a century after the grandfathers and great-grandfathers of the parties mentioned in the papyri had fled into Egypt with Jeremiah; and secondly, that the order of intercalation at that time was not dissimilar to that in use to-day.

In drawing any conclusions, one may put aside possible errors of the scribe. It is highly improbable that in the first line of original and important deeds like these papyri the scribe would make such errors as would be common in copies.

These deductions do not harmonise with the views of the late distinguished chronologist M. Oppert. It may be assumed that what was current with the Jews at Babylon during the Captivity would have been continued by them in their subsequent migration. M. Oppert states that the apparition of the crescent moon signalised the commencement of the month, and in a paper "Sur l'ancien Calendrier Perse,"* he claims to have proved that the Babylonians had no fixed system for their calendar until after the year B.C. 367; that prior to that period the 19-year cycle was in use, but the intercalary months were inserted without any order, and solely on astrological grounds; and that it was the Greek influence which gave to Babylon a fixed system, assigning to each year of the cycle its particular character, whether common or embolismic, and he denies the correctness of Dr. Mahler's conclusions.

This view can hardly be sustained, for in making the Babylonian date B.C. 523, 14th Tammuz, the basis of the appended table, it is most improbable that we should arrive at such coincidence of the Egyptian and Jewish dates of the papyri if there had been no fixed system at all. The table connects in a systematic manner

* Oriental Congress, 1897. In this paper he calculates October 6th as the 1st day of Tishri, B.C. 521, as it is found in the present table.

Babylonian dates with the dates used by the Jews at Syene over a century later; and, notwithstanding M. Oppert's characteristic remark that "on fait l'histoire avec les livres historiques et non pas avec les éclipses," the rock upon which this investigation is built is the lunar eclipse at Babylon in the 7th year of Cambyses.

Table of the 1st Tishri from B.C. 523 *to* B.C. 406.

Year B.C.	Julian Period.	Lunar Cycle.	Days.	1st Thoth.	1st Tishri.	G.M.T. New ☾.
523	4191	16	354	Jan. 1	Sept. 29	Sept. 27·57
522	2	17	384		Sept. 18	17·05
521 *β*	3	18	354	Dec. 31	Oct. 6	5·13
520	4	19	384		Sept. 25	24·80
519	5	1	354		Oct. 14	13·85
518	6	2	354		Oct. 3	3·27
517 *β*	7	3	384	Dec. 30	Sept. 21	21·41
516	8	4	354		Oct. 10	10·16
515	9	5	355		Sept. 29	29·17
514	4200	6	384		Sept. 19	18·46
513 *β*	1	7	354	Dec. 29	Oct. 7	6·47
512	2	8	384		Sept. 26	26·11
511	3	9	354		Oct. 15	15·18
510	4	10	355		Oct. 4	4·77
509 *β*	5	11	384	Dec. 28	Sept. 23	23·13
508	6	12	354		Oct. 12	11·95
507	7	13	354		Oct. 1	30·93
506	8	14	384		Sept. 20	20·02
505 *β*	9	15	355	Dec. 27	Oct. 8	7·91
504	4210	16	354		Sept. 28	27·41
503	1	17	384		Sept. 17	17·07
502	2	18	354		Oct. 6	6·15
501 *β*	3	19	384	Dec. 26	Sept. 24	24·69
500	4	1	355		Oct. 13	13·61
499	5	2	354		Oct. 3	2·72
498	6	3	384		Sept. 22	21·72
497 *β*	7	4	354	Dec. 25	Oct. 10	9·50
496	8	5	355		Sept. 29	28·81
495	9	6	384		Sept. 19	18·38
494	4220	7	354		Oct. 8	7·47
493 *β*	1	8	384	Dec. 24	Sept. 26	26·10
492	2	9	354		Oct. 15	15·12
491	3	10	355		Oct. 4	4·45
490	4	11	384		Sept. 24	23·51
489 *β*	5	12	354	Dec. 23	Oct. 12	11·25
488	6	13	354		Oct. 1	30·34
487	7	14	384		Sept. 20	Sept. 19·72

Year B.C.	Julian Period.	Lunar Cycle.	Days.	1st Thoth.	1st Tishri.	G.M.T. New ☾.
486	4228	15	355	Dec. 23	Oct. 9	Sept. 8·77
485 β	9	16	354	Dec. 22	Sept. 28	27·42
484	4230	17	384		Sept. 17	17·06
483	1	18	354		Oct. 6	6·03
482	2	19	384		Sept. 25	25·26
481 β	3	1	355	Dec. 21	Oct. 13	13·04
480	4	2	354		Oct. 3	2·03
479	5	3	384		Sept. 22	21·21
478	6	4	354		Oct. 11	10·15
477 β	7	5	355	Dec. 20	Sept. 29	28·72
476	8	6	384		Sept. 19	18·41
475	9	7	354		Oct. 8	7·47
474	4240	8	384		Sept. 27	26·91
473 β	1	9	354	Dec. 19	Oct. 15	14·78
472	2	10	355		Oct. 4	3·81
471	3	11	384		Sept. 24	22·83
470	4	12	354		Oct. 13	11·67
469 β	5	13	354	Dec. 18	Oct. 1	30·08
468	6	14	384		Sept. 20	19·71
467	7	15	355		Oct. 9	8·78
466	8	16	354		Sept. 29	28·40
465 β	9	17	384	Dec. 17	Sept. 17	16·77
464	4250	18	354		Oct. 6	5·59
463	1	19	384		Sept. 25	24·59
462	2	1	354		Oct. 14	13·35
461 β	3	2	354	Dec. 16	Oct. 2	1·54
460	4	3	384		Sept. 21	21·02
459	5	4	354		Oct. 10	10·09
458	6	5	355		Sept. 29	29·76
457 β	7	6	384	Dec. 15	Sept. 18	18·33
456	8	7	354		Oct. 7	7·25
455	9	8	384		Sept. 26	26·39
454	4260	9	354		Oct. 15	15·15
453 β	1	10	355	Dec. 14	Oct. 3	3·15
452	2	11	384		Sept. 23	22·44
451	3	12	354		Oct. 12	11·44
450	4	13	354		Oct. 1	1·08
449 β	5	14	384	Dec. 13	Sept. 19	19·72
448	6	15	355		Oct. 8	8·75
447	7	16	354		Sept. 28	28·11
446	8	17	384		Sept. 17	17·18
445 β	9	18	354	Dec. 12	Oct. 5	4·91
444	4270	19	384		Sept. 24	23·99
443	1	1	355		Oct. 13	Sept. 12·89

Year B.C.	Julian Period.	Lunar Cycle.	Days.	1st Thoth.	1st Tishri.	G.M.T. New ☾.
442	4272	2	354	Dec. 12	Oct. 3	Sept. 2·38
441 *β*	3	3	384	Dec. 11	Sept. 21	21·04
440	4	4	354		Oct. 10	10·12
439	5	5	355		Sept. 29	29·66
438	6	6	384		Sept. 19	18·93
437 *β*	7	7	354	Dec. 10	Oct. 7	6·72
436	8	8	384		Sept. 26	25·70
435	9	9	354		Oct. 15	14·49
434	4280	10	355		Oct. 4	3·78
433 *β*	1	11	384	Dec. 9	Sept. 23	22·35
432	2	12	354		Oct. 12	11·44
431	3	13	354		Oct. 1	1·07
430	4	14	384		Sept. 20	20·55
429 *β*	5	15	355	Dec. 8	Oct. 8	8·43
428	6	16	354		Sept. 28	27·48
427	7	17	384		Sept. 17	16·48
426	8	18	354		Oct. 6	5·31
425 *β*	9	19	384	Dec. 7	Sept. 24	23·70
424	4290	1	355		Oct. 13	12·74
423	1	2	354		Oct. 3	2·39
422	2	3	384		Sept. 22	22·03
421 *β*	3	4	354	Dec. 6	Oct. 10	10·01
420	4	5	355		Sept. 29	29·24
419	5	6	384		Sept. 19	18·26
418	6	7	354		Oct. 8	7·02
417 *β*	7	8	384	Dec. 5	Sept. 26	25·18
416	8	9	354		Oct. 15	14·12
415	9	10	355		Oct. 4	3·70
414	4300	11	384		Sept. 24	23·38
413 *β*	1	12	354	Dec. 4	Oct. 12	11·43
412	2	13	354		Oct. 1	30·45
411	3	14	384		Sept. 20	20·05
410	4	15	355		Oct. 9	8·81
409 *β*	5	16	354	Dec. 3	Sept. 28	26·80
408	6	17	384		Sept. 17	16·07
407	7	18	354		Oct. 6	5·04
406	8	19	384		Sept. 25	Sept. 24·67

32 *Tavistock Square, London, W.C.:*
1908 *March* 11.

Observations of Saturn's Ring at the time of its Disappearance in 1907, made with the 40-in. Refractor of the Yerkes Observatory. By E. E. Barnard. (Plates 9, 10.)

The position of Saturn for observation of the phenomena connected with the disappearance of the ring has been unusually favourable this year, though the south declination (5°) of the planet was somewhat unfortunate for northern observers.

At the reappearance of the ring in October 1891, Saturn rose only about two hours before the Sun, and the resulting low position and the approach of daylight made it very unfavourable for the observation of such a delicate thing as the ring when it is placed edge on towards us. At that time, with the 36-in. and 12-in. refractors of the Lick Observatory, and under such conditions, I was unable to see the ring (*M.N.* for April 1892, vol. lii. p. 419).

According to Professor Hermann Struve, the following table represents the dates of the present disappearances and reappearances of the ring of Saturn.*

1907.	Apr. 17.	Disappearance.	The Earth in the plane of the rings.
,,	July 26.	Reappearance.	The Sun in the plane of the rings.
,,	Oct. 4.	Disappearance.	The Earth in the plane of the rings.
1908.	Jan. 7.	Reappearance.	The Earth in the plane of the rings.

On account of the position of Saturn in the direction of the Sun, the disappearance of the ring in April was invisible from the Earth. The reappearance in January next will be favourable for observation.

The present paper deals with the reappearance and disappearance of the ring in July and October of this year (1907). The times are central standard time, 6^h slow of G.M.T.

The unusually bad spring weather prevented early observations with the 40-in. telescope. The planet was observed, however, as frequently as the weather and other circumstances would permit. It was supposed the "reappearance" of the ring, when the Sun passed through its plane on or about July 26, would be a definite phenomenon, and that the time of this reappearance could be determined with some sort of precision. But the reappearance was a remarkably gradual phenomenon, and there was no possible means of telling when it occurred. The ring simply very slowly and gradually got brighter, and for several days it was impossible to tell that any change had taken place; and then it became bright and almost linear. It was not, however, at this time that the greatest interest lay. It was sometime previous to the reappearance of the ring that the most important phenomena were visible.

When the planet was examined on July 2 the entire surface of the ring was easily seen, though the Sun was not then shining on its visible surface. Where it was projected on the sky, the ring

* Publications of the Astronomical Society of the Pacific, No. 114, June 10, 1907.

appeared as a greyish hazy or nebulous strip, which was not well defined under the best conditions. It was about 1·5 times as broad as the trace of the ring on the ball. Nothing could be seen of the sunlit edge of the ring, which must have been too thin to be visible. There were two nebulous condensations of greater brightness on the ring at each side of the planet. These were quite conspicuous, but were ill defined and nebulous, and of a pale grey colour. The ring and condensations were so pronounced that they were strongly visible at $15^h\ 50^m$ (36^m before sunrise), when the approaching daylight was very bright. They were still visible as late as $16^h\ 2^m$ (24^m before sunrise), and, though faint, could have been followed a little later yet. Rough measures were made of the distances of these condensations on the preceding side of the planet from the preceding limb.

The Centre of the most Distant Spot from the Preceding Limb.	The Centre of the Nearer Spot from the Preceding Limb.
7″·7(1) [7″·6]	2″·9(1) [2″·8]

One setting was made for the extreme diameter of the ring, and this gave 40″·9 [40″·0];* while two settings gave 18″·85 for the equatorial diameter of the planet.

At $15^h\ 4^m$ a faint satellite (Enceladus) following was visible on the south side of the ring, close to the second condensation on that side. The satellite and condensation were of the same brightness, which will give some idea of the brightness of the ring and condensations. The trace of the ring across the ball was not black. The seeing was not good enough to tell whether part of this trace was due to the shadow of the ring, which in all probability at that time did not reach to the ball. The width of this trace was measured by setting the wires so that their outer edges were separated by the width of the trace—a correction afterwards being applied for the thickness of the wires = 0″·10. The width was 0″·62(1) at $14^h\ 50^m$. The ring, as projected on the sky, seemed to be a little south of the trace.

Position of the trace of the ring on the ball at $15^h\ 30^m$:—

From South Limb.	From North Limb.
8″·83(3) [8″·63]	7″·84(3) [7″·66]

This trace was without any visible irregularities.

July 5. $15^h\ 0^m$. The condensations were again visible. The nearer condensations were estimated to be $1-\frac{1}{2}$ times as bright as the distant ones. One setting gave for the width of the trace of the ring on the ball 0″·70. I could see nothing of the sunlit edge of the ring.

The seeing was very poor, and the sky clouded a few minutes later and did not again clear.

* The square brackets throughout this paper mean that the enclosed values have been reduced to the mean distance of Saturn from the Sun. The numbers in parentheses are the number of settings of the micrometer wires. The times are central standard times, $6^h\ 0^m$ slow of Greenwich.

July 6. $14^h\ 30^m$. The ring was rather faint—very much fainter, apparently, than on July 2, and perhaps somewhat narrower.

Width of trace on ball $0''{\cdot}62(2)$.

Position of the trace :—

From South Limb.	From North Limb.
$8''{\cdot}79(3)$ [$8''{\cdot}53$]	$8''{\cdot}02(3)$ [$7''{\cdot}79$]

There was a faint dark belt south of the trace of the ring, $6''{\cdot}6(1)$ from the south limb. The condensations on the ring were visible as before. Nothing could be seen of the sunlit edge of the ring.

July 12. $14^h\ 10^m$. The feebly luminous ring was visible as a thin nebulous strip on each side of the planet. Seeing very bad. There was a satellite preceding and another (Enceladus) following the ring. When best seen, the ring was about midway in brightness between these satellites. It was very thin and of a pale nebulous colour. The sunlit edge could not be seen. As late as $15^h\ 35^m$ (57^m before sunrise) the ring was still distinctly visible.

July 23. $13^h\ 50^m$. Sky thick. Seeing very bad. The ring was dimly visible as a faint thin line. The sunlit edge could not be seen.

July 24. Full moon. Seeing poor. $14^h\ 20^m$. The full extent of the ring was visible as a narrow, almost thread-like strip. It was faint and nebulous, without any irregularities. A faint satellite (Enceladus) was close following the following edge of the ring; when best seen, the ring was perhaps one magnitude less bright than this satellite. There was no appearance of direct sunlight falling on the ring or its edge. The following ansa seemed to be the most distinct—perhaps a little brighter.

July 25. Full moon. $15^h\ 10^m$. The ring fairly well seen as a faint nebulous strip. $15^h\ 15^m$. There was a small satellite (Enceladus) preceding the preceding end of the ring; it was 1–½ magnitude brighter than the ring, which was of a faint grey colour. Seeing fair. With an occulter in the eyepiece the ring was quite easily visible. There was no appearance of direct sunlight, however, but the same pale illumination seen previously. There did not seem to be any irregularities on the ring, which was very narrow, and quite distinct when best seen. The trace of the ring on the ball was not black. At $15^h\ 45^m$ ($1^h\ 0^m$ before sunrise) the ring could still be fairly well seen.

July 26. $15^h\ 0^m$. Seeing fair. Slight haze. Bright moonlight. The ring was a little brighter, but whether due to less moonlight or not, it was not possible to tell. A small satellite (Enceladus) near and preceding the preceding end of the ring was but little brighter than the ring, perhaps as much as half a magnitude.

$15^h\ 15^m$. The ring was decidedly brighter than on the 25th. This was perhaps due to the direct sunlight shining on it. It was very thin. A faint satellite (Tethys) followed the ball, and

was a little south of the ring. It was but a little brighter than the ring near it.

Width of trace of ring on ball = 0″·45(4).

15h 30m. The small satellite (Tethys) following was much brighter than the ring near it—perhaps one magnitude brighter. The ring on the sky was broader than the trace

A power of 700 was then applied. With this the ring was pale and nebulous. The illumination did not then look like direct sunlight.

Position of trace of ring on ball at 15h 45m—

From South Limb.	From North Limb.
8″·77(2) [8″·22]	8″·23(2) [7″·72]

The thickness of the trace was 0″·52(4).

16h 0m (45m before sunrise). The ring was still easily seen. It was but little less bright than the satellite (Enceladus) preceding.

16h 2m. The ring was still faintly visible. The sky was white from daylight.

July 28. 11h 40m. The ring was bright, but much less bright than the ball of the planet. The illumination looked like true sunlight. The projection of the ring on the sky was south of the projection on the ball.

14h 30m. Thickness of trace on ball 0″·44(4). The illumination of the ring did not look like direct sunlight. It was a pale grey nebulous light.

15h 35m. The ring could not be traced up to the ball, but was discontinuous for some few seconds.

15h 45m. The following ansa seemed to be a little thicker than the preceding. A very faint belt on the ball was seen north. The ring on the sky was about twice as thick as its trace on the ball.

August 6. 13h 45m. Trace of ring on ball :—

From South Limb.	From North Limb.
8″·68(3) [8″·04]	8″·37(3) [7″·89]

Measures were very poor—seeing very bad.

August 11. 12h 40m. Position of trace :—

From North Limb.	From South Limb.
8″·25(3) [7″·59]	9″·37(3) [8″·63]

15h 0m. The ring was much less bright than the ball. It looked like a bar of light on each side of the planet. Seeing very poor.

September 1. 11h 5m. A faint belt was visible on each side of the trace of the ring.

September 3. 13h 0m. The trace on ball seemed to be thinner than usual, and did not seem very distinct. When best seen the ring was clean cut, like a bar sharply pointed at the ends. The thickness of the ring on the sky was roughly 0″·67(1).

September 8. 10h 45m. The trace of the ring on the ball was

occasionally seen. It was very narrow. The ring was bright and narrow.

September 10. $12^h\ 5^m$. The trace of the ring was very faint and narrow. The planet was occasionally well seen, but the trace on the ball could scarcely be made out.

October 1. $11^h\ 6^m$. Seeing bad. The ring was of a pale ashy colour and comparatively faint. It was not very thin, and looked like a nebulous strip.

October 4. $8^h\ 30^m$. The ring was entirely invisible, with magnifying powers of 460 and 700.

$8^h\ 40^m$. Feeble traces of the ring following the planet were suspected, but not certain. The presence of Titan near interfered. A satellite preceding the planet prevented any chance of seeing the ring on that side. There was a faint satellite (Mimas) following, which seemed to be on the ring. Seeing poor.

$9^h\ 10^m$. With an occulter in the field, in the moments of best seeing, feeble traces of the ring were visible following.

$9^h\ 25^m$. With occulter, the following part of the ring was feebly glimpsed, but it was very difficult.

At $9^h\ 45^m$ a hexagonal diaphragm was put on over the O G. This collected the stray light into six rays, leaving clearer sky between. An occulter was used in the eyepiece. With this, very feeble traces of the ring could be seen following, but not near the planet. It could also be seen feebly once in a while preceding. In moments of steadiness, the trace of the ring or shadow of the ring on the globe could be seen. But this would have been easy if the seeing had been good.

$10^h\ 0^m$. The seeing had got too bad to do anything with the planet.

$11^h\ 40^m$. The diaphragm removed, occulter on. The seeing very bad; could see nothing of the ring in the steadiest moments.

October 5. $10^h\ 0^m$. A few minutes' opportunity was offered to observe Saturn with the 40-in. The seeing was fair, and I think better than on the 4th. The ring was faintly visible with the occulter. It seemed to be decidedly brighter than on the 4th. It was very slender and faint. A satellite was close preceding the planet. The shadow of the ring on the ball was sharply marked, but the seeing was not good enough to show it black. The ring was much fainter than a faint satellite (Mimas) near its following end.

October 6. $9^h\ 25^m$. The seeing was extremely bad, with a very high wind.

$10^h\ 30^m$. Seeing very bad, but by glimpses with the occulter the ring could be seen occasionally very feebly. Even with good seeing the ring would have been faint.

$10^h\ 50^m$. In moments of steadiness, fairly good glimpses of the ring were had. It was certainly brighter than on the 4th, and perhaps somewhat brighter than on the 5th. The shadow on the ball could also be seen.

The planet was examined later, but the seeing was too bad for any observations of value.

October 8. 9^h 0^m. The ring could only be seen by occultation of the planet. It was very faint and narrow. The south belt across the planet was far more conspicuous than the north one, which was rather difficult.

Position of the shadow on the ball. 9^h 10^m:—

From South Limb.	From North Limb.
8″·92(3) [8″·08]	8″·98(3) [8″·14]

Very uncertain—the planet very faint in haze.

11^h 0^m. I could not see anything of the ring with Saturn uncovered in the field, but with occulter I could see it at both sides of the planet very easily, though it was faint. It was straight and very narrow, with no irregularities. A faint satellite at the following side interfered. The ring was easier than at the previous observations since its disappearance. The south belt was very diffused to the south, while the north belt was diffused to the north. The equatorial space between these two belts was the brightest region of the planet.

11^h 25^m. Thickness of shadow on ball was 0″·69(2). Distance of south belt from shadow = 2″·20(2). A satellite following half way to end of ring was very much brighter than the ring.

October 13. 8^h 25^m. The ring could be seen with Saturn uncovered in the field. It was thin and rather faint. A satellite following and another preceding.

Micrometer positions on the ball:—

South belt from south limb	6″·77(2)	South belt from north limb	10″·94(2)
Shadow from south limb	8·81(2)	Shadow from north limb	8·25(2)
North belt from south limb	10·64(2)	North belt from north limb	7·30(2)

There was a faint broad shading parallel to the equator in the northern hemisphere. The southern edge of this was 13″·9(2) from the S. limb, and 3″·8(2) from the N. limb. The seeing was very bad. This last marking was a very faint and diffused belt. Though the ring was faint, it could be distinctly seen with 460 diameters, with Saturn uncovered in the field, while with the occulter it was seen quite easily on both sides.

11^h 0^m. The ring was easily visible without occultation of the planet. it was very thin, but by occultation it was quite conspicuous. There were two regions of greater brightness on both the preceding and following ansa [the condensations]. The seeing was good.

Thickness of the shadow 0″·28(2). The equatorial region was brighter than any other part of the planet. The north belt was fainter than the south one.

October 28. 6^h 30^m. The ring was distinctly visible. The two luminous spots were quite conspicuous on each ansa.

November 3. 6^h 30^m. The ring and condensations were easily seen. The outer condensations were the brighter. These were so distinct that they looked almost as definite as an ill-

defined satellite would be if projected on the ring. Measures were made at $7^h\ 10^m$.

From the following limb:—

The distant one.	The near one.
7″·88(7) [7″·34]	3″·04(6) [2″·84]

From the preceding limb:—

The distant one.	The near one.
8″·14(5) [7″·59]	2″·68(5) [2″·50]

Distance between the centres of the two following condensations:—

4″·40(4) [4″·10]

Thickness of the trace of ring on the ball:—

0″·71(4) [0″·66]

The position of the centre of the trace was measured, but the seeing was so poor that the result was uncertain.

From North Limb.	From South Limb.
9″·21(2) [8″·59]	8″·82(2) [8″·23]

$7^h\ 50^m$. The inner condensation was a little the brighter in each case. It looked as if it might be double, but the seeing was not good enough to decide. Between the two condensations the ring was almost discontinuous. From the inner condensation to the ball the ring was easily seen to join up to the ball, and was but little less bright than the adjacent condensation. The outer ones seemed to be more brightly condensed, or like very small blurred stars. The inner condensations were about as bright as the faint satellite (Tethys) preceding, and somewhat brighter than the distant ones (condensations).

Assuming that these luminous places were symmetrical with respect to the ball, and taking the means of the measures, we find the distance of the nearest condensations from the limb was

2″·86 [2″·67]

Distance of the outer condensations from limb was

8″·01 [7″·46]

November 5. Very bad sky—thick, with misty clouds.

$10^h\ 17^m$. In moments of steadiness and clearness the distant condensation in the preceding ansa looked like a small, ill-defined satellite. There was a faint satellite (Enceladus) close to it, and it was not possible from their appearance to tell which was the satellite. They were the same size, same form, and same brightness—small hazy spots. The nearer condensation was the brighter, and at times it seemed to be double. The space between it and the ball was nearly as bright as the condensation. The ring between the distant and near condensations was faint and almost discontinuous. The full extent of the ring was seen with difficulty because of the poor sky.

At $10^h\ 22^m$ the following measures were made between the preceding limb and the condensations:—

Distant condensation.	Near condensation.
8″·17(3) [7″·63]	2″·82(2) [2″·63]

At $10^h\ 31^m$ the distant condensation and the faint satellite appeared as one. On the following side the nearer condensation was decidedly brighter than the distant one, say by half a magnitude or more. The inner condensation was certainly double. Perhaps a faint satellite was close to it. The trace of the ring on the ball did not appear black, but the seeing was too poor to be sure.

November 12. $5^h\ 45^m$. The condensations were readily visible. There were two considerable satellites of equal brightness, one near the preceding and the other near the following end of the ring.

$5^h\ 50^m$. The condensations were bright and nearly equal. The inner ones were bright up to the ball. The space between them was almost discontinuous.

$6^h\ 30^m$. Seeing very poor. At this time there was a faint satellite (Mimas) between the bright satellite on the following side and the end of the ring. The outer condensations seemed to be perhaps a little brighter than the inner ones, and possibly a little larger. They were all very conspicuous. The outer one, following, was at least two times as bright as Mimas. The small satellite itself (Mimas) was about the same brightness as the ring between the two condensations. There was a faint satellite (Enceladus) close to the preceding end of the ring, which was nearly as bright as the outer condensation on that side. It was a little north, following the condensation. Occasionally the full extent of the ring was seen up to near Mimas.

$7^h\ 0^m$. The faint satellite (Enceladus) preceding and the outer condensations were together at this time, and made quite a large spot. Distance between the two condensations on the following ansa:—

$$= 5''\cdot04(4)\ [4''\cdot76]$$

Roughly, the length of the outer condensation was—

$$2''\cdot10(4)\ [1''\cdot98]$$

It was perhaps $2\frac{1}{2}$ times as long as broad, and diffused rather abruptly on each side. I should say that the outer condensation was $\frac{1}{2}$ as bright as the bright satellite following.

$7^h\ 20^m$. The faint satellite (Mimas) following was going back at this time, and was making a double bright spot with the outer condensation. The faint satellite (Enceladus) preceding was then between the two preceding condensations and very slightly north of the line between them. Could not make out anything on the ball except the trace which was seen only once in a while, though it was conspicuous enough when seen. The inner condensation seemed to go up to the ball without much if any loss of light.

The seeing did not permit any accurate measures from the limbs, but the following results were obtained.

Distance of preceding condensation from preceding limb—

$$7''{\cdot}66(6)\,[7''{\cdot}23]$$

The limb was only a blur, from bad seeing. With the planet unobscured in the field, the condensations were quite conspicuous.

$7^h\,40^m$. At this time the faint satellite (Mimas) was between the two condensations following, but nearer the distant one.

Distance from following limb to outer condensation—

$$7''{\cdot}72(8)\,[7''{\cdot}29]$$

At the time of these measures the faint satellite (Mimas) following was close preceding the outer condensation.

The outer and the inner condensations seemed to be of the same brightness. It looked as if the preceding condensation on the preceding ansa was not quite as bright as the one on the following ansa, but the presence of the faint satellite (Mimas) near the following one may have been the cause of this. A half moon, with bad seeing, made the sky very bright.

The Condensations.

The following table contains the measures of the distances of these condensations from the preceding and following limbs of Saturn. They are reduced to the mean distance (9·5389) of Saturn from the Sun, and are therefore comparable.

Measures of the Condensations.

	Preceding.		Following.	
	Near.	Distant.	Near.	Distant.
	"	"	"	"
1907 July 2	2·85	7·56	...	...
Nov. 3	2·50	7·59	2·84	7·34
5	...	...	2·64	7·63
12	...	7·23	...	7·29
Mean	2·68	7·46	2·74	7·42

It is evident that the condensations are symmetrical with respect to the centre of Saturn. The individual differences are not large when we take account of the indefinite character of the phenomenon.

I have at various times compared the brightness of these condensations with that of Mimas, Enceladus, and Tethys. The following approximate values of the magnitudes of these satellites reduced to the mean distance of Saturn have been supplied me

by Mr. Parkhurst from his unreduced measures of the magnitudes of the satellites of Saturn :—

Mimas . . .	12·3 magnitude.
Enceladus . .	11·7 ,,
Tethys . . .	10·2 ,,

Taking the positions of the condensations from the measures, and comparing them with the dimensions of the ring system (from my own measures in *M.N.*, lvi. p. 171), it is seen at once (Plate 10, fig. 2) that the outer condensations are located on the outer part of the inner bright ring, in a region that is the brightest of the entire Saturnian system. The inner ones apparently fall on the crape ring. These condensations, therefore, would seem to be on the brightest and the faintest parts of the system. As this light is unquestionably transmitted through the material of the rings, it is evident that the above facts are opposed to each other if we assume that the condensations are caused by the crape ring and the outer part of the inner bright ring, or the thinnest and densest portions of the ring system. It is improbable that the two similar phenomena are produced by exactly opposite conditions. In the crape ring there are so few particles that the sunlight would readily pass through with little or no scattering effect or augmentation by additional reflection. It is known that a narrow portion of the inner bright ring is very much brighter than any other part of the rings or ball. The explanation of this is doubtless due to a denser collection in this zone of the small bodies that form the rings—just as the scarcity of them produces the duskiness of the crape ring for the want of material to reflect light. It is probable, therefore, if they are not too densely crowded to let the sunlight shine through among them, that when seen from the under side they will appear brighter than the other portions of the ring by repeatedly reflecting and scattering the transmitted light. Looking at the diagram, it would seem that the outer condensations are due to the inner bright ring alone. It would appear most probable that the outer bright part of the inner bright ring is responsible for both condensations. On this supposition, the inner condensations are due to the two brighter portions of the inner bright ring seen in perspective. Just why there should be a loss of illumination between the condensations is not quite clear at present, unless it is that the greater depth of the rings in perspective in some way militates against either the transmission or reflection—or both—of the solar rays in that direction.

In the description, I have used the word 'condensation' for these luminous appearances, although I believe it to be misleading. Though these bright places look decidedly broader than the thin trace of the ring on the sky, and appear to be condensations *on* the ring, I think it is simply a matter of contrast or irradiation, and that they are not any broader in reality than the trace of the ring where they occur. I have come to this conclusion from the drawings that I have made, and which are communicated with this

paper, where the ring has simply been darkened between the points of greater brightness without changing its original outline. The result is that, when looked at from a distance, the bright places appear broader than the ring, and look like the 'condensations' as seen in the sky. I think, if the definition were perfect, it would be at once evident that this explanation is the true one. That they are not material conden-ations is shown by the fact that they entirely disappear when the ring is edge on toward us, at which time they should be most conspicuous if they were masses on the ring.

There is one fact that seems to have been brought out strongly at this disappearance of the rings. The sunlight can sift through them just as it does in the case of the crape ring, only to a very much less degree. The rings were visible when there was no direct sunlight shining upon the surface presented to us—the Sun being on the opposite side to that of the Earth. This phenomenon might be explained by either of two suppositions: that the rings are self-luminous, or that the sunlight sifts through among the particles composing them and thus makes them visible. Inasmuch as the rings have been proved to consist of discrete particles, their temperature cannot be high, and they are therefore not likely to be self-luminous. The second supposition seems to be a simple and sufficient explanation. This is not improbable, for the rings are extremely thin, as no trace of the illuminated edge could be seen at any time while they were supposed to be invisible. Notwithstanding the fact that the sunlight may thus penetrate entirely through even the bright rings, it does not do so with sufficient intensity to illuminate a satellite when in their shadow; for although Japetus was visible at the eclipse of November 1, 1889 (*M.N.*, vol. l. pp. 107–110), through the entire shadow of the crape ring, it was wholly invisible in a 12-in. telescope when it was in the shadow of the bright rings. In fact, though the rings are translucent, they are not transparent.

I have excluded the idea that the illumination is due to reflection of the sunlight from the ball of Saturn, for the further part of the ring was visible—the entire surface—which could not be so by reflection from Saturn.

P.S.—1908 *March* 3.—In looking over my observations of 1907 July 2, I find a note which has a bearing on Professor Aitken's measures of two condensations at the place of the inner ones. The note, which is attached to the sketch of the inner condensation preceding, says: "Possibly here there are two condensations." It will be seen, in my later notes, I also suspect the inner condensation of being double; but the seeing was never so good that I would be sure enough of it to make measures.

Previous Disappearances of the Rings.

I have gathered the following facts from the American Ephemeris and elsewhere about some previous disappearances of the rings:—

In 1848 the Earth passed the plane of the ring in April, going south. On September 3 the Sun passed the plane, going south. On September 13 the Earth passed back north, and on January 19, 1849, the Earth once more went south with the Sun. Therefore between April 12 or 13 and September 3 the Earth and Sun were on opposite sides of the ring. This was again the case between September 13 and January 19.

1861 November 22. The ring disappeared by the Earth passing through its plane to the north—the Sun being some $2\frac{1}{2}^\circ$ south.

In 1862 it reappeared on January 31, the Earth passing through the plane of the ring to the south. On May 17 it disappeared by the Sun passing through the plane to the north. It reappeared on August 12, the Earth passing to the north.

1878. The ring disappeared on February 6, the Sun passing to the south. It reappeared March 1 by the Earth passing to the south.

In 1891 the ring disappeared, September 22, by the Earth passing to the north. It reappeared, October 30, by the Sun passing to the north.

1892, about May 20, the Earth was less than $\frac{1}{2}^\circ$ north of the plane of the rings, but it did not come any nearer, and the ring did not disappear.

1907. The American Ephemeris does not give the times of disappearance and reappearance, but one can deduce them from the "Apparent Elements of Saturn's Ring," p. 515. I have already given Professor Struve values of these quantities for the present apparition of the planet. For completeness of this list I will repeat them here.

1907 April 17. The ring disappeared by the Earth going south.

July 26. It reappeared by the Sun passing south.

October 4. The ring disappeared by the Earth passing north.

1908 January 7. It reappears, the Earth passing south.

The following table, containing the mean times of meridian transit and the declinations of Saturn at the critical times for the phenomena of the disappearance and reappearance of the ring, may be of interest in connection with the present and for comparison in future disappearances of the ring. The last column indicates the cause of the disappearance or reappearance.

Date.	M.T. of Transit.	Declination.	With reference to Plane of Ring.
	h m	° ′	
1848 Apr. 22	21 24	−5 17	⊕ going S.
Sept. 3	12 44	−5 5	☉ ,, S.
,, 12-13*	12 6	−5 22	⊕ ,, N.
1849 Jan. 19	3 36	−5 20	⊕ ,, S.
1861 Nov. 22	19 20	+5 15	⊕ ,, N.
1862 Jan. 31	14 48	+5 15	⊕ ,, S.

* The meridian passage and declination are given for the 12th.

Date.	M.T. of Transit. h m	Declination. ° ′	With reference to Plane of Ring.
1862 May 17	7 30	+7 38	⊙ ,, N.
Aug. 12	2 8	+5 17	⊕ ,, N.
1878 Feb. 6	2 15	−6 14	⊙ ,, S.
Mar. 1	0 54	−5 9	⊕ ,, S.
1891 Sept. 22	23 24	+5 6	⊕ ,, N.
Oct. 30	21 11	+3 25	⊙ ,, N.
1907 April 17	21 54	−4 48	⊕ ,, S.
July 26	15 40	−3 12	⊙ ,, S.
Oct. 4	10 47	−5 6	⊕ ,, N.
1908 Jan. 7	4 30	−5 3	⊕ ,, S.

From an inspection of this table it will be seen that the present disappearance of Saturn's ring has been an extremely favourable one—the most favourable in forty-five years—comparing in this respect with the disappearances of 1848–9.

In 1848–9 and 1861–2 there were two disappearances and two reappearances each of the ring. In 1878 there was only one disappearance, and the corresponding reappearance, both very unfavourably situated. The ring was invisible for about twenty-two days.

In 1891 there was only one disappearance and reappearance. The conditions were likewise very unfavourable. The ring was invisible some thirty-eight days.

At the present apparition of the planet there were two disappearances and two reappearances.

There is an important series of observations of Saturn made with the 15-inch refractor of the Harvard College Observatory by the Bonds at the disappearance of the ring in 1848. These observations form part of volume ii. of the *Harvard College Observatory Annals*.

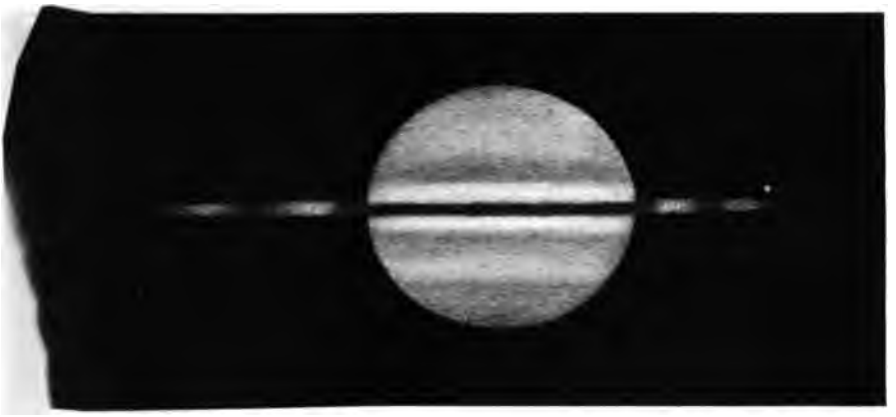
Some of the drawings of Saturn given in the above volume show the ring and condensations essentially as they have appeared here in the past six months.

The conditions of the disappearances and reappearances of the ring in 1848 were almost identical with those we have just witnessed.

In all his observations Bond assumed that what he saw was the sunlit edge of the ring. The explanation of its visibility was therefore an easy one. The observations of the present disappearances have shown, however, that it was not the edge which we saw, but the very oblique unilluminated surface of the ring. The minor axis of the ring about the 1st of July was some 2″.

Bond's explanation of the bright markings, condensations, or knots seen on the ring during its "invisibility" was very ingenious, but it depended upon the edge of the ring being seen through the Cassini division. The extreme thinness of the ring and the largeness of the condensations make this explanation unsatisfactory.

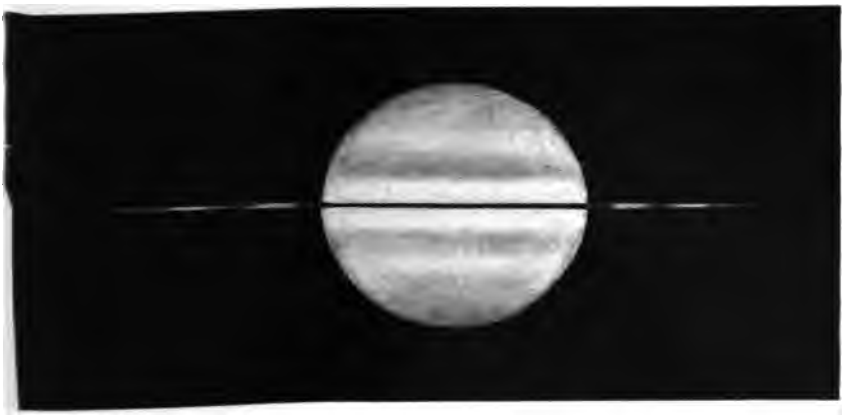
S



N

1907 JULY 2, 15 H. 30 M.

S



N

1907 OCT. 13, 11 H. 30 M.

S

N

1907 NOV. 3, 7 H. 50 M.

In *Monthly Notices* for November 14, 1862, vol. xxiii. pp. 87–88, there are some observations of Saturn at the time of the disappearance of the ring in that year made at Greenwich and Pulkowa. In both these there are distinct references to luminous appendages, which seem roughly to be the same seen at the observations of this year—1907. The descriptions are not quite in satisfactory accord with their appearance as seen to-day. The ring was supposed to have disappeared on May 17, 1862, by the Sun going north, and to have remained invisible until August 12. At Pulkowa the descriptions seem to indicate only two appendages, one on the preceding, the other on the following ansa. They did not seem to have been perfectly symmetrical.

The Pulkowa observations are :—

"1862 May 20. The aspect of the luminous appendages has not varied since last night, with this difference only—that I estimated the length of the preceding side as 0·65 and the following as 0·5 of the planet's diameter.

"1862 May 21. Images good. Estimated extent of luminous appendages :—Preceding side 0·6, following side 0·4, of the planet's diameter; also the intensity of the light appears much more feeble on the following side. The size of these appendages increased in the neighbourhood of the planet, giving them the form of sharp wedges.

"1862 May 22. Images less favourable than yesterday. Extent of luminous appendages on preceding side 0·6 and on following side 0·5 of the planet's diameter.

"1862 June 3. Image very bad, yet the luminous appendages are still distinctly visible. It appears that the length of the following ansa is a little the greater, but this is not certain."

The Greenwich observations are :—

"1862 May 17. The ring beyond the planet at times just visible on the left of the disk, but on the right only a small faint spot could be seen in the plane of the ring, about $\frac{1}{3}$ the diameter of the disk from the planet's limb. I should have suspected this was a satellite but for its elongated shape. Sky very hazy.

"1862 May 19. Saturn very well seen. The ring distinctly visible on the left of the disk, and fairly but not so distinctly visible on the right. Where it crosses the disk, the under edge is much sharper and better defined than the upper edge.

"1862 May 20. Saturn very well seen; the ring distinctly visible, appearing brighter than it did last night. The Greenwich observations were made by Mr. Carpenter."

There are also some observations by Mr. Wray in the same number of the *Monthly Notices*, in which he saw something of the kind in December of 1861 and January of 1862 with a 7-in. refractor.

Yerkes Observatory,
Williams Bay, Wis.:
1907 *November* 25.

Additional Observations of the Disappearances and Reappearances of the Rings of Saturn in 1907–8, *made with the* 40-*inch Refractor of the Yerkes Observatory.* By E. E. Barnard.

1907 Nov. 23—with 12-inch telescope, $6^h\ 15^m$, power 150±. I could see the two condensations on the preceding part of the ring. They were fairly distinct, but faint. Could scarcely see them on the following side.

$7^h\ 30^m$—using a higher power. The ring and condensations were not so well seen. A satellite closing in following, mixed its light with the condensations on that side. With the low power could see the two condensations on the preceding ansa very distinctly. They appeared equal in brightness. The inner one joined up to the planet with but little change of brightness. There was no doubt but that the following ansa and condensations were fainter than the preceding. The satellite interfered, however. The sky was clear and good, but the image was unsteady. The trace on the ball was apparently dark and without irregularities.

Nov. 24. $5^h\ 11^m$. The condensations were easy. There were two satellites, one preceding and one following, that interfered. The condensations were of equal brightness, both preceding and following. The inner one preceding was bright to the ball. The entire ring was visible with or without the occulter. Seeing = 2 (on a scale of 5). Lost in clouds.

$8^h\ 46^m$. Thinning down near Saturn. The following ansa was certainly the brighter. There was a faint satellite (Mimas) following the following end of the ring. The outer condensation near it was twice as bright as the satellite. Seeing = 1. The full extent of the ring was quite conspicuous when best seen.

$9^h\ 8^m$—with the 12-inch telescope. Could see the condensations. Both Mr. Sullivan and I decided that the following side was the brighter. Seeing poor with 12-inch. (Back to 40-inch.) There was no question but that the following ansa was the brighter. The ring between the condensations was faint. The outer condensation following was two times as bright as the faint satellite following (Mimas). In each case the outer and inner condensations were of equal brightness. Seeing very bad.

Nov. 25. $4^h\ 55^m$. The condensations were of equal brightness. The space between them was almost discontinuous.

$5^h\ 25^m$. Distance between the centres of the preceding condensations—

$$4''{\cdot}60\ (6)\ [4''{\cdot}44].$$

Distance between the centres of the following condensations—

$$4''{\cdot}51\ (6)\ [4''{\cdot}35].$$

Seeing very poor. Went to the 12-inch telescope. Could faintly see the rings and condensations—very faint compared with the view in the 40-inch.

Nov. 26. $5^h\ 55^m$. Seeing fair for moments. The condensations

were very bright, but the ring between them was very faint, and all but discontinuous.

$6^h\ 25^m$. The small satellite following was the same brightness as the inner condensation, but very slightly brighter than the outer one. The following ansa was considerably brighter than the preceding. I could see the ring beyond the outer condensation following,—I think, all of it, but it was extremely faint. The condensations were about the thickness of the diameter of the bright satellite following. The inner condensations were a little brighter at their outer ends. Seeing = 3.

$6^h\ 45^m$. The bright satellite was half a magnitude brighter than the outer condensation.

$6^h\ 50^m$. The outer condensation was exactly midway between the satellite and the following limb. The following ansa and condensations were certainly brighter than the preceding. They were all conspicuous with the planet unobscured. When seen best the condensations appeared to become narrower.

Nov. 28. $5^h\ 4^m$. Seeing very poor. The ring and condensations were conspicuous.

$5^h\ 24^m$. There was a small satellite (Tethys) just north of the ring, and about midway between the condensations following. I think the following ansa was the brighter.

$6^h\ 49^m$. There was a small satellite between the two condensations preceding, nearer the outer one. There was a bright satellite close preceding the end of the ring in line with it. There was a similar satellite at the preceding end of the ring a little north. These satellites were half a magnitude brighter than the condensations. I think the inner condensation following was slightly brighter than the outer one.

$6^h\ 59^m$. Seeing very poor. There was a very faint satellite (Enceladus?) then visible at the preceding end of the ring, close south following the brighter satellite. The seeing was very poor. I could not make out any details on the ball.

$8^h\ 54^m$. The seeing better, though the planet was low. The following ansa was, I think, the brighter. The inner and outer condensations were of the same brightness. The inner ones joined up to the ball with almost full brightness. The space between them looked to be a little less than on some previous nights.

Dec. 3. $7^h\ 42^m$. The condensations were quite bright.

Distance from preceding condensation to following limb—

$$27''{\cdot}25\ (8)\ [26{\cdot}63].$$

From preceding limb to outer condensation—

$$7''{\cdot}55\ (8)\ [7{\cdot}38].$$

The inner condensation was bright up to the planet. Could not make much out of it on account of the blurring. I don't think there had been any change in the condensations. They were brighter than the small satellite (Tethys?) preceding.

Dec. 5. $5^h\ 46^m$. The condensations were bright in moments of steadiness. The following ansa was the brighter.

$6^h\ 11^m$. I went over to the 12-inch telescope. The seeing was very poor. I could faintly see the following ansa, but could get only feeble traces of part of the preceding ansa. I could not be certain of seeing the condensations. Used low and high powers. The trace of the ring on the ball, or the shadow, was fairly well seen.

Back to 40-inch. The ansæ were conspicuous without occulter, as were also the condensations, though the seeing was very bad.

$6^h\ 21^m$. In moments of steadiness could see the ring continuous between the two condensations.

Dec. 10. $7^h\ 50^m$. Through clouds, seeing poor. The ring and condensations were easy. A small satellite was $3''$–$4''$ following the following end of the ring. The outer condensation was brighter than this satellite.

Dec. 12. $4^h\ 38^m$. The definition was fair in moments. The ring and condensations were quite easily seen.

$4^h\ 58^m$. Seeing = 3. The ring was easily continuous between the condensations. Could see the ring beyond the outer condensations as a slender thread of light.

Distance from preceding limb to outer condensation,	$7''\cdot78$ (6) [$7''\cdot73$]
„ following „ „ „	$7''\cdot84$ (6) [$7''\cdot79$]
Length of outer condensation following,	$2''\cdot30$ (7) [$2''\cdot28$]
Width of space between condensations following,	$2''\cdot32$ (7) [$2''\cdot30$]

The ring and condensations following seemed to be brighter than those preceding. Sometimes the central part of the outer condensation, following, looked like an ill-defined satellite. A sketch showed it to be 2 to 3 times as long as it was broad. The inner condensation following was continuous up to the ball, but there was a brighter place at its outer end. The outer and inner condensations seemed to be of equal brightness. The ring between them was 6 or 8 times as faint as the condensation and looked much thinner. I could not see anything of the shadow of the ball.

$5^h\ 45^m$. The seeing has been 3 all along. The thickness of the outer condensation following was $\frac{1}{3}$ or $\frac{1}{4}$ its length. A sketch shows that close up to the ball the following inner condensation was quite faint for a second or so.

$7^h\ 6^m$. Dione was in conjunction close north with the outer condensation following. A small satellite (Enceladus?) following the following end of the ring was several times fainter than the condensations.

Dec. 25. $4^h\ 48^m$. The condensations, and the ring in general, were easily visible, but they were much thinner than at the last observation. Could see the full length of the ring. It was very narrow and faint at the ends beyond the outer condensation. The ring on both sides could be seen between the two condensations, though faint.

$5^h\ 8^m$. The inner condensation was bright nearly up to the ball, but, perhaps by contrast, it was faint near the ball. Seeing = 3. The trace of the ring on the sky was about on a line with the south edge of the shadow on the ball. The seeing was not quite good enough to tell if the shadow of the ring was irregular. It appeared clear and straight in moments of best seeing. A sketch says that the following inner condensation was not continuous close to the ball.

Estimated with the micrometer wire. The thickness of the condensations, or the ring at these points, was about $1\frac{1}{2}$ times the thickness of the wire, while the outer slender end of the ring was perhaps $\frac{1}{2}$ or $\frac{1}{3}$ the thickness of the wire [thickness of the wire $= 0''{\cdot}1$].

$5^h\ 38^m$. A small satellite was at this time exactly midway between the condensations. It was about one diameter of itself south of the line of the ring. The ring at the following side was not continuous up to the ball, but there was a faint place from the ball out for, say, $1''$. Question if this was the shadow of the ball on the ring. I could not be certain of this effect at the preceding side.

1908 Jan. 2. $5^h\ 50^m$. The ring was very thin. There was a satellite at each end. The one following was very close to the ring. There was a very faint satellite close north of the ring, half way out, following. Seeing = 3. Without occultation it was almost impossible to see any trace of the ring on the sky. The condensations were feebly seen as slightly brighter parts of the ring. The ring and condensations were slightly brighter than the faint satellite, but were much less bright than the satellite at the following end of the ring. Same at $6^h\ 30^m$.

Jan. 5. $5^h\ 0^m$. The sky good. Seeing = 2. Not dark enough to see the ring.

$5^h\ 10^m$. Could see the ring very faintly with the occulter.

$5^h\ 20^m$. The sky was not entirely dark, yet could see the ring faintly, but not well.

$5^h\ 40^m$. The ring was very faint and quite thin. I think there was a feeble intensification of the ring at the places of the condensations; that is, it was possibly very slightly brighter at those points, but not thicker. The shadow of the ring on the ball was black and strong. The ring was much more difficult than on Jan. 2. I could not see anything of it without the occulter.

Jan. 6. $5^h\ 0^m$. The sky too bright to see the ring. Seeing very poor.

$6^h\ 0^m$. A close watch was kept until this time with occulter. The seeing had steadied a little. During the best moments could not see anything of the ring. Seeing at best = 2. There was considerable glow from want of steadiness. Watched at both sides of the planet. It is possible that with better seeing I might have seen it, but it is doubtful. It must have been excessively faint.

Then went to the 12-inch telescope, but the seeing was poor. Tried high and low powers, but could not see anything of the ring.

Jan. 7. $5^h\ 15^m$. Seeing = 2. The ring was quite easily visible by occultation. It was linear, but rather faint on the following side. On the preceding side could see it well. There was a bright satellite at the preceding end of the ring. The ring must have been much brighter than on the 6th, for I could see it when the seeing was worse than it was at times on that date —when I could not see it at all. It looked hazy and ill-defined. There were no traces of condensations. It was a straight bar, its full extent perfectly uniformly illuminated clear to the end. It was of an ashy color, rather thickish and fuzzy.

$5^h\ 30^m$. There was a faint satellite 6″ or 8″ preceding the preceding end of the ring. It was just as bright as the ring. The sky hazed over. Went to the 12-inch telescope with Mr. Fox. He got a glimpse of the planet in a clear place between the clouds, and saw the ring. Thick sky prevented my seeing the ring with the 12-inch. Moonlight made the hazy sky too bright.

Jan. 8. $6^h\ 15^m$—with 12-inch telescope. The planet seen through break in clouds. Could see the ring faintly both following and preceding. Seeing poor—haze and clouds.

From these observations it would appear that the Earth must have passed the plane of the ring sometime between 1908 January, 5 days $6^h\ 0^m$ and January 7 days $5^h\ 15^m$. Perhaps this occurred not far from the time of the observation of January 6.

The observation of January 7 would seem to indicate that the plane had already been passed some hours before, from the distinctness with which the ring could be seen at that time.

The observation of January the 6th, though not under the best conditions, again showed how thin the ring must really be, for it was not visible with the light of the planet occulted.

I think, however, a more effective criterion of its thinness was offered at the various times when the ring was observed with the Sun and Earth on opposite sides of it. At such times, though the very oblique surface of the ring was visible, nothing could be seen of the edge of the ring itself, which should have been seen as a thin rim of light. This sunlit edge of the ring was not visible at any time, though it was always looked for carefully when the seeing was best.

As it may be important for the aid of others in any investigation of the phenomena presented by the ring in 1907, I may be permitted to copy here my previous measures of the Saturnian system; to these I will add the measures of the condensations. They are all reduced to the mean distance of Saturn from the Sun = 9·5389. (See *M.N.*, vol. lvi. p. 171.)

Table of Measures of the System of Saturn.

Equatorial diameter of Saturn . .	17″·800	Radius	8″·900
Outer diameter of outer ring . .	40·108	,,	20·054
Inner diameter of outer ring . .	35·046	,,	17·523
Centre of Cassini division . .	34·517	,,	17·258
Outer diameter of inner ring . .	33·988	,,	16·994
Inner diameter of inner ring . .	25·647	,,	12·823
Inner diameter of crape ring . .	20·528	,,	10·264
Width of Cassini division . .	0·529		

Inner condensations from limb . . .	2″·706	{ from centre of planet }	11″·606
Outer edge of inner condensations from limb	...	,,	12·900
Inner edge of outer condensations ,,	...	,,	15·200
Centre of outer condensations . ,,	7·438	,,	16·338
Outer edge of outer condensations ,,	...	,,	17·480

The polar diameter of Saturn was 16″·241.

The measures of December 12 made the length of the outer condensations = [2″·28], and the distance between their nearest edges = [2″·30]. From these and the tabulated position of the centre of the outer condensations we have the values of the positions of the ends of the condensations as given in the table. The observations show that the condensations on the preceding and following ansæ were perfectly symmetrical with respect to the centre of Saturn.

If now we compare the full extent of the outer condensations with the Cassini division we have—

Outer edge of outer condensation from centre .	17″·48
Outer edge of the Cassini division . . .	17·52

Likewise we have for the inner condensations—

Outer edge of the inner condensation . . .	12″·90
Outer edge of the crape ring	12·82

These would seem to connect the condensations directly with the Cassini division and with the crape ring.

They are opposed to the supposition that the bright part of the inner bright ring is responsible for the outer condensations, for they show that the outer condensations extended beyond this ring.

I believe it has been shown that the Cassini division must be free of particles, because of the disturbing action of certain of the satellites. If this were not so, and the Cassini division were filled with particles as closely clustered as they are in the crape ring, a satisfactory explanation of the condensations would be that they

were simply due to the sunlight shining through and illuminating the particles in the crape ring for the inner condensations, and a similar effect of the Sun shining through the Cassini division and illuminating the particles in it would produce the outer condensations.

The fact that the inner and outer condensations were essentially of the same intensity would require that the particles should be as closely clustered in the Cassini division as they are in the crape ring.

With the assumption that particles do exist in the Cassini division, the above explanation of the condensations would be satisfactory.

In any attempt to connect the inner condensations with the crape ring, it may be important to state that these condensations have always appeared much brighter than the crape ring has ever appeared to me—even allowing for that ring being ordinarily seen between two bright regions, the inner bright ring and the ball of Saturn.

In conclusion, I am greatly indebted to Professor Frost for the extra time with the 40-inch to observe the phenomena of the disappearances and the reappearances of the ring of Saturn, and for the kind interest he has shown in the work.

Yerkes Observatory,
Williams Bay, Wis.:
1908 *January* 10.

A few Observations of the Planet Saturn and his Rings in the Years 1897–1904. By E. E. Barnard. (Plate 11.)

In connection with the papers on the disappearance of the ring of Saturn, I have thought it might be interesting to include in a separate short paper some other notes on the planet.

I have a number of observations of Saturn made here in the past ten years. Some of these are perhaps worthy of record, as they bear on the appearance of the planet with the large telescope when the rings were wide open, and I have collected a few of them for this paper.

1897 May 24. The north polar cap of Saturn was of a bluish dark colour—not well seen. It was bordered by a light belt.

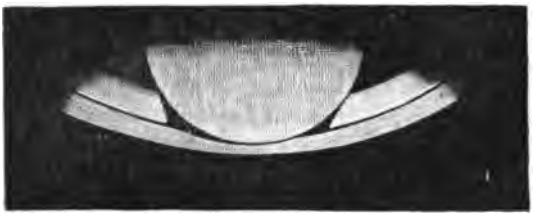
1897 July 1. (With the 12-in. telescope.) The rings were opened so that their outer edges seemed to be exactly coincident with the polar limb of Saturn. It appeared perfectly so on this date, and also on June 29. The shadow of the ball on the rings seemed to be "squarey" where it struck the Cassini division. The polar cap was dark grey and not large. It was bordered by a light space, then a delicate dark narrow belt. There was no light equatorial zone.

I think this angular or "squarey" form of the shadow of the ball on the ring was simply caused by the shadow falling on the

Cassini division and producing thus a kind of "black drop" effect by the junction of the black shadow with the black space.

1898 February 26. The shadow of the ball on the rings was not a uniform curve, but of an angular outline, as shown in the

South.



Form of the shadow of the ball on the Ring of Saturn, 1898 February 26.

sketch. The north pole was dark—not black. The ball was clearly seen through the crape ring.

1898 March 6. The shadow of the ball on the rings preceding seemed "squarey." The north pole was dark.

1898 April 20. There was no definite dark polar cap such as was seen last year, but the polar regions were dark. The planet was beautifully seen through the crape ring. The shadow of the ball on the rings looked "squarey."

1898 July 7. The trace of the crape ring was hazy at both edges, or rather the inner edge of the bright ring was ill defined where it crossed the ball, but it was well defined on the sky between the ring and the ball. The crape ring was easily seen and well defined on the sky, but faint where it approached the ball. The two northern belts were more in contrast, or darker near the limbs of the planet. The inner bright ring was brightest toward the Cassini division, where the brightness was rather narrow. I could not see with certainty any division in the outer ring. There seemed to be a dusky shading where the Encke division is usually shown. The polar cap was darker than the darkest part of the ball.

The definition was superb. I have never seen the planet better, nor have I seen so much detail upon it before. The belts, as shown on the drawing (Plate 11), were seen with certainty. The ball was easily visible through the crape ring.

1899 April 7. $16\frac{1}{2}^{h}$. A heavy diffused belt at the equator. The north pole seemed dark. Seeing poor.

1899 April 25. The north pole was not very dark. The black, well-defined cap was not present. It was a little dusky at the pole.

1901 October 21. $17^{h}\ 50^{m}$. There was no polar cap. There seemed to be a luminous appearance near the north pole, at a point inside the following limb near the shadow; seeing fair, but planet very low.

1904 June 17. There was a heavy, diffused, broad dark belt north of the equator.

1904 July 2. The north belt was very heavy and diffused.

1904 July 4. The north belt was heavy and broad.

1904 July 11. The north polar region was of a light yellow colour.

1904 August 27. The north polar region was light-coloured.

1904 December 5. Light at the equator, with broad dusky region toward the north, but all the north region was lighter.

I have purposely withheld (for another paper) the observations of Saturn in 1903, at the time of the appearance of the white spot on the ball of the planet.

Unless otherwise stated, all the observations in these papers were made with the 40-inch telescope.

Yerkes Observatory,
Williams Bay, Wis.:
1908 *January.*

Photometric Measurements of Saturn, August to December 1907.
By J. M. Baldwin, M.A., 1851 Exhibition Scholar (Melbourne).

(*Communicated by Sir David Gill.*)

During the recent opposition of Saturn, at the suggestion of Professor Müller, I undertook a series of photometric measures of that planet. This opposition has been of particular interest, for, owing to the small elevation of the Earth above the plane of the ring, the reduction to "ring invisible" is subject to very slight uncertainty, and so the variation in the brightness of the spheroid itself with phase angle can be obtained.

Seeliger* has theoretically arrived at an expression for the magnitude of Saturn, in which the magnitude depends on two quantities, the first of which, expressing the light received from the ring, shows a marked variation with phase, while the second, expressing the light from the spheroid, is almost independent of phase. Müller,† on the other hand, has arrived at empirical formulæ for the magnitude, in which the variation in magnitude is proportional to the change in phase. In discussing his observations and those of Zöllner, he remarks (*l.c.*, p. 343), "es scheint also, als ob auch bei gänzlich verschwundenem Ring ein Einfluss der Phase sich geltend mache"; and Pannekoek‡ has further emphasised this point, showing that the observed change of brightness with phase when the elevation of the Earth above

* H. Seeliger, *Abhandl. der Bayer. Akad. der Wissensch.*, Bd. 16, 403, 1888.

† G. Müller, *Publ. des Astrophys. Observ. zu Potsdam* 8, 339 u. 341, 1893.

‡ Ant. Pannekoek, *Ast. Nach.* 4006, Bd. 167, 363, 1905.

the plane of the ring was small are greater than accounted for by Seeliger's theory, and so rendering it probable that the spheroid itself shows changes in brightness with the phase. The object of this series of measurements was therefore to determine which of the two formulæ agreed best with observation.

In order to avoid bias, I was careful not to find out the magnitudes of the variation in phase given by the formulæ; and further, the observations were not reduced at all until after the whole series was completed.

The instrument used throughout was the Zöllner photometer referred to in the Potsdam publications as C III, the objective of which is of 21·5 mm. aperture and 137 mm. focal length. (For description see *Publ. Astr. Obs. zu Potsdam*, Bd. viii., p 17, 1891.) In this instrument the image of the planet differs but little from that of a star. Unfortunately there was no suitable comparison star near Saturn; α Aquilæ, distant some 60°, was chosen as the most convenient and was used throughout. Its magnitude has been taken as 0·96 (*l.c.*, p. 235), so that Saturn should be referred to the same system as former planetary observations here have been. For the extinction correction the mean values for Potsdam have been used.

A typical set of observations and the method of reduction is shown in Table I.

TABLE I.

1907 Sept. 24.	Potsdam Sid. Time.	Quadrant. I.	II.	III.	IV.	Mean I.	Log $\sin^2$ I.	Zenith Extinction. $\frac{d \log}{\sin^2 I}$.	Dist.	Corr.	ΔM.
	h m	°	°	°	°	°			°		
Saturn	23 12	29·1	24·9	29·4	30·5	28·47	9·3566	+111	57·5	+199	+·08
α Aquilæ	17	31·9	26·0	28·7	28·9	28·87	9·3677	...	61·0	...	...
α Aquilæ	19	29·4	24·1	32·8	26·3	28·15	9·3475	...	61·3	...	...
Saturn	22	28·0	26·9	27·3	30·0	28·05	9·3446	+ 29	57·3	+229	+·06
Saturn	25	30·9	27·0	32.8	27·6	29·57	9·3867	−428	57·3	+324	−·03
α Aquilæ	29	27·1	26·1	30·9	28·0	28·02	9·3439	...	62·7	...	...
α Aquilæ	33	30·4	26·1	27·3	27·2	27·75	9·3360	...	63·3	...	...
Saturn	36	32·2	26·1	29·0	29·0	29·07	9·3732	−372	57·2	+372	·00

In a few cases, however, only eight readings for each were taken, and occasionally α Aquilæ was observed first.

The results of the observations are given in Table II.

TABLE II.

	G.M.T.				Z_β.	Z_α.	ΔM.	Corr.	M_0.	a.	A.	A′.	M_M.	O—C.	M_β.	O—C.
		d	h	m	°	°				°	°	°				
1	Aug.	11	10	55	66·0	46·7	+0·22	−·04	1·14	3·77	−1·82	−0·23	0·96	+·18	0·93	+·21
2		14	11	4	63·8	48·1	+0·08	·03	1·01	3·52	1·74	0·27	0·95	+·06	0·93	+·08
3		16	10	30	66·7	46·3	+0·37	·03	1·30	3·34	1·69	0·30	0·95	+·35	0·93	+·37
4		20	10	45	63·4	48·7	+0·08	·02	1·02	2·97	1·56	0·36	0·94	+·08	0·93	+·09
5	Sept.	4	10	30	59·9	53·1	+0·12	·01	1·07	1·46	1·05	0·59	0·89	+·18	0·92	+·15
6	Sept.	10	11	2	57·3	59·9	+0·13	−·01	1·08	0·83	−0·83	−0·68	0·88	+·20	0·92	+·16
7		11	10	29	58·4	56·1	+0·06	·01	1·01	0·73	0·79	0·69	0·87	+·14	0·92	+·09
8		12	10	20	58·6	55·6	−0·02	·01	0·93	0·63	0·76	0·71	0·87	+·06	0·92	+·01
9		13	10	56	57·2	60·8	+0·02	·01	0·97	0·53	0·72	0·72	0·87	+·10	0·92	+·05
10		18	10	31	57·3	60·1	−0·01	·01	0·94	0·28	0·53	0·80	0·86	+·08	0·92	+·02
11	Sept.	22	10	7	57·8	59·0	+0·02	−·01	0·97	0·59	−0·39	−0·86	0·89	+·08	0·92	+·05
12		24	10	21	57·3	62·1	+0·03	·01	0·98	0·80	0·32	0·89	0·90	+·08	0·92	+·06
13		25	9	52	57·9	58·6	−0·09	·01	0·86	0·90	0·28	0·90	0·90	−·04	0·92	−·06
14		27	10	5	57·5	61·4	−0·04	·01	0·91	1·11	0·21	0·93	0·92	−·01	0·93	−·02
15		28	9	2	59·8	53·6	−0·04	·01	0·91	1·21	0·17	0·95	0·92	−·01	0·93	−·02
16	Sept.	29	9	29	58·3	57·6	−0·10	−·01	0·85	1·32	−0·14	−0·96	0·93	−·08	0·93	−·08
17	Oct.	2	9	27	58·0	58·8	+0·02	·01	0·97	1·63	0·04	1·01	0·95	+·02	0·93	+·04
18		3	9	17	58·2	58·1	0·00	·01	0·95	1·74	0·00	1·02	0·95	·00	0·93	+·02
19		8	9	27	57·7	62·1	+0·13	·02	1·07	2·25	+0·16	1·10	0·97	+·10	0·93	+·14
20		11	8	51	58·2	58·8	−0·04	·02	0·90	2·55	0·25	1·14	0·98	−·08	0·92	−·02
21	Oct.	20	8	51	57·9	63·6	+0·16	−·03	1·09	3·39	+0·50	−1·27	1·00	+·09	0·92	+·17
22		24	8	48	58·1	65·5	+0·01	·03	0·94	3·74	0·59	1·33	1·01	−·07	0·92	+·02
23	Nov.	3	8	23	58·5	67·4	+0·19	·05	1·10	4·51	0·77	1·48	1·04	+·06	0·92	+·18
24		27	6	2	58·3	61·0	+0·20	·09	1·07	5·69	0·90	1·84	1·08	−·01	0·91	+·16
25	Dec.	1	4	57	59·7	54·3	+0·13	·09	1·00	5·78	0·87	1·90	1·09	−·09	0·91	+·09

1. Hazy near horizon.
2. Only eight observations of each. Wt. ½.
3. Seeing good; at times cloudy in parts. Wt. ½.
10. Only eight observations of each. Clouded over suddenly. Wt. ½.
11. Light cloud visible near moon.
14. Slightly foggy. Wt. ½.
16. Cloud rising slowly below α Aquilæ. Wt. ½.
18. Eight observations; seeing below α Aquilæ bad. Wt. ½.
21. Somewhat foggy; cloud rising below α Aquilæ. Wt. ½.
22. Seeing not good, especially below α Aquilæ. Wt. ½.
23. α Aquilæ very unsteady.

In Table II., Z_s and Z_a are the zenith distances of Saturn and α Aquilæ respectively for the mean time of observation; ΔM is the observed difference of magnitude (Saturn—α Aquilæ) corrected for extinction; the next column contains the correction to mean opposition, and M_0 is the magnitude for mean opposition; α is the phase angle; A the elevation of the Earth; A′ that of the Sun above the plane of the ring; M_M and M_s are the magnitudes calculated from the formulæ of Müller and Seeliger, in each case using Müller's values for the constants (*l.c.*, pp. 339, 341, and 348); and O—C are the values of $M_0—M_M$ and $M_0—M_s$ respectively.

On examining the column M_0 it is seen that the early observations are irregular; this is probably due to a large extent to want of practice with the instrument, these being the first observations that I had made with it, although I had been using another Zöllner photometer (photometer D) for stellar work for some months. The irregularity may also arise partly from uncertainty in the extinction correction in the first four observations, where the difference in zenith distance is so large (15 to 20 degrees). If the night were not very clear, the difference in extinction would be greater than that given by the Potsdam tables, the values of ΔM would be diminished, and the observed values would agree better with the computed. On account of this irregularity I give the results deduced

(1) from all the observations,
(2) from the observations after opposition (Nos. 10–25),

the conclusions for the latter being, in my opinion, more reliable than from the whole series.

From Table II. it appears—

(1) That if Seeliger's formula were true, the magnitude of Saturn during the observations would be almost constant, becoming very slightly brighter towards the end of the series. The observations, on the other hand, show that the brightness reduced to mean opposition shows a decided decrease after opposition, and so Seeliger's formula cannot hold. If Seeliger's second formula (*l.c.*, p. 489), which assumes a different law for emanation, be used, the results obtained are almost the same, the difference being slightly more marked.

(2) On the other hand, although the earlier observations of the series do not agree well with the results deduced from Müller's formula, yet those after opposition show a very satisfactory agreement. From the column O—C, giving the weight $\frac{1}{2}$ to certain observations as indicated in the table, the mean error is

(*a*) for all observations $\pm 0^{m}\cdot115$ mag.
(*b*) for Nos. 10–25 $\pm 0^{m}\cdot069$,,

If the reduction to "ring invisible" is applied to the mean opposition magnitude (this reduction is $+0^{m}\cdot081$ on August 11,

·000 on October 3, and + ·040 on November 27), and the results substituted in the formula

$$h = h_0 + a\alpha,$$

the values for the constants h_0 and a determined by the method of least squares are

(*a*) for all observations $h_0 = 0{\cdot}967$, $a = {\cdot}0300$, mean error of $h \pm {\cdot}095$
(*b*) Nos. 10–25 $h_0 = 0{\cdot}916$, $a = {\cdot}0313$, mean error of $h \pm {\cdot}064$
while Müller's values are $h_0 = 0{\cdot}877$, $a = {\cdot}0436$.

As before stated, the second set of values for h_0 and a are probably much more reliable than the first set.

It appears then from the observations that the spheroid of Saturn shows a change of brightness with phase, and the change in magnitude can be well represented by the formula

$$h = 0{\cdot}877 + {\cdot}0436a$$

already obtained by Müller from his long series of observations.

Astrophysical Observatory, Potsdam:
1908 Jan. 24.

Reappearance of Saturn's Ring, January 1908.

By R. T. A. Innes.

The following observations of the ghost-ring and of the reappearance of the ring were made here with the 9-inch refractor.

1907 Dec. 25. Ghost-ring seen on both sides, but more distinct on *f.* side. Shadow of ring quite its own breadth N. of equator and very black—no longer brown.

Dec. 28. Ring-like extension very faint, is more distinct on *f.* side—is to S. of shadow.

Dec. 29 and 30. Ghost-ring still visible.

Dec. 31. Just glimpsed (ghost-ring)—4 satellites seen close to Saturn.

1908 Jan. 3. Ghost-ring invisible—Mimas or Enceladus at *p.* elongation.

Jan. 4. No ghost-ring seen.

Jan. 5. No trace of ring-system—one satellite about 4″ and another about 10″ *f.* Saturn—the nearer of these would be half way along the ring had the latter been visible.

Jan. 6. $5^h\ 30^m$ to $6^h\ 5^m$ G.M.T. Ring shadow appreciably narrower—no trace of ring.

Jan. 7. $6^h\ 20^m$ G.M.T. Bad definition through passing thunderclouds. No trace of ring—two satellites, much closer in than Titan, seen closely *f.* Ring shadow narrow.

Jan. 8. $5^h\ 35^m$ G.M.T. (daylight). Ring visible—it is very fine, and not unlike the ghost-ring of a month ago. From my recollection of the disappearance in October 1907, the ring looks as though it had reappeared about six hours earlier. The ring is brighter *f.* the planet—this was verified by using the erecting eye-

piece referred to in a previous communication, when, had the appearance been subjective, there would have been no reversal. Observation was continued until it clouded over at $6^h\ 15^m$ G.M.T. Three satellites *p.* and Titan *f.*

On the same date, at about 6^h G.M.T., Mr. H. E. Wood, M.Sc., made the following note:—"Ring seen distinctly as a sharp well-defined line on either side of the disc. I consider that it was brighter on the *f.* side. I did not see any points of increased brightness in the trace of the ring."

Jan. 9. Ring brighter—*f.* part decidedly thicker than *p.* part.

Jan. 11. *f.* part of ring is the thicker, but with the erecting eyepiece this is not so certain.

Johannesburg:
1908 *January* 13.

Note on the discovery of a Moving Object near Jupiter (1908 CJ).

(*Communicated by P. H. Cowell, F.R.S., for the Astronomer Royal.*)

Whilst examining a photograph of Jupiter's satellites VI and VII, taken on 1908 February 28 with the 30-inch reflector, a faint object was noticed by Mr. Melotte not far from the sixth satellite.

Photographs taken previously were then examined, and the object was identified and traced back as far as January 27. In all there are eight photographs taken on eight nights. The object is slightly brighter than J. VII, and its motion relatively to Jupiter during the month over which the observations extend is $+70^s{\cdot}58$ in R.A. and $+11'\ 39''{\cdot}2$ in Decl. At present the material available is insufficient to show conclusively whether it is a new satellite or a minor planet.

Provisional positions were obtained by referring the image on each plate to two stars chosen from the *Astronomische Gesellschaft* Catalogue.

More accurate places will be determined in the course of the reduction of the photographs of J. VI and VII.

Observations of Moving Object near Jupiter.

Date and G.M.T.	Apparent R.A.	Apparent Dec.	Exposure.	Object—Jupiter. R.A.	Object—Jupiter. Dec.
1908. d h m	h m s	° ′ ″		m s	′ ″
Jan. 27 12 44	8 45 51·78	+18 5 3·2	64 min.	−0 8·32	−43 15·8
Feb. 1 11 52	43 20·49	17 35·7	111 min.	+0 2·01	41 41·7
3 10 27	42 21·99	22 25·2	80 min.	6·47	41 4·7
22 10 57	33 35·48	19 5 48·9	100 min.	48·45	33 59·0
23 8 34	33 14·23	7 36·6	42 min.	50·56	33 36·7
24 12 25	32 47·26	9 51·2	80 min.	53·04	33 9·3
27 10 59	31 42·99	15 17·9	70 min.	1 0·04	31 59·8
28 11 29	31 21·67	17 5·3	80 min.	+1 2·26	−31 36·6

Observations of the Transit of Mercury at Mauritius, 1907 November 14. By T. F. Claxton.

On the afternoon of the transit the weather was cloudy, but the Sun emerged from behind clouds about one minute before the internal contact at ingress, which occurred at $13^d\ 22^h\ 25^m\ 19^s$ G.M.T., whereas the computed time from N.A. data was $13^d\ 22^h\ 25^m\ 47^s$ G.M.T. The telescope used was a 6-inch equatorial by Cooke, with a diagonal eyepiece, power 80.

The Sun's limb was boiling considerably; the definition of the limb was bad, but of the spots at the centre and near the limb good.

Mercury appeared as a clear-cut black disc, perfectly circular, with no white spot or fringe.

No flashing across of cusps was detected.

Almost immediately after internal contact at ingress the Sun became obscured, and was not visible again until about 15 minutes before internal contact at egress. A careful watch was then made for any distortion, white spot, or disc, but none could be detected in the equatorial.

On examining the planet in the 2-inch telescope of the theodolite, however, with an eyepiece of power 40, undoubted distortion was noticed; but on rotating the graduated coloured glass it was found that the distortion rotated with it, indicating that it was an effect of the glass. No white spot or disc was noticed.

Mr. W. P. Ebbels, who continued watching the planet with the equatorial while I was observing with the theodolite, detected no distortion, white spot, or fringe.

The observed times of egress were as follows:—

	W. P. Ebbels with equatorial.	T. F. Claxton with theodolite.	Computed from N.A. data.
	d h m s	d h m s	d h m s
Internal contact .	14 1 46 24 G.M.T.	14 1 46 31 G.M.T.	14 1 46 40 G.M.T.
Bisected by Sun's limb	47 37 ,,	47 39 ,,	
External contact .	48 36 ,,	48 34 ,,	14 1 49 21 ,,

Mr. A. Walter took 11 photographs of the Sun with the photo-heliograph near the times of ingress and egress, copies of five of which are presented to the Society.

The Perturbations of Halley's Comet in the Past. Third Paper. The Period from 1066 *to* 1301. By P. H. Cowell and A. C. D. Crommelin.

We have again to acknowledge the continued kind assistance of Dr. Smart and Messrs. F. R. Cripps and Thos. Wright, who have done a large portion of the mechanical quadratures for this period.

We commence by indicating some small corrections to the values of n at various returns which we gave in the first paper; these arise chiefly from the introduction of the definite integral for Venus and the Earth from the table on p. 386. The corrected values are, 1531 46″·339, 1456 46″·021, 1378 45″·622, 1301 44″·862.

We next give the full table of perturbations for the revolution 1222 to 1301. It will be seen that the effect of the change of n and the action of the smaller planets has an appreciable influence on our former result.

Revolution 1222 *to* 1301.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0°– 30°	+″·0069	...″	+198″
,,	30–330	−·0013	...	− 50
,,	330–360	−·0070	...	0
Earth	0– 30	+·0045	...	+128
,,	30–330	−·0049	...	+ 31
,,	330–360	+·0089	...	0
Jupiter	0– 90	−·2444	−186	−7040
,,	90–270	−·0175	−167	+7894
,,	270–360	+·5694	−186	− 343
Saturn	0– 90	−·0590	+ 24	−1555
,,	90–270	+·0569	− 37	+1437
,,	270–360	−·2366	+ 16	+ 35
Uranus	0–360	−·0147	− 1	− 34
Neptune	0–360	+·0010	− 6	+ 219
	Sums	+·0622	−543	+ 920

Hence n at $1222 = 44''\cdot862 - ''\cdot062 = 44''\cdot800$.

$$\text{Theoretical period in days} = \frac{1296000 - 920}{44\cdot800} = 28908^{d}\cdot0.$$

Calculated date of perihelion passage 1222 August 30·7 = Julian day 2167635·7. This is 17 days later than our former provisional calculated value, and 9 days later than the day we adopted for the perihelion passage. As the computations for the revolution 1145 – 1222 indicated a still later date for the perihelion passage in 1222, we have reconsidered our interpretation of the observa-

tions of that year and decided to abandon our conjecture that the date of passage of Yew She Te should be mid-August instead of mid-September, though we still think it probable that the comet was seen in August by some of the Chinese observers, since we cannot otherwise explain the statement that it was seen in China for two months ending October 8.

We now put the perihelion passage on 1222 September 10, with a probable error of several days; the conditions were thus practically the same as at the return of 1682, when the comet was circumpolar in Ursa Major in mid-August; we suppose now that the phrase "La Lune joignit la comète" refers not to the date in mid-August when the comet was first seen, but to September 13, when the two bodies were really close together.

A further advantage of the new date is that it puts the comet considerably nearer Antares on October 8 than the former value did; this accords with the Chinese account. We pass now to the preceding revolution; as before, we provisionally took Hind's value of the perihelion passage, viz. 1145 April 19, for the purpose of calculating the perturbations, and deduce the following quantities:—

Revolution 1145 *to* 1222.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0° – 30°	+″·0091	...″	+275″
,,	30–330	+·0063	...	+157
,,	330–360	−·0021	...	0
Earth	0– 30	−·0109	...	−290
,,	30–330	−·0006	...	+209
,,	330–360	+·0064	...	0
Jupiter	0– 90	+·8690	−181	+24797
,,	90–270	−·0506	−277	−3501
,,	270–360	−·9932	+256	+153
Saturn	0– 90	+·1276	−89	+3661
,,	90–270	−·0411	+56	+921
,,	270–360	+·0158	−72	−38
Uranus	0–360	−·0118	...	−42
Neptune	0–360	−·0015	...	−46
	Sums	−·0776	−307	+26256

Now allowing for the fact that the adopted perihelion passage in 1222 is 11 days later than that previously calculated, the adjusted value of u at 1222 = 44″·820; hence that at 1145 is 44″·820 + ″·078 = 44″·898. And calculated period in days $= \frac{1296000 - 26256}{44·898} =$

28280·6 days. Now 1222 September 10 = Julian day 2167646. Hence the calculated date of the preceding passage = 2139365·4 = 1145 April 6·4. This is 12·6 days earlier than Hind's date, a quantity sufficiently small to make it certain that Hind's identification of the comet of 1145 with Halley's is correct. Hind's date may be a few days in error, the observations being too vague to fix it with precision. We take the elements as follows (on the old system), ϖ being computed from the perturbations and Ω from our assumed uniform motion,

$$\left.\begin{aligned} T &= 1145 \text{ April } 19. \\ \varpi &= 294^{\circ}\cdot5 \\ \Omega &= 39\cdot9 \\ i &= 16\cdot5 \\ q &= 0\cdot6 \end{aligned}\right\} \text{Equinox of } 1145.$$

Motion retrograde.

We now pass to the preceding revolution, Hind's date of perihelion passage in 1066 being provisionally adopted for the purpose of computing the perturbations.

Revolution 1066 *to* 1145.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
	° °	"	"	"
Venus	0– 30	+·0013	...	+ 39
,,	30–330	−·0087	...	− 57
,,	330–360	−·0093	...	0
Earth	0– 30	−·0167	...	− 479
,,	30–330	+·0079	...	+ 190
,,	330–360	−·0096	...	0
Jupiter	0– 90	−·0614	+250	−1845
,,	90–270	+·0710	−204	+7655
,,	270–360	+·2486	−401	− 68
Saturn	0– 90	+·0264	− 8	+ 773
,,	90–270	−·0222	−102	− 604
,,	270–360	+·0096	+ 58	− 2
Uranus	0–360	−·0050	...	0
	Sums	+·2319	−407	+5602

Allowing for the fact that the adopted perihelion passage in 1145 is 12 days later than that previously calculated, the adjusted value of n at 1145 is 44″·918. Hence that at 1066 is 44″·918 − ″·232 = 44″·686. And calculated period in days $= \frac{1296000 - 5602}{44\cdot686} =$

28877·0 days. Now 1145 April 19 = Julian day 2139378. Hence the calculated date of the preceding passage is 2110501 = 1066 March 28. This is only 4 days earlier than Hind's date, so that his identification of the famous comet that preceded the Norman Conquest with Halley's comet is fully established. His elements, however, require to be modified, and the following elements, in which ϖ is given its calculated value, and Ω its value by our formula, satisfy the observations with sufficient accuracy, considering the vagueness of the descriptions.

$$\left.\begin{array}{rl} T &= 1066 \text{ March } 27. \\ \varpi &= 293^\circ\cdot3 \\ \Omega &= 38\cdot6 \\ i &= 16\cdot5 \\ q &= 0\cdot6 \end{array}\right\} \text{Equinox of 1066.}$$

According to these elements, the comet would be a morning star, in longitude 343°, latitude +9°, near θ Piscium on April 2, would pass to the evening side of the Sun on April 24, when it would be in Gemini and extremely near the Earth; finally, on May 15 it would be in longitude 148°, latitude −10°, and would then be nearly stationary, its place on May 30 being longitude 149°, latitude −8°. These places accord well with the description from Pingré: "Au jour Ki-ouey (Ap. 2) elle était dans la constellation Che (α, β de Pégase). On le voyait le matin; sa longueur était 7°; elle alla fort vîte ver l'est . . . . le jour Sin-se (Ap. 24) on commença à la voir de soir; . . . . dans ce même temps on l'observa dans les Gemeaux . . . . La comète, en Chine cessa de paraître dans la constellation Tchang (entre le cœur de l'Hydre et la Coupe)."

The conditions in 1145 and 1066 were not unlike those predicted for the return of 1910. In each case the comet was discovered as a morning star, then lost in the Sun's rays; on its emergence it was near the Earth and moved with great rapidity, finally becoming stationary in the neighbourhood of Hydra, where it was lost to view.

Erratum,—*M.N.*, vol. lxviii., No. 2, p. 123.

Revolution 1456 *to* 1531. Venus 330°–360° $\int dn$,

for +″·040 *read* −″·009. Sum of column to be altered accordingly.

The Perturbations of Halley's Comet, 1759–1910.

By P. H. Cowell and A. C. D. Crommelin.

The perturbations by Jupiter and Saturn from 1835 to 1910 were investigated in *M.N.*, vol. lxvii., Nos. 6 and 8. The present paper investigates those produced by Venus, the Earth, and Neptune, for the two periods 1759–1835 and 1835–1910.

As the perturbations by Venus and the Earth at the beginning of the period 1759–1835 have only a very trifling influence on the date of the next return, de Pontécoulant's results have been utilised; they are given in *Conn. des Temps*, 1838, p. 118, and *Mémoire sur le calcul des perturbations de la comète de Halley*, 1829, p. 70.

The sum of his values of dn for Venus from values of u 0^g to 20^g (or $0°$ to $18°$) is $+''\cdot00315$ in sexagesimal seconds. As he has only included the direct action of Venus on the comet, we have to add the action of Venus on the Sun, which is given below. The co-ordinates of Venus and the Earth are taken throughout from our table, *M.N.*, vol. lxviii., No. 2, p. 113. $\sqrt{1-e^2}$ is taken as $=0\cdot254$. The Julian days of perihelion in 1759, 1835 are 2363592·57, 2391598·94.

Table of the Action of Venus on the Sun, 1759, *for* $u = 0°$ *to* $18°$.

u.	x'.	y'.	$\frac{1}{r'^3}$.	$-\frac{x'}{r'^3}\sin u$.	$-\frac{y'}{r'^3}\sqrt{1-e^2}\cos u$.
0°	+·06	−·71	2·63	·000	+·236*
1	+·01	·71	2·63	−·001	·473
2	−·04	·72	2·64	+·004	·481
3	·09	·71	2·64	·012	·473
4	·14	·71	2·64	·026	·473
5	·19	·69	2·65	·044	·461
6	·24	·68	2·65	·067	·454
7	·30	·66	2·66	·098	·442
8	·35	·63	2·67	·129	·422
9	·40	·60	2·68	·167	·403
10	·45	·55	2·68	·211	·366
11	·50	·50	2·69	·256	·333
12	·55	·44	2·69	·308	·293
13	·60	·37	2·69	·362	·247
14	·63	·29	2·70	·411	·192
15	·66	·19	2·70	·461	·125
16	·68	−·09	2·70	·508	+·058
17	·68	+·02	2·70	·537	−·012
18	−·66	+·14	2·70	+·550	−·092

* Denotes that the quantity has been divided by 2.

$$\int dn = \text{sum of } \left(-\frac{x'}{r'^3}\sin u + \frac{y'}{r'^3}\sqrt{1-e^2}\cos u\right) = -1\cdot678.$$

The reducing factor to reduce $\int dn$ to seconds of arc $= \frac{6\pi}{365{\cdot}256} a^{\frac{1}{2}} m' du$ (*M.N.*, lxvii., 2, p. 122). The log of this for Venus = 7·2870.

Hence $\int dn$ (0° to 18°, Sun action) = −″·00325.

Hence total $\int dn$ (0° to 18°) = +″·00315 − ″·00325 = −″·0001.

The following is a computation of the direct and indirect action of Venus from $u = 19°$ to $u = 30°$.

u	$x'-x$.	$y'-y$.	z'.	$\frac{x'-x}{\rho^3}$.	$\frac{x'}{r'^3}$.	$\frac{y'-y}{\rho^3}$.	$\frac{y'}{r'^3}$.	dn.
19°	− ·22	−1·22	+·26	−·108	−1·660	−·597	+ ·726	+·822
20	− ·05	1·17	·25	−·029	1·480	·684	1·050	·908
21	+ ·15	1·13	·24	+·095	1·236	·713	1·343	·962
22	·39	1·11	·21	·231	·908	·656	1·604	·958
23	·67	1·10	·17	·305	·479	·500	1·809	·845
24	·96	1·15	·12	·284	·053	·341	1·880	·640
25	1·27	1·23	+·05	·230	+ ·422	·222	1·847	·395
26	1·58	1·36	−·02	·174	·865	·150	1·680	+·114
27	1·87	1·54	·09	·131	1·280	·108	1·380	−·185
28	2·13	1·78	·16	·099	1·590	·083	·935	·471
29	2·36	2·06	·21	·076	1·760	·067	+·386	·715
30	+2·51	−2·38	−·25	+·060	+1·760	−·057	−·259	−·447*

* Denotes that the quantity has been divided by 2.

$$dn = \left(\frac{x'-x}{\rho^3} - \frac{x'}{r'^3}\right) \sin u - \left(\frac{y'-y}{\rho^3} - \frac{y'}{r'^3}\right) \sqrt{1-e^2} \cos u.$$

Sum of $dn = +3{\cdot}826 = +″{\cdot}0074$ after multiplication by factor whose log is 7·2870. Hence total $\int dn$ (0° to 30°) = +″·0073.

For the direct action of the Earth on the comet from $u = 0°$ to $u = 30°$ in 1759 we have used de Pontécoulant's figures, revising his values between $u = 13^g$ and $u = 18^g$, where we detected an error. Our revised values for dn are $13^g - ‴{\cdot}01330$, $14^g - ‴{\cdot}02585$, $15^g - ‴{\cdot}01581$, $16^g + ‴{\cdot}06098$, $17^g + ‴{\cdot}02172$, $18^g + ‴{\cdot}00996$ (the symbol ‴ denotes centesimal seconds). Further, his value of log m' for Earth should be reduced from 4·4820 to 4·4781. With these corrections $\int dn$ (0° to 30°) = +″·02003 in sexagesimal seconds. To this we have added the action of the Earth on the Sun, the investigation of which is given below.

u.	x'.	y'.	$-\frac{x'}{r'^3}\sin u$.	$-\frac{y'}{r'^3}\sqrt{1-e^2}\cos u$.
0°	−·61	+·74	·000	−·095*
1	·59	·76	+·010	·195
2	·56	·79	·020	·202
3	·53	·81	·028	·207
4	·49	·84	·034	·215
5	·44	·87	·038	·219
6	·39	·89	·041	·224
7	·35	·91	·043	·228
8	·30	·94	·042	·236
9	·25	·95	·039	·237
10	·19	·97	·033	·239
11	·13	·98	·025	·242
12	−·07	·99	+·015	·241
13	·00	1·00	·000	·242
14	+·08	1·00	−·019	·241
15	·16	+·99	·041	·235
16	·24	·98	·063	·231
17	·33	·95	·093	·223
18	·42	·92	·124	·214
19	·51	·87	·160	·202
20	·61	·81	·198	·183
21	·70	·73	·240	·165
22	·77	·64	·274	·143
23	·86	·51	·321	·112
24	·92	·38	·358	·083
25	·56	·22	·385	·048
26	·97	+·06	·407	−·014
27	·96	−·13	·418	+·029
28	·91	·31	·413	·067
29	·83	·47	·393	·102
30	+·71	−·64	−·175*	+·069*

* Denotes that the quantity has been divided by 2.

$$\int dn = \text{sum of } \left(-\frac{x'}{r'^3}\sin u + \frac{y'}{r'^3}\sqrt{1-e^2}\cos u\right) = +1{\cdot}135.$$

Multiplying by $\frac{6\pi}{365{\cdot}256}a^1m'du$ whose log is 7·3758 we obtain +″·00270. Hence total $\int dn$ (0° to 30°) = +″·02003 + ″·00270 = +″·0227.

We next give the investigation of the perturbations of the comet by Venus at the return of 1835, including both the direct and indirect portions, and dividing them into the two periods $u = 330°$ to $360°$ and $u = 0°$ to $30°$. The column t gives the number of days before or after perihelion passage.

u.	t.	x.	y.	x'.	y'.	z'.	$\frac{x'-x}{\rho^3}$.	$\frac{x'}{r'^3}$.	$\frac{y'-y}{\rho^3}$.	$\frac{y'}{r'^3}$.	dn.
330°	177·90	−1·835	−2·284	+·595	−·330	−·259	+·0793	+1·5440	+·0638	−·8560	+·265*
331	165·62	1·680	2·214	·446	·523	·237	·1048	1·1610	·0834	1·3620	+·191
332	153·95	1·529	2·144	·256	·652	·191	·1404	·6700	·1174	1·7060	−·160
333	142·89	1·383	2·074	+·051	·711	·128	·1841	+·1343	·1753	1·8720	·485
334	132·43	1·243	2·002	−·147	·705	−·058	·2236	−·3894	·2644	1·8670	·754
335	122·55	1·107	1·931	·324	·643	+·014	·2287	·8628	·3761	1·7120	·941
336	113·23	·976	1·858	·467	·542	·079	·1805	1·2500	·4666	1·4500	1·027
337	104·43	·850	1·784	·574	·411	·137	·0991	1·5410	·4930	1·1040	1·014
338	96·16	·729	1·711	·643	·265	·183	+·0276	1·7305	·4645	·7130	·935
339	88·38	·614	1·637	·675	−·114	·218	−·0167	1·8200	·4170	−·3074	·817
340	81·07	·503	1·562	·676	+·034	·242	·0404	1·8230	·3726	+·0917	·677
341	74·21	·398	1·487	·650	·169	·254	·0517	1·7530	·3400	·4557	·527
342	67·78	·298	1·412	·605	·291	·256	·0574	1·6290	·3182	·7836	·374
343	61·76	·203	1·336	·543	·398	·251	·0598	1·4600	·3049	1·0700	·224
344	56·11	·113	1·259	·472	·488	·240	·0616	1·2670	·2995	1·3100	−·087
345	50·84	−·029	1·182	·395	·559	·225	·0635	*1·0580	·3019	1·4970	+·034
346	45·91	+·050	1·105	·313	·614	·205	·0654	·8364	·3104	1·6410	·140
347	41·29	·124	1·028	·234	·657	·184	·0689	·6240	·3244	1·7520	·227
348	36·97	·192	·950	·155	·686	·161	·0728	·4125	·3431	1·8255	·296
349	32·93	·255	·872	·078	·704	·137	·0788	·2070	·3729	1·8690	·348
350	29·13	·312	·793	−·006	·713	·114	·0866	−·0159	·4101	1·8880	·381
351	25·56	·364	·715	+·063	·714	·089	·0961	+·1665	·4561	1·8870	·399
352	22·20	·411	·636	·127	·710	·066	·1087	·3352	·5156	1·8740	·407
353	19·02	·452	·557	·186	·698	·043	·1256	·4899	·5926	1·8390	·398
354	16·01	·487	·477	·240	·682	·022	·1481	·6308	·6952	1·7925	·352
355	13·13	·518	·398	·290	·663	+·002	·1785	·7612	·8308	1·7405	·311
356	10·36	·543	·318	·339	·641	−·019	·2160	·8886	1·0154	1·6803	·24
357	7·69	·562	·239	·382	·616	·038	·2587	1·0000	1·2763	1·6130	·15
358	5·09	·576	·160	·422	·588	·057	·3427	1·1035	1·6650	1·5370	+·08
359	2·53	·585	−·080	·459	·557	·075	·4504	1·1990	2·2765	1·4550	−·17
360	0·00	+·587	·000	+·495	+·524	−·092	−·5842	+1·2910	+3·3275	+1·3670	−·29

* Indicates that the quantity has been divided by 2.

Sum of $dn = -4·294 = -''·00827$ after multiplication by factor whose log is 7·2844.

Perturbations by Venus, 1835, $u = 0°$ to $30°$.

The values of t and x, y are the same (in reverse order, and with the sign of y changed) as those in the last table.

u.	x'.	y'.	z'.	$\frac{x'-x}{\rho^3}$.	$\frac{x'}{r'^3}$.	$\frac{y'-y}{\rho^3}$.	$\frac{y'}{r'^3}$.	dn.
0°	+·495	+·524	−·092	−·5842	+1·2910	+3·3275	+1·3670	−·248*
1	·527	·489	·109	·7450	1·3720	5·2510	1·2740	1·044
2	·558	·450	·125	−·5700	1·4520	9·1800	1·1710	2·098
3	·584	·409	·141	+2·0200	1·5180	15·6000	1·0634	3·650
4	·610	·364	·157	12·1000	1·5860	+8·2900	·9460	−1·126
5	·632	·316	·172	10·5000	1·6400	−7·5250	·8200	+2·878
6	·652	·264	·188	4·6700	1·6920	6·0350	·6850	2·004
7	·668	·208	·202	2·2500	1·7320	3·6300	·5390	1·111
8	·680	·153	·216	1·2800	1·7610	2·3000	·3963	·609
9	·687	·080	·228	·7720	1·7790	1·5200	·2072	·275
10	·688	+·009	·239	·5120	1·7819	1·0700	+·0233	+·052
11	·682	−·067	·249	·3590	1·7664	·7890	−·1740	−·117
12	·666	·145	·256	·2605	1·7249	·6020	·3756	·249
13	·641	·229	·260	·1950	1·6602	·4743	·5931	·359
14	·603	·315	·260	·1500	1·5618	·3840	·8158	·448
15	·554	·399	·254	·1178	1·4380	·3194	1·0350	·517
16	·488	·481	·244	·0938	1·2700	·2717	1·2515	·563
17	·405	·559	·229	·0757	1·0558	·2357	1·4570	·582
18	·307	·625	·205	·0622	·8024	·2094	1·6337	·572
19	·192	·678	·173	·0518	·5036	·1900	1·7780	·527
20	+·065	·709	·133	·0440	+·1711	·1761	1·8660	·445
21	−·074	·714	·085	·0383	−·1956	·1668	1·8870	·323
22	·219	·687	−·030	·0346	·5811	·1626	1·8235	−·159
23	·362	·622	+·030	·0330	·9654	·1626	1·6590	+·041
24	·491	·518	·091	·0339	1·3150	·1662	1·3870	·266
25	·596	·372	·151	·0387	1·6015	·1745	·9995	·505
26	·663	−·191	·202	·0492	1·7860	·1859	−·5146	·729
27	·677	+·018	·240	·0676	1·8260	·1967	+·0485	·915
28	·629	·236	·257	·0938	1·6950	·1989	·6358	1·026
29	·511	·442	·247	·1198	1·3730	·1816	1·1880	1·027
30	−·329	+·605	+·208	+·1298	−·8798	−·1447	+1·6180	+·445*

* Indicates that the quantity has been divided by 2.

Sum of $dn = -1·144 = -''·00220$ after multiplication by factor whose log is 7·2844.

The next two tables contain the perturbations by the Earth in 1835, the values t, x, y being the same as for Venus.

u.	x'.	y'.	z'.	$\frac{x'-x}{\rho^3}$.	$\frac{x'}{r'^3}$.	$\frac{y'-y}{\rho^3}$.	$\frac{y'}{r'^3}$.	dn.
330°	+·442	+·911	−·030	+·038	+·426	+·053	+·877	+·187*
331	·611	·804	·092	·042	·585	·055	·769	·421
332	·748	·673	·147	·048	·713	·059	·642	·442
333	·848	·525	·194	·055	·806	·064	·499	·439
334	·920	·367	·233	·065	·875	·071	·349	·418
335	·960	·208	·263	·078	·915	·080	·198	·381
336	·973	+·053	·285	·094	·929	·092	+·051	·330
337	·964	−·095	·299	·117	·923	·108	−·091	·269
338	·937	·233	·306	·146	·901	·130	·224	·200
339	·894	·357	·308	·189	·864	·160	·345	·123
340	·841	·470	·304	·247	·816	·201	·456	+·039
341	·778	·568	·297	·335	·759	·261	·553	−·057
342	·710	·653	·286	·465	·697	·350	·641	·167
343	·639	·726	·273	·679	·630	·429	·716	·307
344	·565	·787	·259	1·047	·560	·729	·780	·501
345	·492	·839	·244	1·734	·490	1·140	·835	·805
346	·419	·879	·225	3·690	·418	2·259	·877	1·563
347	·349	·912	·208	6·368	·350	3·281	·914	2·390
348	·282	·938	·191	+9·594	·284	+ 1·279	·945	−2·487
349	·217	·957	·173	−4·989	·219	−11·145	·966	+3·526
350	·155	·971	·156	6·871	·157	7·780	·985	2·918
351	·096	·980	·140	4·130	·098	4·083	·998	1·432
352	+·040	·984	·123	2·582	+·041	2·427	1·003	·722
353	−·013	·987	·107	1·766	−·013	1·629	1·007	·370
354	·062	·986	·092	1·285	·063	1·191	1·009	·174
355	·111	·982	·078	·986	·114	·914	1·007	+·053
356	·157	·976	·064	·787	·161	·740	1·003	−·022
357	·199	·968	·050	·649	·205	622	·997	·072
358	·241	·959	·037	·547	·249	·536	·990	·105
359	·281	·947	·023	·471	·290	·472	·978	·125
360	−·320	−·934	−·010	−·410	−·331	−·422	−·967	−·069*

* Indicates that the quantity has been divided by 2.

Sum of $dn = +3·774 = +''·00891$ after multiplication by factor whose log is 7·3733.

u.	x'.	y'.	z'.	$\frac{x'-x}{\rho^3}$.	$\frac{x'}{r'^3}$.	$\frac{y'-y}{\rho^3}$.	$\frac{y'}{r'^3}$.	dn.
0°	−·320	−·934	−·010	−·410	−·331	−·422	−·967	−·069*
1	·360	·919	+·004	·364	·374	·385	·953	·144
2	·400	·903	·017	·325	·415	·354	·937	·145
3	·439	·883	·031	·295	·457	·330	·918	·141
4	·478	·861	·045	·269	·498	·311	·897	·132
5	·516	·837	·059	·247	·539	·296	·874	·119
6	·556	·809	·075	·228	·581	·281	·845	·105
7	·595	·778	·090	·214	·623	·273	·815	·086
8	·635	·743	·106	·201	·665	·265	·778	·064
9	·676	·705	·123	·189	·709	·258	·740	·040
10	·716	·661	·140	·180	·751	·255	·694	−·010
11	·756	·611	·157	·174	·793	·255	·641	+·022
12	·795	·553	·176	·167	·836	·255	·582	·058
13	·831	·489	·193	·163	·874	·259	·514	·097
14	·863	418	·212	·159	·908	·265	·440	·138
15	·894	·341	·229	·157	·940	·276	·359	·183
16	·918	·253	·246	·156	·966	·292	·266	·230
17	·935	·157	·262	·154	·981	·313	·165	·277
18	·943	−·053	·276	·150	·987	·341	−·055	·328
19	·941	+·059	·288	·144	·983	·379	+·062	·380
20	·925	·176	·297	·130	·964	·428	·183	·430
21	·892	·300	·300	·101	·926	·488	·311	·484
22	·841	·425	·299	−·048	·869	·553	·439	·541
23	·771	·550	·293	+·039	·793	·601	·565	·597
24	·678	·670	·279	·150	·691	·599	·683	·638
25	·563	·780	·256	·249	·569	·525	·789	·647
26	·425	·875	·225	·291	·426	·401	·877	·605
27	·267	·947	·186	·274	·266	·277	·943	·521
28	−·089	·991	·138	·227	−·088	·182	·975	·407
29	+·103	1·000	·081	·177	+·100	·121	·975	·280
30	+·300	+·965	+·019	+·135	+·290	−·083	+·934	+·081*

* Indicates that the quantity has been divided by 2.

Sum of $dn = +5{\cdot}889 = +''{\cdot}01391$ after multiplication by factor whose log is 7·3733.

In the above investigations we have restricted ourselves to the determination of the perturbation of n, the mean daily motion, as the examination of several revolutions has shown that the effect of Venus and the Earth on the other elements of the comet's orbit is always negligible. $\int d\zeta$ is found with sufficient accuracy by multiplying $\int dn$ from 0° to 30° by period in days − 80.

$\int dn$ from 330° to 360° does not give any sensible contribution to $d\zeta$. We have uniformly adopted 330° to 30° as the interval over which the perturbations by Venus and the Earth should be determined by mechanical quadratures. Beyond these limits we use the definite integral method described *M.N.*, vol. lxvii., No. 6, p. 391, neglectiug the quantity denoted by V_4 as insensible.

The following tables have been constructed, enabling the values of $\int dn$, $\int d\zeta$ to be taken out at sight. The argument is the Julian day of the comet passing the positions $u = 30°$, 330°. Multiples of $224^d{\cdot}7023$, $365^d{\cdot}2614$, for Venus and the Earth respectively, must be added to the Julian day to bring it within the limits of the tables. $\int d\zeta$ for $u = 330°$ is always negligible. The tables have been constructed for an average value of a, but they are sufficiently accurate to use without modification for any revolution. The tabular quantities are to be applied algebraically, with the signs of the tables, to those deduced from the mechanical quadratures.

	Venus.				Earth.			
Jul. Day. 239	$u=30°$ $\int dn$.	$\int d\zeta$.	$u=330°$ $\int dn$.		Jul. Day. 239	$u=30°$ $\int dn$.	$\int d\zeta$.	$u=330°$ $\int dn$.
1492	+″·0069	+195″	−″·0013		1494	+″·0002	+ 6″	+″·0008
1501	·0061	172	·0029		1509	·0016	45	·0018
1510	·0051	144	·0041		1524	·0029	82	·0025
1519	·0040	113	·0052		1539	·0043	121	·0029
1528	·0028	79	·0062		1554	·0054	152	·0028
1537	·0015	42	·0069		1569	·0064	180	·0026
1546	+·0003	+ 8	·0073		1584	·0073	206	·0019
1555	−·0008	− 23	·0074		1599	·0078	220	+·0008
1564	·0019	54	·0071		1614	·0079	223	−·0003
1573	·0025	70	·0065		1629	·0076	214	·0018
1582	·0028	79	·0056		1644	·0069	195	·0033
1591	·0030	85	·0046		1659	·0060	169	·0046
1600	·0027	76	·0033		1674	·0049	138	·0059
1609	·0022	62	·0020		1689	·0036	102	·0070
1618	·0014	39	−·0008		1704	·0022	62	·0076
1627	−·0004	− 11	+·0004		1719	+·0008	+ 23	·0080
1636	+·0008	+ 23	·0016		1734	−·0003	− 8	·0079
1645	·0019	54	·0023		1749	·0012	34	·0074
1654	·0032	90	·0028		1764	·0022	62	·0068
1663	·0044	124	·0032		1779	·0026	73	·0056
1672	·0055	155	·0031		1794	·0027	76	·0045
1681	·0064	180	·0026		1809	·0026	73	·0034
1690	·0069	195	·0019		1824	·0022	62	·0020
1699	·0071	200	+·0009		1839	·0012	34	−·0006
1708	+·0071	+200	−·0001		1854	−·0001	− 3	+·0005

Hence we obtain $\int dn$ ♀ 1759, $30 = +''\cdot0004$

⊕ 1759, $30 = +\cdot0035$
♀ 1835, $330 = +\cdot0025$
$30 = -\cdot0002$
⊕ 1835, $330 = -\cdot0051$
$30 = -\cdot0025$

These have to be added to the perturbations already obtained by mechanical quadratures.

We next investigate the perturbations by Neptune for the two revolutions 1759–1835, 1835–1910. The co-ordinates of Neptune are taken from the table, *M.N.*, lxviii., No. 2, p. 120.

Revolution 1759–1835.

u.	Interval in years. t.	x.	y.	r.	x'.	y'.	z'.	$\frac{x'-x}{\rho^3}$.	$\frac{x'}{r'^3}$.	$\frac{y'-y}{\rho^3}$.	$\frac{y'}{r'^3}$.
0°	0·0	+ ·6	0·0	·6	−28·0	+ 5·7	+9·1	− ·00100	− ·00103	+ ·00020	+ ·00021
18	0·2	− ·3	+1·4	1·4	28·0	5·8	9·1	·00108	·00103	·00017	·00021
36	0·7	2·9	2·7	3·9	27·9	6·4	9·2	·00128	·00103	·00019	·00024
54	2·0	6·9	3·7	7·8	27·5	7·8	9·3	·00170	·00101	·00034	·00029
72	4·1	11·9	4·4	12·7	26·7	10·1	9·4	·00237	·00099	·00091	·00037
90	7·4	17·5	4·6	18·1	25·1	13·5	9·5	− ·00222	·00093	·00259	·00050
108	11·8	23·1	4·4	23·5	22·4	17·7	9·2	+ ·00017	·00083	·00313	·00065
126	17·3	28·1	3·7	28·4	18·1	22·4	8·6	·00083	·00067	·00156	·00083
144	23·7	32·1	2·7	32·3	12·2	26·4	7·5	·00062	·00045	·00073	·00097
162	30·9	34·7	+1·4	34·8	− 4·6	29·2	5·7	·00043	− ·00017	·00039	·00108
180	38·3	35·6	0·0	35·6	+ 3·4	29·6	3·3	·00033	+ ·00013	·00025	·00109
198	45·8	34·7	−1·4	34·8	11·5	27·8	+ ·7	·00028	·00042	·00018	·00103
216	53·0	32·1	2·7	32·3	18·2	23·9	−1·9	·00027	·00067	·00014	·00088
234	59·4	28·1	3·7	28·4	23·0	19·0	4·0	·00029	·00085	·00013	·00070
252	64·9	23·1	4·4	23·5	26·2	13·6	5·8	·00034	·00097	·00012	·00050
270	69·3	17·5	4·6	18·1	27·9	8·9	6·9	·00041	·00103	·00012	·00033
288	72·6	11·9	4·4	12·7	28·6	5·4	7·6	·00053	·00106	·00013	·00020
306	74·7	6·9	3·7	7·8	28·8	3·0	8·1	·00069	·00106	·00013	·00011
324	76·0	2·9	2·7	3·9	28·8	1·5	8·3	·00088	·00106	·00012	·00006
342	76·5	− ·3	−1·4	1·4	28·8	0·9	8·4	·00104	·00106	·00008	·00003
360	76·7	+ ·6	0·0	·6	+28·8	+ 0·7	−8·4	+ ·00111	+ ·00106	+ ·00003	+ ·00003

u.	dn.	$dn\times(2\pi-nt)$.	$(xY-yX)\sin u$.	rX.	rXx.	rYy.
0°	·00000	·00000	·00000	+·00001*	·000	·000
18	−·00001	−·00006	+·00002	−·00007	·000	·000
36	·00014	·00087	·00048	·00097	+·003	−·001
54	·00057	·00349	+·00179	·00538	·037	+·001
72	·00135	·00803	−·00030	·01753	·209	·030
90	−·00129	−·00733	·03064	−·02335	+·408	·174
108	+·00114	+·00606	·05876	+·02350	−·543	·257
126	·00132	·00643	−·02107	·04260	1·200	+·077
144	·00058	·00252	+·00283	·03456	1·110	−·021
162	+·00002	+·00008	+·00714	·02088	·725	−·034
180	−·00021	−·00066	·00000	+·00712	−·253	·000
198	−·00017	−·00004	−·00903	−·00487	+·169	+·041
216	+·00009	+·00018	·01335	·01292	·415	·065
234	·00036	·00050	·01126	·01590	·447	·060
252	·00057	·00055	·00571	·01481	·342	·039
270	·00062	·00037	−·00082	·01122	·196	·017
288	·00051	·00017	+·00144	·00673	·080	+·004
306	·00030	·00005	·00122	·00289	·020	−·001
324	+·00010	+·00001	·00039	·00070	+·002	·000
342	·00000	·00000	+·00002	−·00003	·000	·000
360	·00000	·00000	·00000	+·00001*	·000	·000
Sum	+·00187	−·00356	−·13561	+·01134	−1·503	+·708

* Indicates that the quantity has been divided by 2.

X denotes $\frac{x'-x}{\rho^3}-\frac{x'}{r'^3}$. Y denotes $\frac{y'-y}{\rho^3}-\frac{y'}{r'^3}$.

$dn = X\sin u - \sqrt{1-e^2}\,Y\cos u$. $\sqrt{1-e^2}$ is taken as 0·254.

The factor to convert $\int dn$ into seconds of arc $= \frac{6\pi}{365\cdot256}a^3m'du$.

$\log\frac{6\pi}{365\cdot256} = 8\cdot7127$, $\log a = 1\cdot2564$ for revolution 1759–1835, $= 1\cdot2480$ for revolution 1835–1910, $\log m' = 5\cdot7122$, $du = 18°$ in seconds of arc, $\log du = 4\cdot8116$, $\log\frac{1}{e} = 0\cdot0145$.

Hence $\int dn = +''\cdot00137$.

The factor to convert $\int dn \times (2\pi - nt)$ into seconds of arc $= 3a^2 m' du$, giving $-11''{\cdot}6$ for the numerical value of the integral.

$$\int d\varpi = \frac{m'a}{e}\int (x\mathrm{Y} - y\mathrm{X}) \sin u \, du - \frac{m'a}{e}\sqrt{1-e^2}\int r\mathrm{X} du$$
$$= -8''{\cdot}4$$

$$\int d\zeta = -\sqrt{1-e^2}\int d\varpi - 2m'\int r(x\mathrm{X} + y\mathrm{Y}) du + \int dn \times (2\pi - nt)$$
$$= +2''{\cdot}1 + 5''{\cdot}3 - 11''{\cdot}6 = -4''{\cdot}2.$$

Neptune, revolution 1835–1910.

u.	Interval in years. t.	x.	y.	r.	x'.	y'.	z'.	$\frac{x'-x}{\rho^3}$.	$\frac{x'}{r'^3}$.	$\frac{y'-y}{\rho^3}$.	$\frac{y'}{r'^3}$.
0°	0·0	+ ·6	·0	·6	+28·8	+ 0·7	−8·4	+ ·00111	+ ·00106	+ ·00003	+ ·00003
18	0·2	− ·3	+1·4	1·4	28·8	+ 0·5	8·4	·00105	·00106	− ·00003	+ ·00002
36	0·9	2·8	2·7	3·9	28·8	− 0·3	8·5	·00089	·00106	·00008	− ·00001
54	2·0	6·7	3·6	7·6	28·7	1·5	8·7	·00071	·00106	·00010	·00006
72	4·0	11·6	4·3	12·4	28·4	3·8	9·0	·00055	·00105	·00011	·00014
90	7·1	17·1	4·5	17·7	27·7	7·2	9·2	·00043	·00102	·00011	·00027
108	11·4	22·6	4·3	23·0	25·9	12·0	9·5	·00034	·00096	·00012	·00044
126	16·8	27·5	3·6	27·8	22·8	17·2	9·3	·00030	·00084	·00012	·00063
144	23·1	31·4	2·7	31·5	18·0	22·5	8·6	·00028	·00066	·00014	·00083
162	30·0	33·9	+1·4	34·0	11·5	26·8	7·3	·00029	·00042	·00018	·00099
180	37·2	34·8	·0	34·8	+ 4·0	29·3	5·5	·00033	+ ·00015	·00025	·00108
198	44·4	33·9	−1·4	34·0	− 4·1	29·6	3·1	·00043	− ·00015	·00040	·00109
216	51·3	31·4	2·7	31·5	11·5	27·8	−0·7	·00061	·00042	·00076	·00103
234	57·6	27·5	3·6	27·8	17·1	24·6	+1·4	·00080	·00063	·00162	·00091
252	63·0	22·6	4·3	23·0	21·5	20·7	3·3	+ ·00023	·00079	·00348	·00076
270	67·3	17·1	4·5	17·7	24·4	16·9	4·7	− ·00210	·00090	·00357	·00062
288	70·4	11·6	4·3	12·4	26·0	14·0	5·7	·00236	·00096	·00159	·00052
306	72·4	6·7	3·6	7·6	27·0	11·8	6·2	·00173	·00100	·00070	·00044
324	73·5	2·8	2·7	3·9	27·4	10·6	6·5	·00130	·00101	·00042	·00039
342	74·2	− ·3	−1·4	1·4	27·6	9·9	6·7	·00108	·00102	·00034	·00037
360	74·4	+ ·6	·0	·6	−27·6	− 9·8	+6·7	− ·00098	− ·00102	− ·00034	− ·00036

u.	dn.	$dn\times(2\pi-nt)$.	$\frac{(xY-yX)}{\sin u}$	rX.	rXx.	rYy.	$\frac{(xY-yX)}{\cos u}$	rY.
0°	·00000	·00000	·00000	+·00001*	·000	·000	·00000	·00000
18	+·00001	+·00006	·00000	−·00001	·000	·000	+·00003	− 7
36	−·00009	−·00056	+·00038	·00066	+ ·002	−·001	·00052	27
54	·00027	·00165	·00123	·00266	·018	−·001	·00089	− 31
72	·00048	·00286	+·00172	·00620	·072	+·002	+·00056	+ 37
90	·00059	·00335	−·00009	·01044	·179	·013	·00000	283
108	·00056	·00298	·00434	·01426	·322	·032	+·00141	735
126	·00036	·00175	·00975	·01501	·413	·051	·00707	1418
144	−·00008	−·00035	·01215	·01197	·377	·059	·01677	2176
162	+·00016	+·00060	−·00845	−·00442	+ ·150	+·039	·02603	2754
180	+·00021	+·00066	·00000	+·00626	− ·218	·000	·02889	2889
198	−·00001	−·00003	+·00701	·01972	·668	−·033	·02159	2347
216	·00055	·00107	+·00335	·03244	1·020	−·023	+·00462	+ 851
234	·00127	·00179	−·01991	·03975	1·100	+·071	−·01443	− 1975
252	−·00118	−·00113	·06266	+·02346	− ·530	·269	−·02034	6257
270	+·00120	+·00072	·04504	−·02124	+ ·363	·235	·00000	5217
288	·00141	·00047	−·00603	·01736	·201	·057	+·00196	1327
306	·00063	·00010	+·00072	·00555	·038	·007	−·00052	198
324	·00018	+·00001	·00041	·00113	+ ·003	+·000	−·00057	− 11
342	+·00001	·00000	+·00002	−·00008	·000	·000	−·00006	+ 4
360	·00000	·00000	·00000	+·00001*	·000	·000	+·00001	·00000
Sum	−·00163	−·01490	−·15358	+·01069	−1·398	+·776	+·07443	−·01556

$\int dn = -''\cdot00118$, $\int(2\pi - nt)dn = -46''\cdot8$.

* Indicates that the quantity has been divided by 2.

By the formulæ given above $\int d\varpi = -9''\cdot3$

$$\int d\zeta = -46''\cdot8 + 2''\cdot4 + 4''\cdot1 = -40''\cdot3$$

Again, $$\int de = \frac{m'a\sqrt{1-e^2}}{206265}\left[\int(xY - yX)\cos u\, du + \int rY du\right]$$
$$= +\cdot0000043.$$

We have now completed the investigation of the perturbations of the mean motion and perihelion by all the planets whose effect is sensible, with the exception of Uranus. We have decided that it would be waste of labour to repeat de Pontécoulant's work on this planet, seeing that (1) the whole effect on the time of the next return is only 2 days, which is unlikely to be more than 1 day in error; (2) owing to the fact that the period of Uranus is not very different from that of the comet, the perturbations by this planet

change in a fairly uniform progression from one revolution to another. We find that this test is satisfied by de Pontécoulant's Uranus perturbations for the three revolutions 1682–1759, 1759–1835, 1835–1910.

We had originally expected that we should only find it necessary to investigate the revolution 1835–1910, with the single exception of the perturbations by Neptune in the preceding revolution. However, on examination we found large discordances in the different determinations of n in 1835. In particular, the values of de Pontécoulant and Rosenberger differ by $0''\cdot05$, implying a difference of an entire month in the date of the next perihelion passage.

In view of this large discordance it seems necessary to re-investigate the Jupiter and Saturn perturbations for the revolution 1759–1835 with the same rigour as the following revolution. We have as yet only made an approximate investigation of these, using the same abbreviations as we employed in the ancient revolutions. The result is decidedly in favour of the Rosenberger result as the more accurate; in fact, it is on the opposite side of this from the de Pontécoulant result. We suspect that the cause of the latter's error is the manner in which he has rectified the orbit in the fourth quadrant; he has made the eccentric anomaly proceed uninterruptedly in the rectified orbit, which implies a want of continuity in the time.

For example at $u = 300^g$ the time shifts abruptly from 25376^d to 25298^d

,, $u = 325^g$,, ,, ,, 26719^d ,, 26747^d

,, $u = 350^g$,, ,, ,, 27563^d ,, 27545^d

,, $u = 375^g$,, ,, ,, 27896^d ,, 27895^d

In other words, portions of the Jupiter and Saturn perturbations are either omitted or reckoned twice; this would have no sensible effect on the date of perihelion in 1835, but it would have an appreciable effect on the mean motion at that epoch, and consequently on the date of perihelion in 1910.

We regard Rosenberger's mean motion in 1835 as presumably nearer the truth, but we do not consider that it can be safely taken as definitive, since he only carried his computations up to 296° of eccentric anomaly, adopting Damoiseau's results for the rest of the revolution; the shift can scarcely fail to have caused some breach of continuity, in addition to which Damoiseau's date of perihelion passage in 1835 was $11\frac{1}{2}$ days too early, which implies appreciable errors in the Jupiter perturbations shortly before perihelion.

Rosenberger contemplated completing his computations as soon as the date of perihelion was known by observation, but we cannot ascertain that he ever carried out his intention. Should anyone know of later researches published by him, we should be very grateful to be supplied with the reference.

We now collect our results in tabular form, with the premise that the Jupiter and Saturn figures for the first revolution are not yet definitive.

Revolution 1759–1835.

Planet.	Limits of u.			dn.	$d\zeta$.
♀	0°– 30°	(with def. integral)		+″·0077	+″ 215
	330–360	,,	,,	−·0058	0
⊕	0– 30	,,	,,	+·0262	+ 731
	330–360	,,	,,	+·0038	0
♃	0– 90			+·0091	+ 242
	90–270	,,	,,	−·2588	+ 790
	270–360			+·6870	− 108
♄	0– 90			−·0072	− 176
	90–270	,,	,,	+·0381	+2086
	270–360			−·1150	+ 23
♅	0–360			+·0094	+ 229
♆	0–360			+·0014	− 4
			Sums	+·3959	+4028

Hence we have the equation $\frac{1296000'' - 4028''}{n \text{ at } 1759} = 28006{\cdot}37$, the observed period in days. Hence n 1759 = 46″·1314

add ·3959

n 1835 = 46·5273

The other determinations of n in 1835 are Rosenberger 46″·5088, de Pontécoulant (latest value) 46″·4586, Damoiseau 46″·4994, Lehmann (evidently affected by some error) 46″·9916.

Revolution 1835–1910.

Planet.	Limits of u.			dn.	$d\varpi$.	$d\zeta$.	de.
♀	0° 30°	(with def. int.)		−″·0024	″ 0	− ″ 65	0
⊕	0– 30	,,	,,	+·0114	0	+ 309	0
♃	0–360	,,	,,	+·1465	+222	+31986	−·000097
♄	0–360	,,	,,	+·0084	− 52	− 21	−·000010
♅	0–360			+·0105	− 18	+ 107	+·0000041
♆	0–360			−·0012	− 9	− 40	+·0000043
			Sums	+·1732	+143	+32276	−·000099

Hence $\frac{1296000'' - 32276''}{n \text{ at } 1835}$ = period in days.

Using our provisional value of n 46″·5273
this gives 1910 March 29^d·3 for next perihelion passage.

Using Rosenberger's value of n 46″·5088
this gives 1910 April 8^d·8 for next perihelion passage.

Using de Pontécoulant's value of n 46″·4586
this gives 1910 May 7^d·8 for next perihelion passage.

We are inclined to take 1910 Apr. 8·0 as the most probable value, pending the accurate redetermination of the Jupiter and Saturn perturbations from 1759 to 1835.

We have not yet examined the perturbations of the orbit plane, and so we provisionally adopt de Pontécoulant's values of the node and inclination in 1910 for the purpose of computing a search ephemeris.

$$\left.\begin{aligned} T &= 1910\ \text{Apr. } 8\cdot0 \\ \varpi &= 111^\circ\ 32' \\ \Omega &= 57\ 11 \\ i &= 162\ 13 \end{aligned}\right\} 1910\cdot0$$
$$e = 0\cdot96729$$
$$n = 46''\cdot69$$
$$\log a = 1\cdot2539$$

Instantaneous value of period = $76^{y}\cdot00$

$$q = 0\cdot5869$$

These give rise to the equations for the equator

$$\begin{aligned} x &= r[9\cdot9852]\sin(v + 145^\circ\ 38') \\ y &= r[9\cdot9884]\sin(v + 239\ 11) \\ z &= r[9\cdot5350]\sin(v + 188\ 55) \end{aligned}$$

Search Ephemeris for Halley's Comet.

Date.	Interval.	R.A.	N. Dec.	log r.	log Δ.	u.
	d	h m s	° ′			°
1908 Oct. 1·7	54·3	6 46 40	12 23	0·8168	0·8154	311
Nov. 25·0	50·8	6 25 52	11 31	0·7858	0·7217	313
1909 Jan. 14·8	46·8	5 38 52	11 49	0·7537	0·6817	315
Mar. 2·6	43·4	5 7 48	13 10	0·7202	0·7047	317
Apr. 15·0	40·0	5 10 0	14 52	0·6854	0·7317	319
May 25·0	36·7	5 21 4	16 15	0·6489	0·7329	321
June 30·7	33·5	5 41 48	17 6	0·6110	0·7045	323
Aug. 3·2	30·5	6 1 20	17 29	0·5714	0·6472	325
Sept. 2·7	27·8	6 14 52	17 31	0·5300	0·5629	327
Sept. 30·5	25·0	6 16 36	17 20	0·4868	0·4553	329
Oct. 25·5	22·5	5 58 24	17 21	0·4416	0·3235	331
Nov. 17·0	20·2	5 8 8	17 4	0·3944	0·1885	333
Dec. 7·2	17·9	3 42 28	15 16	0·3452	0·0993	335
Dec. 25·1		2 16 4	11 46	0·2940	0·0987	337

To these may be added the following:—

	h m s	° ′
1908 Jan. 3	6 28 10	10 1
Feb. 2	6 11 19	10 35
Mar. 3	6 0 1	11 23

These were computed for comparison with Dr. Wolf's object BN; as they show a distinctly different rate of acceleration, the identity of the object with Halley's comet seems improbable.

From the beginning of 1910, Dr. Smart has kindly computed an ephemeris at 4-day intervals. In view of the considerable uncertainty in the date of perihelion, some abbreviations have been introduced, the computed heliocentric co-ordinates of the comet in 1835 being utilised.

It will be seen that the comet is an evening star till March 17, and after May 11, being a morning star in the interim. The near approach to the Earth on May 12 and the extremely rapid geocentric motion at that time are noteworthy. It is also evident that at the time of greatest brightness, the conditions of observation are much more favourable in the southern hemisphere than in the northern. It may be worth while adding that the comet will probably be near the Sun at the time of the total solar eclipse of 1910 May 8.

Greenwich Noon.	Right Ascension.	Declination.	Δ	Brightness.	Sun's Place. R.A.	Sun's Place. Dec.
1910.	h m s	° ′			h m	°
Jan. 2	1 41 44	+9 30	1·33	1·00	18 49	−23
6	1 28 28	+8 54	1·37	1·00	19 6	−23
10	1 16 56	+8 17	1·42	1·00	19 24	−22
14	1 6 12	+7 46	1·46	1·01	19 41	−21
18	0 57 12	+7 19	1·51	1·02	19 58	−21
22	0 49 0	+6 58	1·55	1·03	20 15	−20
26	0 41 40	+6 37	1·59	1·06	20 32	−19
30	0 35 16	+6 22	1·64	1·09	20 49	−18
Feb. 3	0 29 36	+6 9	1·68	1·13	21 5	−17
7	0 24 28	+6 1	1·72	1·18	21 21	−15
11	0 19 48	+5 53	1·75	1·26	21 37	−14
15	0 15 28	+5 46	1·77	1·35	21 53	−13
19	0 11 24	+5 41	1·79	1·47	22 8	−11
23	0 7 32	+5 37	1·81	1·61	22 24	−10
27	0 3 48	+5 34	1·82	1·80	22 39	− 9
Mar. 3	0 0 0	+5 30	1·81	2·05	22 54	− 7
7	23 58 8	+5 26	1·80	2·37	23 9	− 6
11	52 24	+5 22	1·79	2·74	23 23	− 4
15	48 20	+5 16	1·75	3·28	23 38	− 2
19	23 44 0	+5 8	1·71	3·96	23 53	− 1
23	23 39 32	+4 58	1·65	4·83	0 7	+ 1
27	34 48	+4 47	1·58	5·94	0 22	+ 2

Greenwich Noon.	Right Ascension.	Declination.	Δ	Brightness.	Sun's Place.	
					R.A.	Dec.
1910.	h m s	° ′			h m	°
Mar. 31	23 30 8	+4 32	1·50	7·48	0 36	+ 4
Apr. 4	25 40	+4 14	1·40	8·95	0 51	+ 5
8	23 21 24	+3 55	1·28	10·86	1 6	+ 7
12	23 18 20	+3 35	1·15	13·06	1 20	+ 8
16	16 0	+3 16	1·02	16·22	1 35	+10
20	15 24	+3 1	·87	19·50	1 50	+11
24	17 16	+2 46	·72	25·12	2 5	+13
28	23 23 8	+2 37	·57	28·31	2 20	+14
May 2	23 35 44	+2 29	·41	58·34	2 35	+15
6	0 6 8	+2 37	·26	126·5	2 50	+16
10	2 2 40	+3 3	·08	1112·0	3 6	+17
14	7 21 40	+0 34	·10	660·7	3 22	+18
18	9 9 44	−0 41	·25	94·19	3 37	+19
22	9 44 8	−1 9	·40	33·27	3 53	+20
26	9 59 48	−1 21	·55	15·49	4 9	+21
30	10 9 40	−1 29	·70	8·59	4 26	+22
June 3	10 16 16	−1 37	·85	5·30	4 42	+22
7	10 21 4	−1 47	·99	3·51	4 58	+23
11	10 25 28	−1 56	1·15	2·42	5 15	+23
15	10 29 4	−2 5	1·30	1·74	5 32	+23
19	10 32 48	−2 14	1·42	1·34	5 48	+23
23	10 35 36	−2 29	1·58	1·00	6 5	+23
27	10 38 38	−2 41	1·71	·79	6 22	+23
July 1	10 41 38	−2 53	1·85	·63	6 38	+23
5	10 44 32	−3 8	1·98	·52	6 55	+23
9	10 47 28	−3 23	2·10	·42	7 11	+23
13	10 50 16	−3 38	2·22	·35	7 27	+21

Measures of Southern Binary Stars in 1907. By John Tebbutt.

I have pleasure in forwarding to the Society the results of a series of measures which I have made of the most interesting southern binary stars with the 8-inch equatorial, in the same manner as described in my paper in *Monthly Notices*, vol. lxvi. On the evenings of August 23, 24, 25 and September 15, 16, I attempted to measure the rapid binary Lacaille 7194, but failed to see the companion, although it was readily seen and measured by me in 1902. See *Monthly Notices* for November 1903. The non-visibility of the companion has been confirmed by observations subsequently made at my request at the Sydney Observatory.

Star.	Epoch.	Position Angle.	Distance.	No. of Evenings for Position Angle.	No. of Evenings for Distance.	Hour Angle.
α Centauri	1907·410	213°·8	20″·67	6	6	E.
β 416	·567	274°·2	2″·09	5	5	E.
γ Coronæ Aust.	·617	117°·3	1″·83	4	4	E.
h 5014	·652	...	1″·54	...	1	E.
,,	·660	240°·3	...	3	...	E.
γ Centauri	·673	353°·2	1″·42	5	2	W.
β Muscæ	·673	346°·1	1″·28	5	2	W.
π Lupi	·704	84°·5	1″·42	7	3	W.
μ Lupi	·737	153°·3	1″·83	6	3	W.
γ Circini	·737	70°·2	1″·46	5	3	W.
d Lupi	·812	1°·4	...	2	...	W.
β Phœnicis	·875	12°·2	1″·55	5	3	E.
p Eridani	·907	216°·6	8″·23	3	4	E.
Lacaille 2145	·964	45°·2	1″·89	4	4	E.

The Observatory,
The Peninsula, Windsor, N.S. Wales:
1908 *January* 23.

On the relative numbers of Star Images photographed in different parts of the Plates for the Oxford portion of the Astrographic Catalogue. Second Paper. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In *Monthly Notices*, lxii. p. 434, a preliminary discussion was given of the counts of star images in different parts of plates taken for the *Astrographic Catalogue*; and it was shown that the star density was greatest at a certain distance (r_0) from the centre of the plate and fell off rapidly outside this distance; the value of r_0 for different object-glasses being different,—*e.g.* for Oxford $r_0 = 33'$, for Paris $r_0 = 58'$, for Algiers $r_0 = 49'$, for Toulouse $r_0 = 30'$.

The natural inference is that the value of r_0 depends on the position of the plate, increasing as the plate is pushed further within the focus.

Similar results were found for the Potsdam telescope, with $r_0 = 28'$ (*Astron. Nachrichten*, No. 3817); and for the Greenwich telescope with $r_0 = 38'$ (*Greenwich Astrographic Catalogue*, vol. i., pp. vi and vii). On the other hand, the late M. Loewy has questioned the reality of the phenomenon in his annual report of the Paris Observatory for 1905, in the following terms:—

"D'autres expériences ont eu pour objet de déterminer jusqu'à quel point une même carte pouvait être considérée comme homogène au point de vue des grandeurs. On sait que M. H. H. Turner a été conduit à penser que cette homogénéité laissait à désirer, en faisant, pour un grand nombre d'épreuves publiées, le relevé des nombres d'étoiles qui tombent dans les divers carrés du réseau. Pour contrôler ce résultat par une autre voie, on a photographié un groupe d'étoiles avec des poses, décroissantes et de légers déplacements systématiques, afin d'obtenir des images faciles à identifier, et en même temps à la limite de visibilité. On a ensuite répété l'opération en imprimant à la lunette d'autres déplacements systématiques de manière à donner le même temps de pose aux mêmes étoiles dans diverses parties de la plaque. On a pu ainsi constater que la visibilité des images relatives aux astres les plus faibles est la même dans toutes les régions du champ. Le defaut signalé par M. Turner, et qui a pu se produire à la suite d'une mise au foyer défectueuse, ne semble donc pas très redoutable dans les circonstances actuelles" (p. 10).

One possible interpretation of the discrepancy between M. Loewy's conclusion and those above quoted is that his test was not so delicate a one as the numerical test elsewhere applied. The results obtained by counting star images are consistent and definite, and clearly afford a satisfactory measure of something. The fact that other methods fail to detect that something may therefore only enhance the value of the method which reveals it: there are almost certain to be uses to which a new test can be applied, even though they may not be foreseen at the moment. The quantity

measured has reference to the focal length of the instrument, and therefore to the scale of the photograph; and it is at any rate of interest to see how far such measures throw light on variations of focal length with time or with temperature; or on the inclination of the plate to the axis of the telescope (which may be regarded as variation in focal length for different parts of the plate).

2. Three volumes of the *Oxford Astrographic Catalogue* have now been published, containing measures of $3 \times 160 = 480$ plates, with centres in zones $+31°$, $+30°$, $+29°$. Generous volunteer assistance

A B

2	10	26	50	82	122	170	226	290	362	442	530	626
10	18	34	58	90	130	178	234	298	370	450	538	634
26	34	50	74	106	146	194	250	314	386	466	554	650
50	58	74	98	130	170	218	274	338	410	490	578	674
82	90	106	130	162	202	250	306	370	442	522	610	706
122	130	146	170	202	242	290	346	410	482	562	650	746
170	178	194	218	250	290	338	394	458	530	610	698	794
226	234	250	274	306	346	394	450	514	586	666	754	850
290	298	314	338	370	410	458	514	578	650	730	818	914
362	370	386	410	442	482	530	586	650	722	802	890	986
442	450	466	490	522	562	610	666	730	802	882	970	1066
530	538	554	578	610	650	698	754	818	890	970	1058	1154
626	634	650	674	706	746	794	850	914	986	1066	1154	1250

8 16 24 32 40 48 56 64 72 80 88 96

C D

FIG. 1.

from Miss Riddle has enabled us to count the measures made on these plates on a systematic plan arranged so as to exhibit such variations in focal length. It seemed desirable to discuss these measures in order to decide whether to extend the work to the other four volumes as they appear; and to give a brief summary of the discussion, in order to enable our colleagues to judge of the expediency of undertaking similar work elsewhere.

3. Each plate was divided into four quarters, which will be denoted by XY, xY, Xy, xy. The capital letters refer to large values of x or y,—*i.e.* values exceeding 13·000; the small letters

to values less than 13·000. An alternative designation would thus be

$$XY = NE \quad xY = NW \quad Xy = SE \quad xy = SW$$

Each quarter was then divided up into ten regions, the average distances of which from the plate centre were in the ratios 1, 2, 3 . . . . 10, approximately, the boundaries of the regions following the reseau lines. The dividing lines for one of the quarters are shown in fig. 1. A represents the plate centre and D the corner of the reseau. Representing a side of the reseau by 2, the middle points of the reseau squares along A B will be at distances 1, 3, 5 . . . . 25. It is readily seen that on this scale the numbers written in the squares represent $r^2 = x^2 + y^2$, where x and y are the co-ordinates of the centre of the square referred to A as origin. Now if we take ten regions at distances t, $2t$, $3t$ . . . . $10t$ from A as centre, the value for the outermost will be $100t^2$; and a glance at fig. 1 shows that we must put approximately $100t^2 = 1100$, or t^2 = about 11.

TABLE I.

Letter for Region.	No. of Squares.	Average Value of r^2.	t^2.	r in Minutes of Arc.
a	3	7·3	7·3	7
b	5	28	7·0	13
c	14	76	8·4	22
d	15	150	9·4	31
e	23	248	9·9	39
f	23	363	10·1	48
g	34	508	10·4	56
h	29	673	10·5	65
i	15	864	10·7	73
k	8	1090	10·9	82

The thick lines were drawn with this value of t^2 in mind. The 4th and 5th columns of Table I. show that the grouping might have been better, but this was not noticed until so much work had been done that it seemed better to keep the system unaltered. It is not essential to have the distances in exact arithmetical progression.

4. The counts were then treated as shown in the following example:—

TABLE II.

Total counts in the regions for Plates 2034-2057, *Zone* +31°.

No. squares =	3	5	14	15	23	23	34	29	15	8	169
Region =	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*	*i.*	*k.*	Total.
XY	25	30	101	137	222	276	372	297	132	64	1656
*x*Y	22	42	124	136	266	228	332	247	97	36	1530
X*y*	21	38	106	113	226	205	340	312	112	62	1535
xy	22	38	89	120	193	208	315	236	89	35	1345
	90	148	420	506	907	917	1359	1092	430	197	6066

The number of squares in a region varies considerably, as shown in the top line. The next procedure is to divide by this number, so as to get the average star density in each region. Then to reduce this density to a uniform scale for different batches of plates, the quotients are again divided by 6066/4, the average total number of stars on a single quarter-plate. The intermediate results need not be given here; in Table III. are shown the final values obtained for star density in the different regions.

TABLE III.

Star Density on a uniform scale deduced from Table II.

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*	*i.*	*k.*	Total.
XY	55	40	48	61	64	79	72	68	58	53	598
*x*Y	48	56	59	60	77	65	65	56	43	30	559
X*y*	46	50	50	50	65	59	66	72	50	52	560
xy	48	50	42	53	56	60	61	54	39	29	492
Mean	49	49	50	56	65	66	66	63	47	41	552

5. Now the characteristic feature of the variation in density is the gradual rise up to about *f* and *g* and the fall afterwards. The maximum represents the position of best focus, and it is important to note any variations in this position. For this purpose it is convenient to have a single number of some kind rather than a system of 10, and the indicator adopted for a preliminary investigation is

$$\Delta = (c+d+e) - (g+h+i).$$

It will be seen that an increase in Δ means that the maximum comes nearer *a*,—*i.e.* nearer the plate centre,—and presumably that the plate is further from the object-glass: a decrease in Δ corresponds to the plate being pushed in nearer the O.G.

To get an idea of the physical meaning of Δ, let us take the theoretical formula for star density on a plate given in *Mon. Not.*, lxii. p. 441, viz.—

$$\text{Density} = a\{1 \sim c(r^2 - r_0^2)\}$$

where r is the distance of the point on the plate from the plate centre; r_0 is the distance of the maximum from the centre; and $c = 10^{-4}$ if r and r_0 are expressed in minutes of arc. We may put r_0 successively equal to the selected distances in Table I.; and take $a = 66$, which is the mean maximum, on the system adopted in Table III.; and we then get the densities as in Table IV.

TABLE IV.

Theoretical Star Densities, with different positions of the maximum, and corresponding values of Δ *and* n.

r_0	a.	b.	c.	d.	e.	f.	g.	h.	i.	k.	Δ.	n.
7′	66	65	63	60	56	51	45	38	31	22	+65	8039
13	65	66	64	61	57	52	46	39	32	23	+65	8202
22	63	64	66	63	59	54	48	41	34	25	+65	8508
31	60	61	63	66	62	57	51	44	37	28	+59	8883
39	56	57	59	62	66	61	55	48	41	32	+43	9263
48	51	52	54	57	61	66	60	53	46	37	+13	9508
56	45	46	48	51	55	60	66	59	52	43	−23	9526
65	38	39	41	44	48	53	59	66	59	50	−51	9071
73	31	32	34	37	41	46	52	59	66	57	−65	8210
82	22	23	25	28	32	37	43	50	57	66	−65	6833
No. of squares	3	5	14	15	23	23	34	29	15	8		

6. The values of Δ are shown in the last column but one of Table IV. When r_0 is small or large,—*i.e.* when the best focus is very near the centre or the corner of the plate,—Δ is constant, because the distances represented by $c\ d\ \ldots\ i$ are all on one side of the maximum, and hence $\Delta \equiv (c+d+e)-(g+h+i)$ does not change. But such cases do not occur in practice, the plate being always focussed for some distance from the centre, and then Δ is a convenient index to the position of best focus. In the last column is given, n, the total number of stars on the quarter-plate, obtained by assuming the density constant over each of the regions $a, b\ \ldots\ k$; so that the number of stars in each region is the product of the density by the number of squares in the region as shown in the last line of the table. The values of n so found imply the further assumption that the maximum density is the same (viz. 66 per square) whatever be the position of the plate. This assumption is no doubt incorrect, but it may not be far from the truth. The optical image of a star in focus near the axis of the lens is almost certainly smaller than the best focussed image of a star near the edge of the field. Hence, if the number of stars photographed depended on the size of best image alone, the maximum density would fall off as we left the centre of the field, and might fall off even rapidly. But other factors influence the photographic star image—atmospheric tremor, errors of driving and guiding, etc.—which are independent of the distance from the centre of the field, and the maximum density may not diminish so rapidly as might at first appear. It seems worth while to examine the consequences of assuming it to be constant, at any rate in the first instance. But we shall return to this point later. Our first business is with the *position* of the maximum, as indicated by the quantity Δ.

7. We proceed to tabulate the values of Δ for the four quarters,

arranging all the plates of vols. i., ii., iii. of the Oxford Catalogue in groups, as sufficiently indicated by columns 1, 2, 3, 4 in Table V. The detailed grouping was determined by considerations which need not concern us here; nor need we, in a preliminary survey, pay much attention to the fact that one or two of the individual groups are small either in number of plates or number of stars.

TABLE V.

Mean Values of Δ for Plates in vols. i., ii., iii. *of the Oxford Astrographic Catalogue.*

Zone.	Limiting Nos.	No. of Plates.	No. of Stars.	Values of Δ. XY.	*x*Y.	X*y*.	*xy*.	Whole.
29°	525– 542	16	8099	+48	+37	+42	+59	+46
29	700– 855	14	8589	+27	+34	+10	+28	+25
30	896– 921	15	4293	+25	+42	+51	+17	+34
29	937–1023	32	7300	+51	+49	+52	+15	+42
29	1236–1486	18	7913	+52	+12	+23	+37	+31
Totals and Means		95	36194	+41	+35	+36	+31	+36
30	1564–1585	11	12593	−28	− 5	−44	+15	−16
30	1586–1597	12	5933	−49	− 4	−34	+30	−14
31	1598–1606	8	4877	−76	+ 7	−81	+ 3	−37
Totals and Means		31	23403	−51	− 1	−53	+16	−22
29	1747–1843	20	6706	− 1	+51	−20	+34	+16
31	1834–1849	7	6245	−37	+ 3	−38	−21	−23
29	1850–1900	26	8860	+ 2	+37	−30	+34	+11
30	1865–1915	14	8178	− 6	+38	−30	+27	+ 7
30	1916–1943	26	5989	− 1	+48	−22	+38	+16
Totals and Means		93	35978	− 9	+35	−28	+22	+ 5
30	1946–2060	26	11217	−30	+32	−56	+ 8	−11
31	1973–1996	11	4006	−22	+30	−59	−11	−16
31	1997–2032	26	19984	−29	+37	−64	+ 1	−14
31	2034–2057	24	6066	−25	+32	−23	− 3	− 5
Totals and Means		87	41273	−27	+33	−50	− 1	−11
30	2061–2101	26	7797	+ 1	+12	−30	+42	+ 6
31	2087–2155	15	4513	−39	+46	− 9	+34	+ 8
30	2102–2111	10	2693	0	+29	−27	+17	+ 5
30	2136–2196	16	6726	−13	+25	−47	+26	− 2
31	2202–2234	15	4445	−14	+24	− 1	+40	+12
30	2217–2482	4	1163	+12	−16	−28	+56	+ 6
31	2235–2271	26	6095	− 5	+47	−22	+29	+12
31	2274–2299	12	4871	−33	− 6	−41	+25	−14
Totals and Means		124	38303	−11	+20	−26	+34	+ 4
31	2311–2349	18	4596	+ 2	+21	− 5	+43	+15
29	2450–2500	30	11874	−23	+25	−22	+23	+ 1

8. The reasons for separation at the places indicated by the horizontal lines are as follows:—

(*a*) Between plate 1557 and 1558 (summer of 1900) the new dome was erected. The object-glass was dismounted and the eye end also.

(*b*) After plate 1606, the instrument was devoted to the Eros work. The eye end was taken off to study possible instrumental adaptations.

(*c*) After plate 1943 the O.G. was dismounted for cleaning.

(*d*) Some change seems to have taken place between 2057–2061, or near this date, though nothing is recorded in explanation.

(*e*) At plate 2300 the eye end was dismounted to study adaptation to photographs of the Moon among the stars.

(*f*) After plate 2349 the O.G. was dismounted and taken to Egypt for the eclipse of 1905.

There was apparently a slight change of focal length at each of these epochs, due to accidental causes. It is curious that, on the whole, the value of Δ should show an *increase* from the second group onwards, for this corresponds to the plate being further from the O.G., and not nearer to it, as might have been explained by the gradual wearing of the studs against which it is pressed. No change was made deliberately in the adjustment of these until after all the plates had been taken.

9. It is a reasonable assumption that on each occasion the focal length was changed without disturbing the relationship of the quarters to each other. Hence we may subtract the value of Δ for the whole plate (given in the last column) from each quarter. It will be sufficient here to give the means of groups.

TABLE VI.

Values of Δ for each quarter relative to mean Δ for Plate.

	Limiting Nos.	Weight.	XY.	*x*Y.	X*y*.	*xy*.
I.	525–1486	5	+ 5	− 1	0	− 5
II.	1564–1606	3	−29	+21	−31	+38
III.	1747–1943	5	−14	+30	−33	+17
IV.	1946–2057	5	−16	+44	−39	+10
V.	2061–2299	5	−15	+16	−30	+30
VI.	2311–2349	1	−13	+ 6	−20	+28
VII.	2450–2500	2	−24	+24	−23	+22

There is a decided change after the first group. In the first group the quarters XY and X*y* are slightly positive, corresponding to the X side of the plate (large R.A.*s*) being a little further from the O.G. than the *x* side. In all the later groups the reverse is the

case, and the relationship remains fairly constant. It is true there seems to be a slight diminution in xY and an increase in Xy, but the appearance is chiefly due to the last two groups, which have small weight.

10. The changes in general focal length seem more striking than any relative change. But it is possible to take another view of these changes. Are they possibly seasonal? No deliberate change in focus was made at any of the epochs above mentioned, and at one of them no interference with the instrument is recorded.

11. Let us try the extreme assumption that they are all due to temperature or other seasonal change. In the third column of Table VII. is given the month in which the majority of the plates in the group were taken. Arranging the groups according to these months we have—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
+6	+7	−2	+16	+12	−11	+1	−16	−14		−5	
	+8		+16		−16		−14	−37		+11	+15
	+5		+12		−14			−23			
			+6								

There certainly seems to be something to be said for a seasonal effect. The values of Δ from December to May are all positive with one slight exception, and from June to November all negative. If we express the effect as a simple harmonic term, with a constant added, we should have figures something like those given below:—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
+4	+10	+12	+10	+4	−4	−12	−18	−20	−18	−12	−4

Applying these as corrections with reversed sign to the individual groups, we get the results of Table VII.

TABLE VII.

Possible seasonal change in Δ.

	Uncorrected Δ.	Month.	Correction.	Corrected Δ.
1564–1585	−16	Aug.	+18	+2
1586–1597	−14	Sept.	+20	+6
1598–1606	−37	Sept.	+20	−17
Mean	−22	...	+19	−3
1747–1843	+16	Apr.	−10	+6
1834–1849	−23	Sept.	+20	−3
1850–1900	+11	Nov.	+12	+23
1865–1915	+7	Feb.	−10	−3
1916–1943	+16	Apr.	−10	+6
	+5	...	+1	+6

TABLE VII.—*continued.*

Possible seasonal change in Δ.

	Uncorrected Δ.	Month.	Correction.	Corrected Δ.
1946–2060	−11	June	+ 4	− 7
1973–1996	−16	June	+ 4	−12
1997–2032	−14	Aug.	+18	+ 4
2034–2057	− 5	Nov.	+12	+ 7
	−11	...	+10	− 2
2061–2101	+ 6	Jan.	− 4	+ 2
2087–2155	+ 8	Feb.	−10	− 2
2102–2111	+ 5	Feb.	−10	− 5
2136–2196	− 2	Mar.	−12	−14
2202–2234	+12	Apr.	−10	+ 2
2217–2482	+ 6	Apr.	−10	− 4
2235–2271	+12	May	− 4	+ 8
2274–2299	−14	June	+ 4	−10
	+ 4	...	− 7	− 3
2311–2349	+15	Dec.	+ 4	+19
2450–2500	+ 1	July	+12	+13

The figures can thus be explained either by a seasonal change in focal length, or by a series of discontinuities at the times when either eye end or O.G. were dismounted.

Against the latter explanation there is the fact that no discontinuity was recorded about plate Np. 2057, so that the seasonal effect is somewhat more probable.

The first set of plates (Nos. 525–1486) throws no light on the matter, for the most divergent groups in the last column of Table IV. are the first two, which were both taken chiefly in September, so that we must set down their difference to accident.

Summarising the results so far, we find—

(α) That the focal position of the plate was essentially different in the two periods before and after the erection of the new dome in 1900.

(β) In the first period the plate was further from the object-glass ($\Delta = +36$ for the mean of the four quarters), and satisfactorily normal to the line of collimation (values of Δ nearly same in all quarters).

(γ) In the second period the plate was nearer the O.G. ($\Delta = 0$ or less), and the X side (large R.A.'s) much nearer than the x side (small R.A.'s). The values of Δ for the two sides are about $\Delta = -25$ and $\Delta = +25$ respectively.

Moreover, there are variations in the focal length which may be seasonal, ranging from $\Delta = -20$ in June–September (plate

nearer O.G.) to $\Delta = +12$ in December–April (plate further from O.G.), though these variations may be due to a series of discontinuities.

The result that the plate is further from the O.G. in cold weather and nearer in warm seems a little strange. The telescope tube is no doubt longer in the warm weather; but we are concerned with a differential effect, depending partly on the expansion of the tube and partly on that of the lenses, which may alter their focal length more than the tube alters, and so give an apparently reversed effect. For this reason we cannot look for confirmation (or otherwise) to the observed scale-value of the plate deduced from measures of star images; for this scale value depends on the expansion of tube and plate, perhaps also on that of the reseau.

12. Let us now examine the effect of these differences on the relative number of stars photographed in the different quarter-plates. We find that we have in plates 1564–2500 a mass of tolerably homogeneous material with one chief variable, the mean value of Δ for the whole plate, which probably corresponds to the mean distance of the plate from the O.G. Whether the variations in Δ are due to a seasonal effect or to accidental discontinuities need not concern us if we take Δ itself as the independent variable.

TABLE VIII.

Total number of Stars counted on each Quarter-Plate.

Mean Δ.	Plate Nos.	Zone.	XY.	*x*Y.	X*y*.	*xy*.
−37	1598–1606	31°	1320	1208	1269	1080
−23	1834–1849	31	1528	1571	1582	1564
−16	1564–1585	30	3175	3332	3161	2925
−16	1973–1996	31	1022	965	1070	949
			7045	7076	7082	6518
−14	1586–1597	30	1582	1540	1462	1349
−14	1997–2032	31	5201	5075	5000	4708
−14	2274–2299	31	1289	1187	1266	1129
−11	1946–2060	30	2951	2743	2901	2622
− 5	2034–2057	31	1656	1530	1535	1345
− 2	2136–2196	30	1820	1525	1813	1568
+ 1	2450–2500	29	3247	3011	2874	2742
			17746	16611	16851	15463
+ 6	2061–2101	30	1972	1891	2019	1915
+ 5	2102–2111	30	752	653	675	613
+ 6	2217–2482	30	314	293	298	258
+11	1850–1900	29	2308	2190	2250	2108
+ 7	1865–1915	30	2290	2023	2101	1764
+ 8	2087–2155	31	1193	1174	1130	1006
			8829	8224	8473	7664

TABLE VIII.—*continued.*

Total number of Stars counted on each Quarter-Plate.

Mean Δ.	Plate Nos.	Zone.	XY.	*x*Y.	X*y*.	*xy*.
+12	2235–2271	31°	1560	1494	1598	1443
+12	2202–2234	31	1185	1103	1147	1010
+16	1747–1843	29	1679	1630	1755	1642
+15	2311–2349	31	1188	1194	1099	1115
+16	1916–1943	30	1647	1449	1527	1366
			7259	6870	7126	6576

Reducing the numbers to percentages we find for the means of groups—

TABLE IX.

Mean Δ.	XY.	*x*Y.	X*y*.	*xy*.
−23	101·6	102·1	102·2	94·1
− 8	106·4	99·7	101·1	92·8
+ 7	106·4	99·1	102·0	92·3
+14	104·3	98·7	102·4	94·5
Mean	104·7	99·9	101·9	93·4

13. We have now to correct these numbers for the changes in Δ. Table IV. gives in the last two columns the values of n (the whole number of stars on a quarter-plate) for different values of Δ on certain assumptions specified in § 6. We may make a small table of the *percentage* correction to n as follows:—

Δ =	+ 60	+ 50	+ 40	+ 30	+ 20	+ 10	0	− 10	− 20	− 30	− 40	− 50	− 60
Corrn. to n p.c. =	+7·5	+4·5	+2·8	+1·7	+0·9	+0·3	+0·1	0·0	+0·1	+1·0	+2·5	+4·5	+9·7

Now the mean values of Δ for the groups of Table IX. and for each quarter-plate, and the consequent corrections to percentage, would be as follows:—

TABLE X.

Group (Table IX.)	Mean Value of Δ.				Correction to Percentage.				Mean.
	XY.	*x*Y.	X*y*.	*xy*.	XY.	*x*Y.	X*y*.	*xy*.	
I.	−41	+ 9	−55	− 4	+2·7	+0·2	+7·1	0·0	+2·5
II.	−27	+20	−41	+13	+0·7	+0·9	+2·7	+0·4	+1·2
III.	− 5	+24	−26	+34	0·0	+1·2	+0·5	+2·1	+0·9
IV.	− 4	+38	−14	+36	0·0	+2·7	0·0	+2·4	+1·3

Subtracting the mean shown in the last column from the separate corrections for each quarter-plate (so as to keep the mean for the whole-plate zero), the numbers of Table IX. corrected would then be as in Table XI.

TABLE XI.

Table IX. *corrected to uniform* $\Delta = -10$.

	XY.	xY.	Xy.	xy.
	101·8	99·8	106·8	91·6
	105·9	99·4	102·6	92·0
	105·5	99·4	101·6	93·5
	102·9	100·1	101·2	95·5
Mean	104·0	99·7	103·1	93·2

14. It is clear that we have not removed the chief part of the differences shown in Table IX. by this process, and we must look to other causes for them. One such cause is undoubtedly *tilt* of

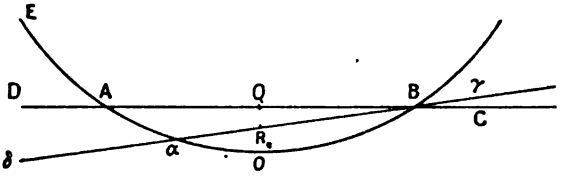


Fig. 2.

the plate, of which no account has yet been taken in dealing with the number of stars as distinct from the position of best focus. That the tilt seriously affects the number of stars can be seen from elementary geometrical considerations. Let E A O B (fig 2) represent a section of the curved field and D A B C a position of the plate normal to the axis.

Now if the plate be tilted to the position $\delta\,\alpha\,B\,\gamma$, it is clear that every portion of the half Q C or R γ is brought *closer* to the curve of good focus, and this side of the plate will thus contain more stars all over. On the other side, regarding the plate as first moved parallel to itself to cut the curve at α, and then tilted, we have in what precedes taken account of the movement of translation, but not of the tilt; and the tilt clearly moves every part of this other half-plate *further* from the curve of good focus, so that we lose stars all over it. The effect of tilt is thus greater than that of translation, because by moving the plate parallel to itself we gain in one part and lose in another, whereas by tilt we either gain or lose all over the half-plate. We must qualify this statement a little when we extend the argument to two dimensions, so as to deal with a curved surface instead of the curved arc shown in the figure. But the general nature of the phenomenon remains; and we see that *when a plate is tilted, the side nearer the O.G. gains at the expense of the side more remote.*

15. We thus see how parts of the large differences of Tables IX. and XI. probably arise. The quarter-plates XY and Xy, which have Δ negative, are nearer the O.G. and gain stars

from their opposites xy and xY. But it is not necessary to proceed to a quantitative estimation to see that we cannot in this way explain the whole of the differences. There must be some other contributing cause which makes the Y quadrants 4 per cent. richer in stars than the y quadrants. The difference between the mean Δ for the Y quadrants and the y quadrants is so slight that tilt cannot explain this considerable excess; and, while reserving the study of tilt for a future paper, we may proceed here briefly to consider the possible causes of an excess of stars in the north half of a plate which cannot be explained (so far as can be seen at present) by the position of the plate.

16. The excess of stars in the N half of the plate may be due to any combination of the following causes:—

(α) Optical performance of the O.G., including possible inclination to the line of collimation.

(β) Increased atmospheric absorption for S stars.

(γ) Actual increase in number of stars as we go northwards. When we combine all the plates in a zone, we eliminate (to a large extent) variations of uniformity in R.A., but a change with declination may be persistent.

17. It is possible to estimate the approximate magnitude of cause (β). The formula for atmospheric absorption at Oxford of visual rays is given in *Mem. R.A.S.*, vol. xlvii., as 0·25 sec. Z.D. We may put Z.D. = 25°; and the variation of sec. Z.D. for 1°, which is the distance between N and S halves of the plate, is about 0·01. The correction is thus about ·0025 magnitude. Now Newcomb gives (in his book, *The Stars: a Study of the Universe*, p. 283) the number of stars of different magnitudes as in the second column of Table XII., whence we get the totals of that magnitude and brighter as in the third column, from which we

TABLE XII.

Mag.	No. of Stars.	Total = N.	log N.	Diff.
6·5	2	2	0·30	·30
7·0	2	4	0·60	·30
7·5	4	8	0·90	·38
8·0	11	19	1·28	·25
8·5	15	34	1·53	·27
9·0	29	63	1·80	·18
9·5	33	96	1·98	·15
10·0	39	135	2·13	·17
10·5	64	199	2·30	·20
11·0	115	314	2·50	

form log N and its differences as in the 4th and 5th columns. It appears that at magnitude 11, log N is increasing at the rate

of about ·4 per magnitude, so that the increase in log N for ·0025 mag. would be about ·001. This would increase 100 stars to 100·23 (since log 100·23 = 1·001), so that a difference of atmospheric absorption indicated by visual observations would only make a difference of 0·23 per cent. between the upper and lower halves of a plate. Photographic absorption would need to be twenty times as large to explain the observed difference, which is not likely. Sir W. Abney emphasises (*Mon. Not.*, xlvii. p. 265) the necessity for knowing the "spectrum value" of the plates employed if we wish to determine the photographed absorption, but the limits between which it varies are not likely to be wider than from one to four times the visual absorption.

18. It occurred to me that we might get some indication of the photographic absorption by comparing in some way the plates taken at different hour-angles; for instance, in the formula given at the heading of each of our plates for determining magnitude from measures of diameters,

$$\text{mag.} = a - b\sqrt{d},$$

the constant a would vary with the atmospheric absorption. But a little examination showed that the differences of Z.D. between plates taken otherwise under the same conditions were too small to afford a trustworthy indication of the phenomenon, which is swamped by a number of other larger variations. A single example will serve to show this. In Table XIII. are shown in the first three columns the Decl. and R.A. of the plate centre and the hour-angle at the middle of the exposure; in the fourth column is given the Z.D. of the centre; and in the fifth the value of the constant a of the above formula. If these values of a are arranged according to Z.D. we find—

No. Plates.	Mean Z.D.	Mean a.
7	35°·4	16·43
6	41°·0	16·78

i.e. the paradoxical result that fainter magnitudes (by 0·38 magnitude) are shown on the plates of 6° greater Z.D. Suspicion arose that the person who measured the plate might influence the result; and on collecting the results for the three measurers shown in the sixth column, it was found that their mean values of a were

BG.	EG.	S.
$16\cdot84_5$	$16\cdot40_5$	$16\cdot50_3$

TABLE XIII.

Plates taken on the night of 1903 *October* 30.

Decl.	R.A.	H.A.	Z.D.	a.	Measurer.	Corrn.
+31°	3·31	1·39 E	28°	16·2	EG	+·2
,,	5·55	3·41 ,,	45	16·5	EG	+·2
,,	6· 4	3·30 ,,	43	17·7	BG	−·2
,,	6·13	3·13 ,,	41	17·0	BG	−·2
,,	6·22	3· 3 ,,	39	16·3	BG	−·2
,,	6·58	3·10 ,,	40	16·3	S	+·1
,,	7· 7	3· 0 ,,	39	16·9	S	+·1
,,	7·25	2·57 ,,	38	16·6	EG	+·2
,,	7·34	2·41 ,,	36	16·8	BG	−·2
,,	8· 1	2·49 ,,	37	16·4	BG	−·2
,,	8·28	2·55 ,,	38	16·2	EG	+·2
,,	8·46	2·39 ,,	36	16·3	S	+·1
,,	8·55	2·31 ,,	35	16·5	EG	+·2

but on applying corrections as in the last column the 16·43 only became 16·50 and the 16·78 became 16·75, so that the anomaly was not removed. It seems better to determine the photographic absorption by special observations rather than to seek it in the plates already taken, in which its effect was consciously or unconsciously minimised.

19. We may now turn to cause (γ), the distribution of stars in the sky. That the density increases as we go northwards in the Oxford zones is shown, for instance, in Argelander's *Durchmusterung*. His numbers per square degree for zones +28°, +29°, +30°, +31°, are 14·9, 16·2, 16·4, 16·4; so that the differences per cent. between the north half and the south half of plates in zones with centres +29°, +30°, +31°, here discussed, would be 8·1, 1·3, 0·0; mean, 3·1. This is comparable with our 4·0 per cent., and the difference is what we might expect, considering that Argelander only goes to magnitude 10. But it is unsatisfactory that the difference is chiefly in one zone. We may put this down to accidental error, but the evidence is only sufficient to state a case for inquiry. In *Monthly Notices*, vol. lx., plate 2 (opposite p. 16), Mr. Bellamy has given a curve showing the approximate variation in stellar density (for the Oxford zones) with Galactic latitude somewhat as follows:—

TABLE XIV.

Gal. Lat.	0°.	10°.	20°.	30°.	40°.	50°.	60°.	70°.	80°.	90°.
N = No. of stars	800	460	310	230	180	155	135	120	110	105
Differences	340	150	80	50	25	20	15	10	5	
Log N	2·90	2·66	2·49	2·36	2·26	2·19	2·13	2·08	2·04	2·02
Differences	0·36	0·17	0·13	0·10	0·07	0·06	0·05	0·04	0·02	

20. There are two important points lying away from the curve, but we will neglect these in the following brief and general remarks. The Galaxy cuts the parallel of 30° Dec. about $5\frac{1}{2}^h$ and $19\frac{1}{2}^h$.

(A) At these crossing points the changes of stellar density are very large, but *if they are symmetrical* on opposite sides of the Galaxy there should be a rough compensation. On one side the N half of a plate will be richer, on the other the S half. We must be prepared, however, to find this compensation break down if the Milky Way is unsymmetrical.

(B) In the neighbourhood of 0^h the Oxford zones are near the Milky Way, south of it, and nearly parallel to it. Thus to pass from the southern half of a plate to the north is to approach the Milky Way, and consequently to find more stars. In these R.A.'s, therefore, we may expect to find the main contributions to the discrepancy between N and S halves of a plate. What is the maximum contribution we can expect from them? Suppose they ran all the way from 20^h to 5^h (9 hours of R.A. out of the 24), parallel to the Milky Way and 15° from it. We see from Table XIV. that log N increases 0·17 in 10° of Galactic latitude, or 0·017 in 1°; so that 100 stars on the S half of a plate would become 104 on the N half, due to approach to the Galaxy. This is an increase of 4 per cent., which is about what is required; but it becomes inadequate when we take 9/24 of it in order to distribute it round the whole zone. This fraction 9/24 corresponds to uniform distribution in R.A., which is, of course, not in strict accordance with the facts; but weighting each hour by the number of stars in it gives a nearly equal fraction, since the rich Galactic portions are chiefly in the non-contributing R.A.'s.

(C) For the remainder of the zone contributes nothing, *or goes the other way.* We have spoken of the crossing points in (A) above; from R.A. 5^h–6^h, for instance, the increase northwards is rapid for the first half hour, but is balanced by an equally rapid diminution when the central line is crossed. Reserving for future investigation the effects of possible asymmetry, we may put aside this crossing region as contributing nothing. Proceeding to greater R.A.'s, we are now north of the Galaxy, and thus stars decrease in number as we go northwards. The quantity N–S will thus be negative. But the zone +30° is a small circle of the sphere, and presently its sharper curvature brings it perpendicular to the Galaxy, when the difference N–S will be zero, and then positive again. It is positive for about an hour only (near $12\frac{1}{2}^h$) and then is negative again up to 19^h (say), when we get the other crossing point (19^h–20^h).

21. These facts can, of course, be expressed in tabular form, and are given below in Table XV.; but it seemed desirable to call attention to them also in general terms, because general considerations seem to show that we cannot fairly expect, by improved knowledge of the details, to explain the whole of the difference N–S by actual distribution of the stars in Galactic latitude alone. The figures given below are only approximate: they could doubt-

less be improved, for instance, by using Professor Kapteyn's results in No. 18 of his publications, which have become available by his kindness in sending an advance copy since Table XV. was constructed. But the change would not be great. One of the most important details left outstanding is that of the symmetry or want of symmetry of the Galaxy on opposite sides; but Professor Kapteyn's paper does not touch this point, since he has not yet undertaken any discussion of distribution in Galactic longitude.

TABLE XV.

R.A.	Gal. Lat. (Zone +30°).	Calcd. N–S. per cent.	Observed (N–S) per cent. 31°.	30°.	29°.	Mean.
0·5	−33°	+3	+ 8	− 6	− 3	0
1·5	−31	+3	+10	−12	+13	+ 4
2·5	−27	+3	+ 8	0	+ 8	+ 5
3·5	−20	+3	+24	−17	+18	+ 8
4·5	−11	+4	+ 8	+30	+34	+24
5·5	0	...	+ 3	+21	+24	+16
6·5	+11	−2	0	−10	− 2	− 4
7·5	+23	−1	+15	− 4	− 5	+ 2
8·5	+35	0	+ 9	− 6	+ 4	+ 2
9·5	+48	0	+ 5	− 1	− 5	0
10·5	+61	0	+ 5	− 3	+ 9	+ 4
11·5	+74	0	+ 4	+10	− 2	+ 4
12·5	+86	+1	+ 4	− 5	0	0
13·5	+79	0	0	+ 3	− 2	0
14·5	+66	0	− 4	+ 5	+ 6	+ 2
15·5	+53	0	+ 8	+11	−11	+ 3
16·5	+40	−1	− 7	+10	+22	+ 8
17·5	+28	−1	+ 1	− 1	+ 3	+ 1
18·5	+16	−2	+ 7	+ 7	+ 3	+ 6
19·5	+ 4	...	+ 2	− 4	+18	+ 5
20·5	− 7	+3	+ 1	+15	+14	+10
21·5	−16	+3	+11	+ 4	+ 9	+ 8
22·5	−25	+3	+ 2	+12	+ 9	+ 8
23·5	−30	+3	+15	+11	+ 8	+11

22. Percentages are given in Table XV. rather than total numbers of stars, in order to avoid undue influence from the Galaxy. It will be seen that the observed percentage is from $20^h{\cdot}5$ to $23^h{\cdot}5$ +9, and from $0^h{\cdot}5$ to $4^h{\cdot}5$ it is +8; the mean for the whole 9 hours being +8·7, as against a theoretical +3·1. For the portion $6^h{\cdot}5$ to $18^h{\cdot}5$ the observed N–S percentage is +2·2, as against a theoretical −0·5. Hence the mean observed N–S is throughout in excess of the theoretical; or, in other

words, only a part of it can be set down to star distribution; the rest must be due either to atmospheric absorption or unsymmetrical performance of the object-glass. But these latter causes do not vary with the R.A. If we estimate their combined effect as 2 or 3 per cent. (being guided by the figures for $6^h{\cdot}5$ to $18^h{\cdot}5$, where the effect of stellar distribution is small), we get 6 or 7 per cent. for the observed effect of stellar distribution in R.A. $20^h{\cdot}5$ to $4^h{\cdot}5$ instead of about 3. This suggests that the rate of diminution with Galactic latitude has been considerably underestimated in those regions. A scrutiny of the figures shows that this is quite possibly the case, the reason being that the rate of diminution varies considerably in different Galactic longitudes. But a complete discussion of this point cannot be given here.

Summary.

§§ 1, 2. Introductory.

§§ 3, 4. Method of measuring the average star density on each quarter-plate, at different distances from the centre.

§ 5. Definition of Δ, which indicates the distance of maximum density from the centre. Table of values of star density and total number of stars for different values of Δ.

§ 7. List of observed values of Δ, in order of date of exposure of plates, in vols. i., ii., and iii.

§§ 8–11. Variations of focal length indicated by changes of mean Δ may be due to discontinuous disturbances of the plate, but are more probably due to seasonal changes in focal length.

But there is certainly one discontinuity, at the erection of the new dome in 1900. Before this the plate was satisfactorily normal to the telescope axis; afterwards it was apparently tilted in R.A., greater R.A.'s being nearer the O.G.

§§ 12–14. In consequence of this tilt, the side of greater R.A. is richer in stars; but the detailed study of the effect of tilt is reserved for a separate investigation.

§ 15. But there is an excess of stars in the N half of the plate, which is 4 per cent. richer than the S half, and this cannot be due to tilt.

§§ 17, 18. Visual atmospheric absorption would give an excess of 0·23 per cent. in the N half: photographic absorption may be greater, but its value cannot be well determined from the Oxford plates, which were taken at nearly constant Z.D. A special investigation of it is desirable.

§§ 19–21. The distribution of the stars in Galactic latitude alone explains a part, but cannot explain the whole of the 4 per cent. excess in the N half of the plates. The general nature of the distribution in Galactic longitude is such as to explain another portion of the excess, but quantitative estimates cannot yet be made.

Further Considerations on the Correlations of Stellar Characters. By Winifred Gibson, B.Sc., formerly Jessel Scholar, University College, London, and Karl Pearson, F.R.S., University College, London. (With Six Diagrams.)

(1) *Introductory.*—In a paper communicated to the Society last year (*Monthly Notices*, vol. lxvi. p. 445), modern statistical methods were used for the first time to determine the numerical relationships between various star characters. The object of the present paper is to deduce further similar relationships, and to deal with some of the same relationships on the basis of wider data. The general characters with which we have to deal, and which are more or less accurately known for larger or smaller stellar populations, are (1) magnitude, (2) colour, (3) spectral class, (4) proper motion, (5) parallax, (6) position. In any attempt to look upon the stellar universe as an orderly whole, the relationships between these characters must be of fundamental importance. To determine their numerical values is the first stage by which we pass from chaos to an organised and locally differentiated cosmos. The aid which the statistician may venture to offer the trained astronomer in this respect may, perhaps, be illustrated by reference to some recent work. Since the publication of the first paper above referred to, two memoirs, both of considerable importance, have appeared. The first is that of Messrs. Chase, Smith, and Elkin ("Parallax Investigations on 163 Stars, mainly of large Proper Motion," *Transactions, Yale Univ. Observatory*, vol. ii. pp. 1–207). This memoir deals with the relationship of proper motion, magnitude, parallax, and spectral class, among much else of great value, but not bearing on the topics we have at present in hand.

A second memoir of less scope but of considerable interest is that of Mr. W. S. Franks ("The Relation between Star Colours and Spectra," *Monthly Notices*, vol. lxvii. pp. 539–42). Now, these memoirs more than suffice to show that the distribution of star characters is not one of mere random association. The characters occur in a *correlated* manner, and this in itself is suggestive of the cosmos being a differentiated organisation. Even if we at once admit that we might anticipate that parallax would be related to proper motion, or even to magnitude, or, again, that colour and spectral group would be found in association, it is less obvious that spectral class will be found related to magnitude, proper motion, and parallax.

Yet, even when we see these relationships indicated in the above and other memoirs, there appears to be something lacking, which it is, perhaps, possible for modern statistical methods to supply. It would not be possible from the above type of classificatory work to determine the intensity of the relationship between the characters under consideration. For example: Is parallax more closely associated with magnitude or with spectral class? Or, again: Is magnitude more closely related to distance

than to chemical composition? To say that the latter is nearly four times as influential as the former is to crystallise at once our general ideas on magnitude.

Accordingly, it seems possible that modern statistical methods may be of some aid in determining the intensity of relationship between various stellar characters; in appreciating what, to adopt a term from biometry, may be spoken of as the organic correlations of the population. It is not suggested that, any more than in the science referred to, the individual must be lost sight of in a cloud of average relationships. But a knowledge of the extent to which stellar characters are correlated may, if properly used, be helpful in indicating the directions of profitable further analysis.

(2) *Determination of Correlation.*—It may not be out of place here to give a brief summary of the constants by aid of which correlation is determined in modern statistical practice. This is the more important as it is desirable to indicate the limits of their proper application in the case of stellar characters. The methods in use are threefold:—

(i) *Coefficient of Correlation*, usually represented by r. This can only be used effectively if both characters are quantitative and actually measured. Let A and B be the two characters, m_1 and m_2 their mean values, m_1+x, m_2+y, the characters in any pair of individuals, σ_1 and σ_2 the standard deviations (square roots of mean square deviations of either group of characters); then if n be the size of the sample taken of the population,

$$r_{12} = \mathrm{S}(xy)/(n\sigma_1\sigma_2) \quad . \quad . \quad . \quad . \quad (i).$$

v possesses the following properties: it lies between -1 and $+1$, according to the intensity of the relationship; $r_{12}\sigma_1/\sigma_2$ is the slope of the best fitting straight line of the average values of A for a given value of B; and $r_{12}\sigma_2/\sigma_1$ is the slope of the corresponding line for average values of B for given values of A. $\sigma_1\sqrt{1-r_{12}^2}$ is the average standard deviation of arrays of A for given values of B, and $\sigma_2\sqrt{1-r_{12}^2}$ is the average deviation from the straight line of arrays of B for given values of A. When the average values of one character for a given value of a second lie nearly on a straight line the correlation is said to be linear, and in this case the vanishing of r_{12} marks the absolute independence of the two characters. r_{12} is always the same as r_{21}.

All this is completely independent of the nature of the frequency distribution. Provided the characters are quantitatively measurable and the correlation is approximately linear, the correlation coefficient r is pre-eminently the best suited to express the degree of interdependence of the two variables.

For example, it was used in the first paper in determining the relationship between proper motion in R.A. and in declination.

(ii) *The Correlation Ratio*, usually represented by η. If the curve of mean values of A for given values of B depart widely from a straight line, then the vanishing of r does not necessarily mean that

the characters A and B are unrelated. It would only signify that the best fitting straight line was horizontal. In order to cover this case the correlation ratio η has been introduced. Let Σ_1 be the standard deviation of the means of arrays of A for given values of B, each array mean being weighted with its total frequency; then,

$$\eta_{12} = \Sigma_1/\sigma_1 \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

η possesses the following characters: it lies between 0 and 1, being always zero if the characters are independent, and unity if they are absolutely related or causal. η always lies between r and 1 and is equal to r when the correlation is linear. $\sigma_1\sqrt{1-\eta_{12}^2}$ is the mean standard deviation of arrays of A for a given B. $\sigma_1\sqrt{\eta_{12}^2-r_{12}^2}$ is the mean square deviation of the curve of means from its best fitting straight line. η_{12} is not necessarily equal to η_{21}. When the correlation is approximately linear, we have $\eta_{12}=\eta_{21}=r_{12}$ very closely, and this relationship holds for a very wide range of physical and organic variables.

η, the correlation ratio, will clearly be of service when one variable, say B, is not quantitative, but classificatory, because in this case we can determine Σ_1 and σ_1 although we have no quantitative measures of B. We ought, however, to have fairly fine groupings of B. Such cases are those of stellar spectra and stellar colour. On the other hand, η_{12} and η_{21} may be found in order to measure the degree of divergence of the correlation from linearity. As illustration of this, we may consider magnitude. If we replace it by amount of light, this might possibly give a true quantitative scale; but not only does the reduction involve some doubt, but it introduces extremely laborious calculations compared to the simplicity of magnitude proper. Accordingly the η method, classifying according to magnitude, seems a suitable method of approaching the problem.

(iii) *Coefficient of mean square Contingency*, usually represented by C_1.

Let the arrangement of any table of two variables—*e.g.* Mr. Franks' tables of colour and spectral class—be purely classificatory. Let the frequency of any class a of A in the population N be n_a and of the class b of B be n_b; let the frequency of individuals combining both classes be n_{ab}, then

$$\phi^2 = \mathrm{S}\left\{\left(n_{ab} - \frac{n_a n_b}{\mathrm{N}}\right)^2 \Big/ \frac{n_a n_b}{\mathrm{N}}\right\}\frac{1}{\mathrm{N}}. \quad . \quad . \quad \text{(iii)}$$

is termed the mean square contingency, and it clearly vanishes if the distribution of the two characters be independent.

$C_1 = \sqrt{\phi^2/(1+\phi^2)}$ is termed the coefficient of mean square contingency, and it measures the deviation of the two characters from independence. It approaches unity if the characters are causal, and is zero if they are independent.

The whole of the above brief résumé of the usual statistical methods of dealing with correlation is independent of any assump-

tion as to the distributions of frequency following the normal or Gaussian law of variation. Should they do so, however, we have

$$r = \eta = C_1 \quad . \quad . \quad . \quad . \quad . \quad (iv)$$

or all our three methods of determining the intensity of correlation merge theoretically into a single value.

We say theoretically, because the truth of (iv) depends upon our replacing summation by integrals, or it is the limit with sufficiently fine grouping. By actually testing (iv) on fairly Gaussian material it will be found that even moderately coarse classifications give us quite close results. Deviations from (iv) arise, not wholly, but chiefly through the characters dealt with being non-Gaussian in distribution of variability. We venture to think that possibly astronomers have been too ready to assume that all types of variability follow the Gaussian law of distribution, and that the assumption that star characters follow it requires statistical justification. It is, perhaps, rather dangerous to start with the doctrine that they must, and then deduce rather sweeping conclusions from the fact that they do not. In the present investigation the correlation coefficient, the correlation ratio, and the coefficient of mean square contingency will be used according to the nature of the statistics with which we have to deal.

The probable error of r is calculated from

$$p.e \text{ of } r = \cdot 67449(1 - r^2)/\sqrt{N}$$

and of η from

$$p.e \text{ of } \eta = \cdot 67449(1 - \eta^2)/\sqrt{N}.$$

These values are not the absolutely correct values, but are sufficiently close for most practical purposes.*

The probable error of C_1 is troublesome to calculate,† but if C_1 be two to three times greater than $\cdot 67449/\sqrt{N_1}$, this being the maximum value of the probable error, *i.e.* that when $C_1 = 0$, it will undoubtedly be significant.

(3) *Correlations with Stellar Colour.*—(i) *Colour and Magnitude.*—In the earlier memoir stellar colour and magnitude were correlated, using for this purpose the catalogue of star colours contained in vol. ix. of the *Annals of the Cape Observatory*. The stars included in that list range in magnitude from 4 to 10, being 4 to 9 for visual and 6 to 10 for photographic magnitude. The colours ranged from yellow to red, and contained no blue or green or white. In order to include green and blue stars we used, at the suggestion of Mr. T. W. Backhouse, the star catalogue in vol. xiv. of the *Harvard Annals*. That list includes stars of photographic magnitude $-1\cdot5$ to 7, and contains about 3·6 per cent. of blue and green stars. A table of contingency for magnitude and colour in

* For probable errors of r and η, see Pearson, "On the General Theory of Skew Correlation," pp. 19 and 20, *Drapers' Research Memoirs*, Dulau & Co.

† Blakeman and Pearson, "On the Probable Error of Mean Square Contingency," *Biometrika*, vol. v. pp. 191 *et seq.*

the case of 2834 stars is given below (Table I.). In grouping for contingency we reduced this to a 6 × 9-fold table, but the means were calculated on the basis of Table I. The results reached were as follows:—

	Previous Table. (*Monthly Notices*, vol. lxvi., p. 455.)	Present Table.
Mean magnitude . . .	8·58 ± ·05	5·15 ± ·012
Standard deviation . .	·835 ± ·032	·906 ± ·008
Coefficient of contingency	·304 (± ·05)	·271 (± ·01)

Thus while the two groups of stars differ very widely in mean character, we see that the dependence of one character on the other is essentially the same in the two samples. The reduction of the

TABLE I.

*Contingency Table for Colour and Photographic Magnitude.**

Magnitude.	Colour. Blue.	Green.	White.	Yellow.	Orange.	Red.	Totals.
−1·5 to −1	...	...	1	...	...	...	1
−1 to −·5	...	...	...	...	...	...	0
−·5 to 0	...	...	...	...	...	...	0
0 to ·5	2	...	1	1	...	...	4
·5 to 1	...	...	1	...	1	...	2
1 to 1·5	...	...	3	2	2	...	7
1·5 to 2	...	...	6	...	...	...	6
2 to 2·5	...	1	11	12	1	...	25
2·5 to 3	...	6	9	12	3	1	31
3 to 3·5	1	5	16	39	8	...	69
3·5 to 4	...	9	32	68	12	...	121
4 to 4·5	1	17	61	121	27	...	227
4·5 to 5	4	15	112	191	54	8	384
5 to 5·5	8	16	208	371	111	9	723
5·5 to 6	4	6	239	425	140	13	827
6 to 6·5	5	1	82	192	77	5	362
6·5 to 7	...	...	10	24	9	1	44
7 to 7·5	1	...	...	...	...	...	1
Totals,	26	76	792	1458	445	37	2834

* In the magnitude class, the group includes the first and excludes the second value given. Thus all stars being enumerated to one decimal in magnitude, the group 4 to 4·5 contains 4·0, 4·1, 4·2, 4·3 and 4·4 stars or centres at 4·2. The blank lines indicate the magnitude groups actually used in deducing the contingency coefficient from a 6 × 9-fold table.

mean magnitude by three classes, the introduction of blue, green, and white stars, has not essentially altered the numerical relationship between magnitude and colour. There are several points about this which deserve fuller consideration. Magnitude is often taken as a true variable. When dealing with the relationship of magnitude to parallax in the former paper it was shown that magnitude *used as a numerical variate*, unless we convert it into light units, does not give very satisfactory results, and that, further, the curves of means such as we find for stellar characters are generally widely different from straight lines. In the present case we cannot calculate the correlation coefficient r, because there is no quantitative scale of colour; we can, however, find the correlation ratio η on the assumption that magnitude is a true scalar quantity, which, of course, it is not. We find numerically $\eta = \cdot 17 \pm \cdot 01$. The results given on p. 456 of the former paper (for visual magnitude) lead to $\eta = \cdot 13 \pm \cdot 05$. Thus, again, the two results are alike within the range of probable error. But we note at once that if magnitude be used as a true quantitative character we shall not get results for C_1 and η which are really comparable. Indeed, a little thought will show that a mean magnitude is something having little physical meaning, and the ratio of two mean magnitudes, which is the nature of η, may often be deceptive.* We must accordingly anticipate erratic results when magnitude is used as a numerical variate, and we shall endeavour to adopt it only as an index to classification.

Breaking, however, for a moment through this good rule we may note that the mean magnitudes of the colour arrays are—

Colour.	Mag.	No.	Colour.	Mag.	No.
Red .	5·38	37	Green	4·38	76
Orange	5·31	445	Blue	4·97	26
Yellow	5·18	1458	White	5·09	792

Whole series: Mag. 5·15 No. 2834

We see at once from this result that, within the limits of probable error, the mean magnitude of the white stars is the mean magnitude of the whole group of stars, or this group may be omitted when we determine the influence of deviation from mean colour on deviation from mean magnitude. The presence or absence of white stars does not affect the contingency. In the accompanying Diagram I.† a graph has been drawn of the variation of mean magnitude with

* The mean magnitude of two stars of magnitudes 5 and 10 is 7·5, but the magnitude of the star which has their mean amount of light is 5·74. The mean magnitude of two stars of magnitude 14 and 1 is also 7·5, but the magnitude of the star with their mean amount of light is 1·75. The ratio of the means found in the first manner is unity, in the second manner more than 3. Yet we perpetually see mean magnitudes compared and the difference between them asserted to be of significance or non-significance for the character of stellar groups.

† This diagram was taken out of its place in this paper and published, without the data on which it is based, in a letter to *Nature*, Oct. 17, 1907.

colour. To obtain a reasonable scale of colour, we have used that of the solar spectrum, plotting first a curve of mean magnitude, and then replacing it by a curve of star luminosity, the scale of luminosity being that in which the unit of luminosity is that of a star of magnitude 10. We have naturally omitted the white stars. (If the white stars be omitted from the Harvard data the value of the contingency is ·297, a value in absolute accordance with that found for the Cape stars in the earlier paper.) The graph shows at once a somewhat striking result,—the distribution of luminosity among the coloured stars gives a curve remarkably similar to that of luminosity in the solar spectrum if we shift the solar curve towards the violet end of the spectrum. How far is the method of determining magnitude influential in this matter? We have at present

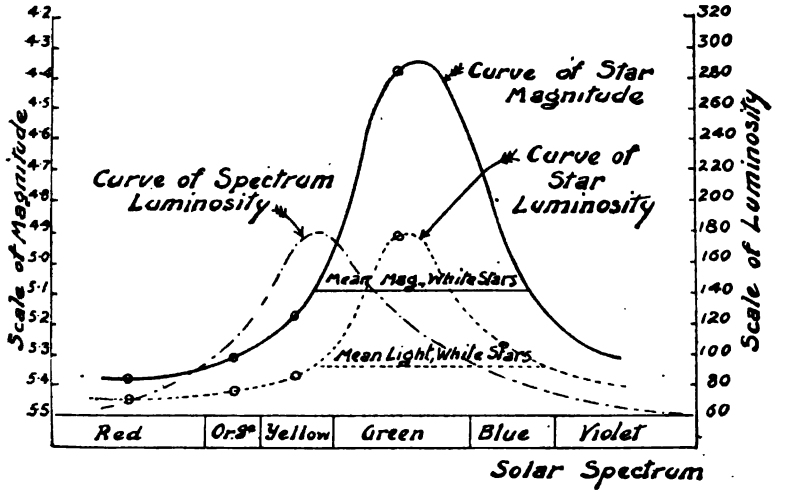


DIAGRAM I.

in hand the Potsdam colour observations, and hope shortly to publish further results on this point.

(ii) *Colour and Spectral Class.* — As far as we are aware, no classifications of stellar colour according to parallax or proper motion have yet been made. Mr. Franks has recently published three tables * giving the classification of colour according to spectral class. These tables, as they stand, are actually contingency tables, but the smallness of some of the groups and the extremely laborious process of working out 7 × 12-fold contingency tables has led us to concentrate the material in rather larger colour groups, keeping, however, the spectral classes the same. We have taken as our colour groups (*a*) O, (*b*) YG^{1}, Y^{1}, O*r*Y^{1} (practically the "white" group), (*c*) Y^{2}, (*d*) O*r*Y^{2}, O*r*^{2}, (*e*) Y^{3}, O*r*Y^{3}, O*r*^{3}, (*f*) O*r*R^{3}, R^{3}, thus making six colour groups or a 7 × 6-fold table. We owe to Dr. A.

* *Loc. cit.*, pp. 539-541.

Lee, of the Biometric Laboratory, University College, London, the calculation of the constants for Mr. Franks' three tables, with the following results:—

Correlation of Colour and Spectral Class.

432 Southern Stars from $-25°$ to S. Pole .	$C_1 = \cdot 71$
928 Northern Stars from $-25°$ to N. Pole .	$C_1 = \cdot 74$
1360 combined results	$C_1 = \cdot 71$

These results are of much interest and in very close accordance. There is no substantial difference between northern and southern stars in the relationship of colour and spectrum, but, as we might have anticipated, there is a very high relationship between the two characters. The relationship is more than double that which we have found between magnitude and colour. We shall see later that spectral class is far more clearly associated with magnitude than colour is.

Accordingly, we may sum up our first results on colour as follows:—The colour of a star depends to some extent on its magnitude, but in a far more marked manner on its spectral class.

It seems unnecessary to reproduce Mr. Franks' southern and northern star tables. His total star table is given here, with the groupings adopted, and in brackets the excess or defect of each group from the independent probability expectation.

TABLE II.

Contingency of Colour and Spectral Class.

Type.	O.	YG^1, Y^1, OrY^1.	Y^2.	OrY^2, Or^2.	Y^3, OrY^3, Or^3.	OrR^2, R^2.	Totals.
Helium stars	125	146	8	3	0	0	282
	(+63·62)	(+42·95)	(−36·37)	(−34·74)	(−33·18)	(−2·28)	
Hydrogen stars	168	195	14	0	0	0	377
	(+85·95)	(+57·23)	(−45·32)	(−50·45)	(−44·35)	(−3·05)	
α Carinæ	3	97	23	8	6	0	137
	(−26·82)	(+46·93)	(+1·44)	(−10·33)	(−10·12)	(−1·11)	
Solar stars	0	41	77	33	29	0	180
	(−39·18)	(−24·78)	(+48·68)	(+8·91)	(+7·82)	(−1·46)	
Arcturus	0	15	86	77	63	0	241
	(−52·45)	(−73·07)	(+48·08)	(+44·75)	(+34·65)	(−1·95)	
Aldebaran	0	0	4	22	43	6	75
	(−16·32)	(−27·41)	(−7·80)	(+11·96)	(+34·18)	(+5·39)	
Betelgeuse	0	3	2	39	19	5	68
	(−14·80)	(−21·85)	(−8·70)	(+29·9)	(+11·00)	(+4·45)	
Totals	296	497	214	182	160	11	1360

The great regularity of the differences certainly speaks for the excellence of the spectral and colour classes adopted by Mr. Franks.

(4) *Correlations with Spectral Class.*—We have seen in the pre-

ceding section that the association of spectral class and colour is markedly high, giving the mean square contingency coefficient $C_1 = \cdot 71$. Data for the correlations of spectral class with magnitude, proper motion, and parallax are given in the Yale Observatory memoir already cited.* It is true that the number of stars is small, but the total of 98 is larger than we were able to deal with when considering parallax in the first memoir. Further, some of the spectral classes have so small a frequency that, for statistical purposes, it was absolutely needful to group certain classes together. The classes actually used in the notation of the Draper Catalogue were—

Class A	=	13 Stars.
Class E	=	12 ,,
Classes F and G	=	34 ,,
Classes H and I	=	29 ,,
Class K	=	5 ,,
Classes M and Q	=	5 ,,

Practically, no weight can be placed on the last two results. Now parallax and proper motion are true quantitative characters, and η is accordingly the proper constant to calculate in these cases. It has also been found for magnitude. We again owe the whole numerical work to Dr. Lee. The results come out as follows :—

Correlation Ratio of Spectral Class and Parallax,		$\eta = \cdot 36 \pm \cdot 06$
,, ,, ,,	Proper Motion,	$\eta = \cdot 39 \pm \cdot 06$
,, ,, ,,	Magnitude	$\eta = \cdot 68 \pm \cdot 04$

All these results are significant compared with their probable errors, but before we discuss them it is well to ascertain whether the last result will be confirmed if we drop the idea of magnitude as a scalar quantity and proceed only by using it as an index to classification. Table III. is a contingency table for magnitude and spectral class for these 98 stars :—

TABLE III.

Contingency : Spectral Class and Magnitude Class. Draper Catalogue.

Magnitude.	A.	E.	F and G.	H and I.	K, M, and Q.	Totals.
− ·5 to + ·5	1	...	1	...	1	3
+ ·5 to 1·5	2	...	1	...	3	6
1·5 to 2·5	3	...	...	...	...	3
2·5 to 3·5	1	...	2	...	4	7
3·5 to 4·5	...	...	8	2	2	12
4·5 to 5·5	...	1	13	2	...	16
5·5 to 6·5	1	5	9	12	...	27
6·5 to 8	5	6	...	13	...	24
Totals	13	12	34	29	10	98

* *Loc. cit.*, see Table v. p. 204.

In working this table we have grouped in magnitude $-\cdot5$ to $1\cdot5$ and $1\cdot5$ to $3\cdot5$, or dealt only with a 6×5-fold table. The material is too sparse for finer classification. This table gives us $C_1=\cdot69(\pm\cdot07)$. It accordingly fully confirms the value obtained by the η method. It is, perhaps, worth noticing that it is in the first place the K, M, Q group, and then in the next place the F, G group, which contribute most to the value of the mean square contingency, *i.e.* these are the classes which have most markedly differentiated magnitudes. We see that the relationship between parallax and spectral class is only a little more than half that between magnitude and spectral class. Or we conclude that, on the basis of the Yale stars, the present chemical constitution of a star is of considerably greater importance than its distance in determining its magnitude.

It must, however, be remembered that 98 is rather a limited number to base definite statements upon; and as the point is of considerable interest, it is important to consider the matter from other aspects. Before the Yale memoir came to hand we had already formed Table IV., giving the magnitudes and spectral classes of over a thousand stars. It is based on the data provided in vol. xxviii., *Harvard Observatory Annals*. Pickering's 22 spectral classes would give a contingency table of unworkable magnitude, and accordingly we have grouped his spectral classes as follows, making a 7×8-fold table:—

TABLE IV.

Contingency of Magnitude and Spectral Class.

	Spectral Class.							
Magnitude.*	X.	Y.	Z.	V.	U.	W.	S.	Totals.
Below 1	.4	1	2	4	1	1	...	13
1 to 2	6	1	10	5	2	1	1	26
2 to 3	9	...	22	15	3	1	2	52
3 to 4	13	12	34	62	4	9	...	134
4 to 5	70	31	87	153	25	20	...	386
5 to 6	50	24	60	69	5	11	2	221
6 to 7	58	22	51	40	6	5	6	188
7 and above	23	5	40	18	2	1	5	94
Totals	233	96	306	366	48	49	16	1114

Our Z = Pickering's groups 1 to 5; Y = 6; X = 7 to 11; V = 13 to 16; U = 17 to 20; W = 12; T = 21 (does not occur); S = 22.

The mean magnitudes determined from a ·5 grouping of the classes arranged in order of magnitude are as follows:—

* Magnitude 4 to 5, for example, contains all stars of 4th and less than 5th magnitude.

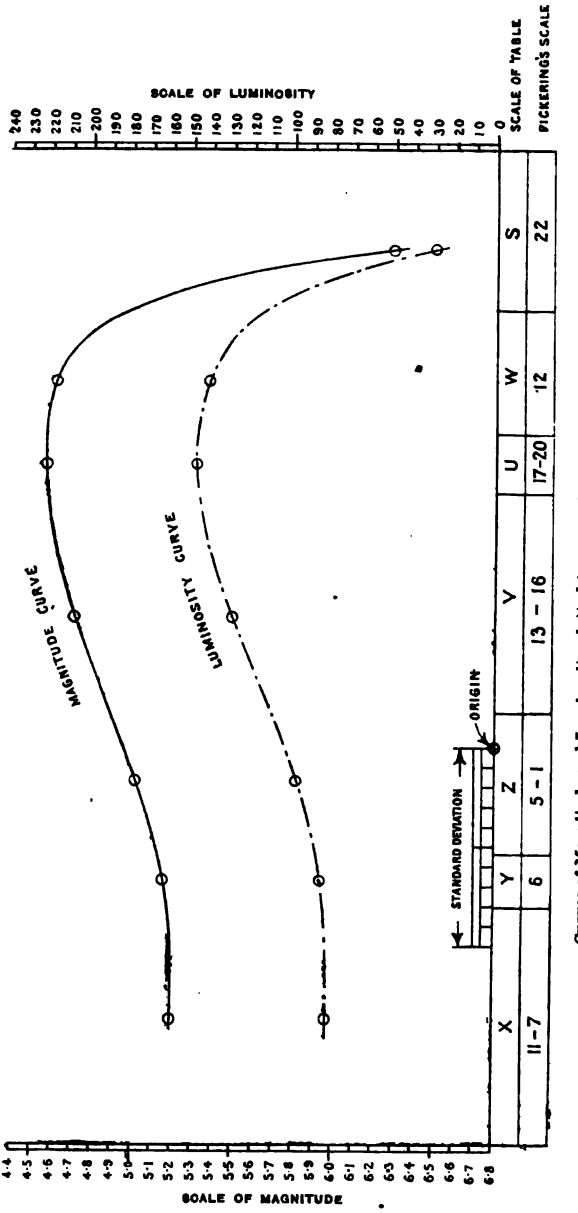
	Mean Magnitude.	No. of Stars.
Class S	6·29	16
„ X	5·20	233
„ Y	5·16	96
„ Z	5·02	306
„ V	4·71	366
„ W	4·62	49
„ U	4·57	48
Whole series	4·95 ± ·03	1114

Standard deviation of whole series = 1·49 ± ·02

The coefficient of mean square contingency for this long series was $C_1 = \cdot 43\ (\pm \cdot 02)$. Thus, while it is not as large as that found for the Draper Catalogue stars given in the Yale memoir, it still points to a fairly close relationship between spectral class and magnitude. It will, of course, be obvious that the order of mean magnitude of the spectral classes given above is not that of Pickering's class numbers, so that it may be presumed that the characters which led to his order of classification are not those which lead to order of magnitude. If we attempt to place our magnitudes in a continuous curve, we note that (i) more than one pair might be interchanged without affecting the limits fixed by the probable errors of random sampling, and (ii) we have no spectral scale upon which to plot the magnitudes or luminosities. The latter difficulty may be overcome in a manner which has proved of some service in other branches of statistical inquiry, namely, by assuming that the total spectral frequencies of each class are portions of a normal curve of frequency, and then plotting on verticals through the means (as deduced from a well-known property of the Gaussian curve) of the corresponding areas. Assuming that the groups, being merely qualitative, may be interchanged in order, it rarely happens that more than one arrangement gives a continuous curve. We have found Diagram II. by this method, plotting both magnitude and luminosity * curves. It will be seen that fairly smooth curves are reached in this manner; they would probably have been better had the U stars had greater frequency, and the V stars been dealt with in smaller groups. The diagram suggests, without of course proving, the likelihood of an ascent to a maximum luminosity from two different ends of a spectral classification.

It appeared probable that a classification of spectral characters based more admittedly on temperature considerations than those of either Secchi or Pickering would lead to a still high correlationship between magnitude and spectral character. Accordingly, appeal was made to Sir Norman Lockyer, who, with great kindness, provided a key index linking Pickering's classes with his own temperature classification. On further consideration, however, it

* The luminosity of the star of the 10th magnitude is taken as the unit and Pogson's value of the constant adopted.



Curves of Magnitude and Luminosity plotted to means for a normal frequency scale.

DIAGRAM II.

seemed probable that somewhat better results would be reached by working with Sir Norman Lockyer's own classified catalogue,* although this involved supplying from other sources (Harvard Photometry, where possible) the magnitudes of the recorded stars. We have from this catalogue formed Table V., giving spectral class and magnitude of 448 stars.

TABLE V.

Contingency: Stellar Magnitude and Spectral Class (Lockyer's Classification).

	Magnitude.							
Class.	Under 1·5.	1·5–2·5.	2·5–3·5.	3·5–4·5.	4·5–5·5.	5·5 and over.	Totals.	Means.
Argonian	...	1	1	...	...	...	2	...
Alnitamian	...	4	1	2	1	...	8	(2·95?)
Crucian	5	8	31	6	2	...	52	2·78
Achernian	1	1	1	8	7	...	18	4·00
Taurian	1	1	3	...	...	...	5	(2·35?)
Algolian	1	1	12	15	1	7	37	3·90
Rigelian	1	1	...	1	...	...	3	...
Markabian	...	4	8	5	1	...	18	3·12
Cygnian	1	...	1	...	...	...	2	...
Sirian	3	11	20	37	24	8	103	3·84
Polarian	...	5	2	6	...	...	13	3·02
Procyonian	2	3	10	20	5	1	41	3·55
Aldebarian	...	8	27	16	1	...	52	3·14
Arcturian	4	7	22	34	9	...	76	3·40
Antarian	2	4	7	3	1	...	17	2·71
Piscian	...	...	...	...	1	...	1	...
Totals	21	59	146	153	53	16	448	3·39

Here 1·5–2·5 signifies 1·5 and under 2·5. In finding the means, the group "under 1·5" was broken up into three groups.

The mean magnitude of the stars in this list was $3{\cdot}39 \pm 0{\cdot}4$, and their standard deviation $1{\cdot}158 \pm {\cdot}026$. They are thus much brighter and less variable than the stars of the two earlier series. The contingency coefficient for this grouping is

$$C_1 = {\cdot}54\ (\pm {\cdot}03).$$

We see accordingly that the association of magnitude and spectral class has been sensibly increased by using Sir Norman Lockyer's classification. The correlation is now about half way between that provided by Pickering's date and the 98 Yale stars.

* *Catalogue of 470 of the brighter Stars classified according to their Chemistry at the Solar Physics Observatory, South Kensington.*

Looking at the column of means, and rearranging it on Lockyer's plan,

Argonian ?

Alnitamian 2·95 ?

2·78	Crucian	Ascending Temperature.	Descending Temperature.	Achernian	4·00
2·35 ?	Taurian			Algolian	3·90
?	Rigelian			Markabian	3·12
?	Cygnian			Sirian	3·84
3·02	Polarian			Procyonian	3·55
3·14	Aldebarian			Arcturian	3·40
2·71	Antarian			...	
	...			Piscian	?

we see that the average magnitudes are greater on the side of "Descending Temperature," and less on the side of "Ascending Temperature," but the order of magnitude cannot be said to fit closely the classification order. The order is chaotic on the "Ascending Temperature" side; and although, with the exception of the Markabian class, it is orderly on the "Descending Temperature" side, the justification for the order does not lie in magnitude. When more material is available, it would seem that a spectral classification according to mean magnitude might be suggestive. All that we can venture to say at present is that magnitude is very far from being independent of spectral class, but that the reason why certain spectral classes have differentiated magnitudes is not evident in any of the classifications here dealt with.

(iii) *Spectral Class with Proper Motion and Parallax.*—We now turn to the relationship between spectral class and proper motion. In this case we have only dealt at present with the table provided in the Yale memoir, but there exists plenty of further data which we hope to work up. The correlation ratio for spectral class and the proper motions of the 98 stars given in that case is

$$\eta = \cdot 39 \pm \cdot 06.$$

Spectral class is thus shown to be definitely associated with proper motion. The association is only about half that of colour, and sensibly less than that of magnitude, still it is quite a considerable relationship. We might, *a priori*, anticipate that the association was an indirect effect of the correlation of spectral class and parallax, but it is easy to prove that this is at least only partly the case. The correlation ratio between parallax and spectral class deduced from the same material is

$$= \cdot 36 \pm .06.$$

This, again, is quite a considerable amount of relationship, but it is not greater but practically equal to the relationship of spectral class and proper motion.

Now if we have three characters, 1, 2, and 3, and if r_{12} be the correlation of the first and second without regard to the third, then

$$\rho_{12} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}}$$

is the "partial" correlation coefficient of the first and second characters for a constant value of the third. In other words, if the relationship between 1 and 2 be solely due to the relationship of both to 3, we should expect that within the array of 1 and 2 corresponding to a constant value of 3, the correlation would be zero between 1 and 2, or $\rho_{12} = 0$. This means that $r_{12} = r_{13} \times r_{23}$ is the test for the correlation of 1 and 2 being solely due to their relationship to 3. Applied to our particular case, if the correlation between spectral class and proper motion be solely due to the relation between both and parallax, we should expect the correlation coefficient of spectral class and proper motion to be equal to the product of the coefficients of parallax with spectral class and parallax with proper motion. Now we have seen that the relationship of spectral class with proper motion and parallax is about equal. Accordingly, we should expect a relationship between proper motion and parallax not very far from perfect, or the correlation coefficient about unity. This is very far indeed from the case; it does not exceed ·3 to ·5. It follows, accordingly, that the relationship of spectral class to proper motion is not an indirect effect of chemical constitution being a function of spatial distribution. It is, of course, partly due to this result, but, to judge from the Yale data, there is a sensible relationship between chemical constitution and intensity of stellar motion in space.

We may form some notion of the amount of this, as follows. Judging from the values of the correlation ratios, we may assume the correlation coefficients to be not very different from ·4 for spectral class with proper motion or with parallax. The correlation between proper motion and parallax is also not far from ·4 (see § 6). Hence we find for the value of the partial correlation of spectral class and proper motion for constant parallax—

$$\rho_{12} = \frac{\cdot 4 - \cdot 4 \times \cdot 4}{\sqrt{1 - (\cdot 4)^2}\sqrt{1 - (\cdot 4)^2}} = \cdot 2857 = \cdot 3, \text{ say.}$$

Thus, in rough numbers, on the data at present available, about 25 p.c. only of the relationship between spectral class and proper motion is due to parallax. It is desirable to place in a table (Table VI.) the mean magnitudes, proper motions, and parallaxes, as given by the Yale results grouped as dealt with here. It is perhaps unnecessary to observe that far more extended frequencies are needful before the numbers given can be finally accepted.

While scarcely any weight at all is to be placed on the results for classes K, M and Q, we still see that for the remainder, while the orders of spectral class for parallax and proper motion are more alike than they are in either case to that for magnitude, they still differ considerably from each other.

TABLE VI.

Characters and Spectral Class.

Magnitude.		Parallax.		Proper Motion.	
Mean.	Spectral Class.	Mean.	Spectral Class.	Mean.	Spectral Class.
6·35	E	·084	F and G	·878	K
6·30	H and I	·068	E	·708	E
4·59	F and G	·065	A	·665	F and G
3·95	A	·054	K	·646	H and I
2·54	M and Q	·025	H and I	·421	A
1·88	K	·020	M and Q	·134	M and Q
4·98	All classes.	·057	All classes.	·616	All classes.

To sum up our results for spectral class we may conclude as highly probable—

That there is a very considerable relationship between magnitude and spectral classes, which will probably be found to be intensified if more satisfactory spectral categories can be determined. This relationship, however, is at present sensibly lower than the relationship between colour and existing spectral classes, which is very high. There are, further, fairly considerable relations, in part independent, between spectral class and both parallax and proper motion. It is thus probable that the chemical constitution of a star is not only associated with a certain colour and brilliancy, but also with its spatial position (as determined by its distance from us), its motion in space, and its size (so far as size as well as intensity is an attribute of magnitude).

(5) *Correlations with Magnitude.* — In the present paper we have already seen that magnitude is related to both colour and spectral class. We may note here that the relation of magnitude to spectral class is not due to the relation of both to colour. On the other hand, the relationships between spectral class and both colour and magnitude are so high compared with that between magnitude and colour, that it is highly probable that the partial correlation between colour and magnitude for a given spectral class is *negative*. In other words, if the colour classes were arranged in order of the average magnitude of the stars contained in them, then, if we confined our attention to one spectral class only, say the Arcturus type, the order of magnitude for the colour classes within this spectral class would be *reversed*.

(i) *Magnitude and Parallax.*—In the former paper this relationship was deduced from 72 stars given for parallax by Newcomb. The deficiency of material was fully admitted, and the results as to correlation could only be considered as preliminary. They indicated, however, (i) that we should not expect to find a very high association between magnitude and parallax; (ii) that the correlation coefficient differing widely from both the correlation ratio and the contingency coefficient, the distribution was far from

normal; and (iii) the line of means was not approximately straight, but the parallax rose again with the fainter stars. The Yale memoir provides far better material of more than double the amount, namely, 173 stars. It is desirable to discuss how far this diverges from the previous results. The Yale authors have grouped their arrays of parallax in somewhat irregular magnitude classes and get the following results (Table ii. p. 201, *loc. cit.*):—

TABLE VII.

No of Stars.	Range of Magnitude.	Mean Magnitude.	Mean Parallax.
			″
10	0·0–1·5	0·81	·0950
29	2·0–4·9	3·75*	·0666†
33	5·0–6·2	5·65	·0558
34	6·3–7·0	6·68	·0447
31	7·1–7·9	7·57	·0168
36	8·0–9·0	8·33	·0472

The mean magnitude for the whole series is $6·16 \pm ·11$, with a standard deviation of $2·06 \pm ·07$. The mean magnitude of the Newcomb series was $4·03 \pm ·22$, with a standard deviation of $2·74 \pm ·15$. It contained on an average much brighter stars, but was a more variable sample in brightness. It will be seen that, as in the first sample, parallax tends to increase again with the faintest stars. Working out η for the arrays of parallax corresponding to the above magnitude ranges, we have

$$\eta = ·28 \pm ·05.$$

The relationship, therefore, of magnitude to parallax is about equal to its relation to colour, and not more than half its relation to spectral class.

We can, however, approach the subject from another standpoint: we may inquire into the distribution of magnitude for given arrays of parallax. We have the following table deduced from our author's Table iii. (p. 202):—

TABLE VIII.

No. of Stars.	Range of Parallax.	Mean Parallax.	Mean Magnitude.
7	−0″·13 to −0″·07	−0″·110	7·34
29	−0·06 to −0·00	−0·025	6·36
66	+0·00 to +0·06	+0·031	6·76
44	+0·07 to +0·13	+0·097	6·10
17	+0·14 to +0·20	+0·159	6·34

The mean parallax of the whole series is $0″·0460 \pm ″·0035$, with a standard deviation of $0″·0664 \pm ″·0025$. In the earlier series the mean parallax was $0″·145 \pm ″·011$, with a standard deviation of $0″·134 \pm ·007$, indicating that we were dealing, on the whole, with a nearer set of stars, but with a greater variability in distance. It is obvious that very little can be judged from the mean magni-

* 3·748; the authors have 3·8. † The authors have ″·066.

tudes, especially if we exclude the first two categories of negative parallaxes.* We have included them because the authors do, but it is not easy to see why the 7 stars with the greatest negative parallaxes are to be looked upon as a class with a really lower quantitative parallax than those of the 29 group with a less negative parallax. Since the parallax cannot be below zero, what the results signify is that an average error of $0''{\cdot}11$ was possible in the group of 7, and one of $0''{\cdot}025$, or about a quarter of it, possible in the group of 29 stars. Errors of observation being thus more liable in the 7 group, it might even be reasonable to suppose members of this group to have, on the whole, higher true parallaxes than those of the 29 group; and the appearance of this group *at the top* with the high magnitude 7·34 is somewhat misleading. Leaving it, however, to form part of the series, we find

$$\eta = {\cdot}20 \pm {\cdot}05,$$

or we conclude that the determination of magnitude from parallax is considerably more inaccurate than that of parallax from magnitude. The association has indeed fallen below anything yet dealt with in the present paper. The inequality of the two values of η shows that our distribution is very far from Gaussian. It is clear that in these circumstances contingency is the only method by which we can approach a *unique* measure of the relationship of parallax and magnitude, and accordingly the following contingency table has been prepared connecting the two characters. It gives us

$$C_1 = {\cdot}32\ (\pm {\cdot}05).$$

In the previous memoir for Newcomb's 72 stars we had

$$C_1 = {\cdot}41\ (\pm {\cdot}08).$$

TABLE IX.

Contingency: Magnitude and Parallax.

Magnitude.	Parallax. $-0''{\cdot}13$ to $-0''{\cdot}07$.	$-0''{\cdot}06$ to $0''{\cdot}00$.	$0''{\cdot}00$ to $0''{\cdot}06$.	$0''{\cdot}07$ to $0''{\cdot}13$.	$0''{\cdot}14$ to $0''{\cdot}20$.	Totals.
Below 3·95†	...	4	3	7	2	16
3·95 to 4·95	...	1	5	6	1	13
4·95 to 5·95	...	4	8	4	4	20
5·95 to 6·95	2	9	13	8	3	35
6·95 to 7·95	3	6	21	11	2	43
Above 7·95‡	2	5	16	8	5	36
Totals	7	29	66	44	17	163

* We would place considerable stress on this, because much of the impressiveness of the above table, both visually and when reduced numerically, lies on the place assigned to the group of largest negative parallaxes, with their mean magnitude 7·34. Now this consists of only 7 stars, and the mean magnitude may be merely indicative of greater errors arising with fainter stars.

† With the exception of three stars, 2·3 2·0, and 2·8, all were above 3·0.

‡ All below 9, with the exception of one star 9.

Thus practically the previous result and the present for the contingency agree within the limits of the probable error of the difference. Further, the contingency and correlation ratio results are also in close agreement, assuming we class by magnitude. Now the series we are considering at present is very different from the earlier series: that series contained 20 stars brighter than magnitude 2, while the present contains *none*. It contained only 7 stars above 7·95, while the present series contains 36. It is perfectly correct that in both cases stars have been frequently selected because they had large proper motions, and so may have large parallaxes. But does this really affect the argument that there is comparatively small relationship between magnitude and parallax? In order to do so, it must mean that the partial correlation is much lower than the absolute correlation, and this can only happen if the correlation between magnitude and proper motion is very high. To this point we now turn, because some criticism of the earlier paper has been made on this ground.

(ii) *Magnitude and Proper Motion.*—Any amount of material is forthcoming, of course, on this point, but it seemed advisable to work with the best available proper motions. Accordingly we used the *Catalogue of 627 Principal Standard Stars*, 1904, by Lewis Boss. Omitting variables, we had 303 stars with positive, 317 with negative, proper motions in R.A.* There were 195 stars with positive and 426 stars with negative proper motion in declination. There was no necessity to separate these groups, as we are dealing only with the size and not sign of the proper motions, but it was desirable to separate into random groups in order to test the steadiness of the coefficients, and the positive and negative groups served as well as any others.

Four tables of contingency were formed, given as X., XI., XII. and XIII. below. We owe the working of the contingency coefficients of the first two tables to Mr. A. M. Pritchard, of the Hartley University College, Southampton, and the last two to Dr. Alice Lee.

The statistical constants reached are as follows:—

Nature of Proper Motion.	Mean Magnitude.	Standard Deviation in Magnitude.	Contingency.
Positive in R.A.	3·63±·05	1·16±·03	·30 (±·04)
Negative in R.A.	3·79±·04	1·12±·03	·38 (±·04)
Positive in Declination	3·88±·06	1·16±·04	·36 (±·05)
Negative in Declination	3·72±·04	1·08±·02	·23 (±·03)

The mean degree of relationship between proper motion and magnitude is accordingly ·32.

In Diagrams III. and IV. the means of arrays of magnitude for given classes of proper motion are plotted; it will be seen at once how little influence of an orderly kind a selection of stars by proper motions would have on their magnitude.

* One star out of the 621 non-variables has been overlooked in tabling the proper motions in R.A. It lies in group of magnitude 3 5-4·4.

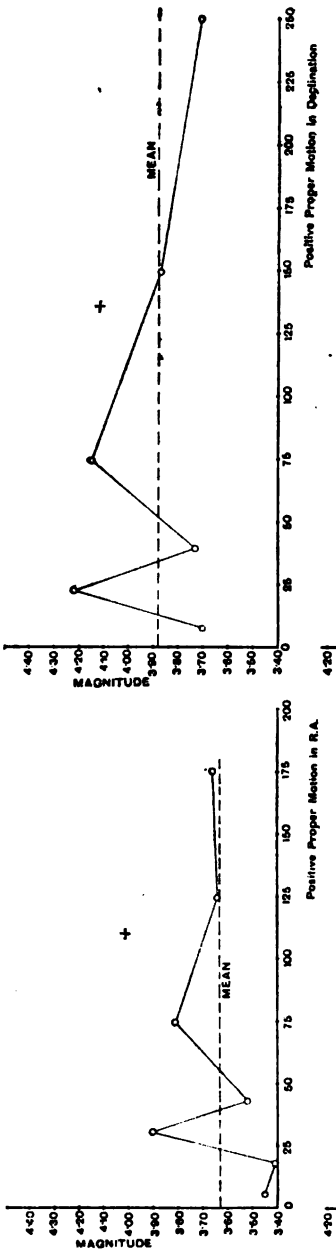


DIAGRAM III

DIAGRAM IV.

Boss, of course, deals only with a selected group of stars, but we can test the result more or less closely on the Yale observations, which give total proper motions for magnitude classes, and magnitudes for total proper motion classes. We owe to Dr. Lee the correlation ratios for the two cases; they are respectively—

Correlation ratio: proper motions for magnitude classes, $\eta = \cdot 43 \pm \cdot 04$
" " magnitudes for proper motion classes, $\eta = \cdot 22 \pm \cdot 05$

The average relationship between the two characters is thus again about ·33. Thus we may safely conclude that the deviation

TABLE X.

Magnitude and Proper Motion in R.A. (μ positive).

	Magnitude.						
	Less than 1·5.	1·5 to 2·4.	2·5 to 3·4.	3·5 to 4·4.	4·5 to 5·4.	5·5 and over.	Totals.
0 to 11	2	9	19	21	11	2	64
12 to 24	2	3	8	23	3	...	39
25 to 36	...	2	5	12	9	...	28
37 to 49	1	3	7	11	3	1	26
50 to 99	1	4	8	31	12	2	58
100 to 149	1	7	8	15	8	3	42
150 and over	3	5	7	19	8	4	46
Totals	10	33	62	132	54	12	303

TABLE XI.

Magnitude and Proper Motion in R.A. (μ negative).

	Magnitude.						
	Less than 1·5.	1·5 to 2·4.	2·5 to 3·4.	3·5 to 4·4.	4·5 to 5·4.	5·5 and over.	Totals.
0 to 11	1	7	19	20	13	4	64
12 to 24	...	1	12	19	17	...	49
25 to 36	2	3	12	10	6	2	35
37 to 49	...	2	7	12	4	...	25
50 to 99	...	2	14	29	10	3	58
100 to 149	...	1	5	15	2	...	23
150 and over	3	6	11	21	13	9	63
Totals	6	22	80	126	65	18	317

TABLE XII.

Magnitude and Proper Motion in Declination (μ' positive).

Proper Motion.	Magnitude. Less than 1·5	1·5 to 2·4.	2·5 to 3·4.	3·5 to 4·4.	4·5 to 5·4.	5·5 and over.	Totals.
0 to 15	2	9	16	20	7	7	61
16 to 29	...	1	7	15	10	4	37
30 to 49	...	2	9	11	3	2	27
50 to 99	...	...	6	11	5	3	25
100 to 199	...	1	6	12	4	1	24
200 and over	2	...	4	11	3	1	21
Totals	4	13	48	80	32	18	195

TABLE XIII.

Magnitude and Proper Motion in Declination (μ' negative).

Proper Motion.	Magnitude. Less than 1·5.	1·5 to 2·4.	2·5 to 3·4.	3·5 to 4·4.	4·5 to 5·4.	5·5 and over.	Totals.
0 to 15	3	10	20	40	20	3	96
16 to 29	2	8	15	29	18	1	73
30 to 49	2	4	26	31	15	3	81
50 to 99	1	5	13	39	21	3	82
100 to 199	2	10	11	18	6	2	49
200 and over	2	5	9	22	7	...	45
Totals	12	42	94	179	87	12	426

from independent variation in the distribution of magnitude and proper motion is not very far from ·35. It is therefore possibly slightly higher than the value found for magnitude and parallax, which is about ·30. It will, we think, be evident that a selection by proper motions would have *comparatively* small effect in modifying the relation found between magnitude and parallax. Some rough estimate of the order of the change can be formed by considering what would be the absolute coefficient of correlation resulting from a partial coefficient of ·30 obtained by selecting stars of a single proper motion only; this will probably give an upper limit. We have

$$\rho_{12} = \frac{r_{12} - \cdot 35 \times r_{23}}{\sqrt{1 - (\cdot 35)^2}\sqrt{1 - r_{23}^2}}$$

Here r_{23} is the correlation between proper motion and parallax. In the former paper for the 72 stars this was found to be about ·4 (p. 449); from the Yale data (see below) it is under ·38. Assuming it = ·38, and putting $\rho_{12} = ·30$, we deduce $r_{12} = ·39$, say. We think, therefore, we may safely assert that the relationship between parallax and magnitude, if no selection by proper motions had taken place, is not likely to prove as great as ·4, and probably lies considerably under this value.

We would suggest, therefore, that the lowness of the magnitude and parallax result obtained in the former paper is not due to any special selection of parallax stars by their proper motions. This could not largely influence the relationship of magnitude and parallax, because proper motion is only moderately correlated with parallax, and still less with magnitude.

(iii) Before we leave the subject of the correlation of magnitude and proper motion, it is worth while noting that it has also been investigated in an entirely different manner. In the above inquiry we have found the correlation ratio for the total proper motion and the magnitude in the case of the Yale stars; we have further calculated the contingency coefficients between two sets of groups of magnitude and proper motion in R.A. and declination respectively from the Boss Catalogue stars. But for other purposes the proper motion in declination, μ', and the proper motion perpendicular to a declination circle, $\mu \cos \delta$, had been found and taken out. Now we are concerned only with the size of the proper motions, and not their sense, at present. We divided, however, our data into two halves for the purpose of checking results. The stars from the equator to the North Pole, 298 in number,* have been taken in one group, and those from the equator to the South Pole, 323 in number, in a second.

Further, to avoid lengthy analysis, we dealt separately with proper motion parallel to the declination circle (μ') and perpendicular to it ($\mu \cos \delta$). Thus we have four series to discuss. The classifications of magnitude were as follows: under 1·5, 1·5 and under 2·5, 2·5 and under 3·5, . . . 6·5 and over. The proper motions are of such a wide range that no grouping was adopted, but the means and standard deviations found by the laborious processes of adding and of squaring. No stars were omitted, although one is sorely tempted to omit those two or three of abnormal proper motions, as undoubtedly abnormal on any probability test. Any selection of this or any other kind must, however, be for our present purpose dangerous, and accordingly we have allowed equal weight to all the available stars in Boss's list.†

The following results were obtained:—

* 6 variable stars in Boss's Catalogue were of necessity omitted.

† It may be of interest to note that, working by correlation methods on the Boss stars I find for the apex of the Sun's way R.A. = 275°, $\delta = +28°·5$, but the requisite conditions for a random distribution are hopelessly unfulfilled. The correlations directly provide a method of approaching the problem of multiple drifts which I hope to deal with later.—K. P.

TABLE XIV.

Northern Stars.

Magnitude.	No.	μ'.	$\mu \cos \delta$.
Under 1·5	9	372·44	214·33
1·5 to 2·4	25	71·08	80·00
2·5 to 3·4	61	71·15	64·59
3·5 to 4·4	137	103·35	76·85
4·5 to 5·4	54	93·70	94·81
5·5 to 6·4	9	66·22	85·00
6·5 and over	3	55·33	50·33
All stars	298	98·83	81·99

The complete constants for the whole of the 298 stars are—

Mean μ' $98{\cdot}83 \pm 7{\cdot}73$. Standard deviation of μ' $196{\cdot}85 \pm 5{\cdot}47$.

Mean $\mu \cos \delta$ $81{\cdot}99 \pm 6{\cdot}66$. Standard deviation of $\mu \cos \delta$ $170{\cdot}40 \pm 4{\cdot}71$.

TABLE XV.

Southern Stars.

Magnitude.	No.	μ'.	$\mu \cos \delta$.
Under 1·5	7	44·29	48·86
1·5 to 2·4	30	78·60	55·93
2·5 to 3·4	81	75·80	58·28
3·5 to 4·4	122	101·11	72·36
4·5 to 5·4	65	119·51	78·05
5·5 to 6·4	13	47·15	35·00
6·5 and over	5	22·40	17·20
All stars	323	91·76	65·58

The complete constants for the whole of the 323 stars are—

Mean μ' $91{\cdot}76 \pm 8{\cdot}87$. Standard deviation of μ' $236{\cdot}33 \pm 6{\cdot}27$.

Mean $\mu \cos \delta$ $65{\cdot}58 \pm 6{\cdot}05$. Standard deviation of $\mu \cos \delta$ $161{\cdot}31 \pm 4{\cdot}28$.

At first sight it might appear that the northern group of stars had a larger proper motion than the southern, but, considering the probable errors, it is doubtful whether any stress can be laid on this. Thus we have for southern and northern groups—

Difference of μ' . . . . . $= 7{\cdot}07 \pm 11{\cdot}77$
Difference of $\mu \cos \delta$ . . . $= 16{\cdot}41 \pm 9{\cdot}00$
Difference of S. D.'s of μ' . $= 38{\cdot}48 \pm 8{\cdot}32$
Difference of S. D's of $\mu \cos \delta$ $= 9{\cdot}18 \pm 6{\cdot}36$

Only in the third case is the difference more than twice its probable error, and it may just be that proper motions in declination in the southern hemisphere are more variable than in the northern, but much larger numbers would have to be used really to demonstrate this.

If, on the other hand, we compare the results for proper motion along and perpendicular to the declination circle, we have—

Northern Stars:

Difference of mean μ' and mean $\mu \cos \delta$ $= 16{\cdot}84 \pm 10{\cdot}20$

Difference of S. D. of μ' and S. D. of $\mu \cos \delta = 27{\cdot}36 \pm 7{\cdot}22$

Southern Stars:

Difference of mean μ' and mean $\mu \cos \delta$ $= 26{\cdot}18 \pm 10{\cdot}74$

Difference of S. D. of μ' and S. D. of $\mu \cos \delta = 75{\cdot}02 \pm 7{\cdot}59$

Three of these differences are more than twice their probable errors; and looking at the values as a whole, it seems not improbable that proper motion in the declination circle is larger and more variable than proper motion perpendicular to it. The point deserves fuller consideration, especially in its relation to the position of the apex of the Sun's way. We shall return to the matter later, from another standpoint.

If we now examine the means of the magnitude arrays, we notice at once some very remarkable points. Regarding the mean values of both proper motion components, we note that, with one exception to be discussed below, the deviations from the mean values of the means of the arrays are small as compared to the corresponding variabilities, but these deviations are remarkably regular.

Taking the southern stars as typical, we find (Diagram V.) a continuous rise in proper motion for both components until stars of about 4·5 magnitude, then the proper motion falls again and becomes still smaller for the faintest stars. The same rule is apparent in the northern stars also, except that in the group of extremely bright stars we have a very large proper motion, which again, however, when we remember size of mean and S. D., is not so big as it appears. Still it is definitely significant, and marks the group of brightest northern stars as significantly different from the corresponding group of southern stars. The Diagram V. will bring out the remarkable character of these magnitude and proper motion curves.

If the determining link between magnitude and proper motion were parallax, surely we might anticipate a uniformly decreasing proper motion with increasing magnitude? Yet we see in all four cases the same phenomenon—the low value of proper motion for the stars of 1·5 to 3·5 magnitude, its growth to a maximum, and then its ultimate fall. It is true the variations are small, but their comparative regularity is very great. It is difficult to see what special selection of these standard stars could have led to this result. It is one that will be tested on much larger masses of material, but Boss's Catalogue was selected on account of the high

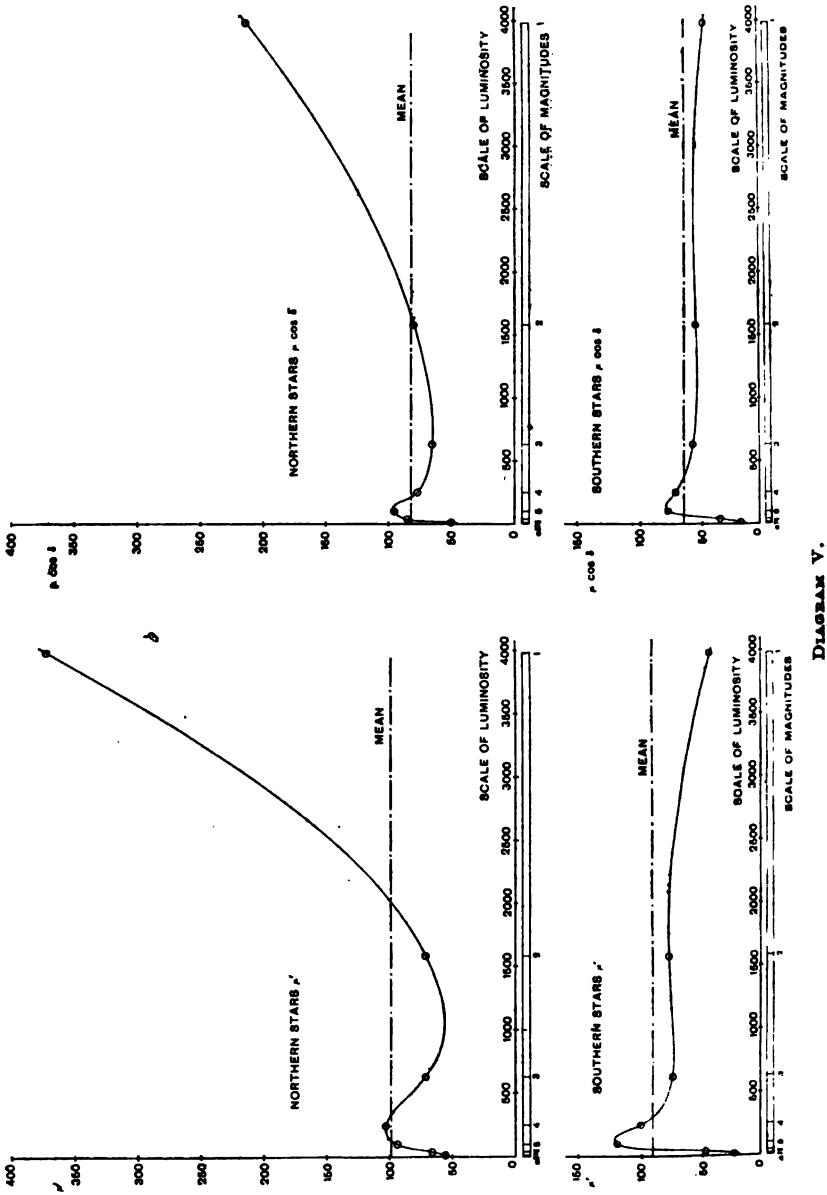

NORTHERN STARS ρ cos δ
MEAN
SCALE OF LUMINOSITY
SCALE OF MAGNITUDES
SOUTHERN STARS ρ cos δ
NORTHERN STARS ρ'
SOUTHERN STARS ρ'

DIAGRAM V.

accuracy of his results and the manageable number of stars provided. We may look at the point from a second aspect. The lines of means are curved, and accordingly the coefficient of correlation is not the proper measure of the association between magnitude and proper motion. It is provided by the correlation ratio η. We have

Northern Stars:

Correlation ratio of magnitude and μ' $\eta = \cdot 26 \pm \cdot 04$

" " " $\mu \cos \delta$, $\eta = \cdot 15 \pm \cdot 04$

Southern Stars:

Correlation ratio of magnitude and μ' $\eta = \cdot 09 \pm \cdot 04$

" " " $\mu \cos \delta$, $\eta = \cdot 08 \pm \cdot 04$

It will thus be seen that the amount of correlation even for the northern stars is only small. Working on the 173 stars of the Yale memoir, we have found the correlation ratio for proper motion in arrays of limited magnitude; it is:

$$\eta = \cdot 22 \pm \cdot 05.$$

Considering that the Yale stars are all northern stars, but that in this case the *total* proper motion has been taken, and not the component proper motions, we see that the agreement, notwithstanding the wide difference of material, is excellent,—well within the limits of the probable errors.

While the correlation is slight for the northern stars, it is practically of little or no service, as far as selection goes, for the southern stars, or we must conclude that—

A selection of stars by their proper motions must have very little influence indeed on the relation between magnitude and parallax.

It will accordingly require some very stringent form of investigation to demonstrate that the form of the parallax-magnitude curve can be sensibly influenced by a proper motion selection.

As far as the present data extends, we would venture to suggest, therefore, that the criticism that the stars dealt with are specially selected is not really a valid one, for this selection would not largely influence the relationship of magnitude and parallax. We must conclude that the present results confirm the early ones, namely, they show that magnitude is a quantity much more closely associated with chemical condition (spectral class) than with distance; indeed, the association with colour is almost as great as the association with distance.

(6) *Correlations with Parallax.*—We have already dealt with the association between parallax and magnitude, colour and spectral class (pp. 422, 423, 428, and 432). The remaining feature is proper motion. This is provided by the Yale observations. Accepting the authors' proper motion classes, the arrays of parallax give us (Table I.),

Correlation ratio, parallaxes for proper motion classes,* $\eta = \cdot 36 \pm \cdot 05$

* This result is for total proper motion in a great circle, but whether we use component proper motions in arcs or again the values in R.A. and δ makes but little difference in the correlation values.

and again (Table III.),

Correlation ratio, proper motions for parallax classes, $\eta = \cdot 38 \pm \cdot 04$

The previous work on the 72 stars gave—

Correlation coefficient, proper motion in R.A. and parallax, $r = \cdot 44 \pm \cdot 06$
" " " in declination and parallax, $r = \cdot 41 \pm \cdot 07$

Considering the size of the probable errors, the present results are reasonably in accordance with the former, or we may take it that the correlation between parallax and proper motion is not far from linear, and of magnitude about ·40. Thus we have the following scheme of relationship:—

Parallax and proper motion . . .	·38
Parallax and spectral class	·36
Parallax and magnitude	·30

Thus, while the distance of a star is sensibly related to its proper motion, this relationship is not really more significant than the relation to spectral class; and it is quite possible that if the spectra were assorted according to parallax, it would be possible to form spectral categories which would give a far higher association between parallax and spectral class than between the former and proper motion. The parallax stars have not yet been dealt with as to colour.

We have seen that, on the basis of the Yale stars, the correlation between parallax and proper motion is nearly linear. It is accordingly of considerable interest to obtain the line giving the mean parallax π_m for a group of stars with a given total proper motion p in a great circle. Let $\bar{\pi}$ be the mean parallax, $\bar{p}$ the mean proper motion, σ_π the standard deviation in parallax, σ_p the standard deviation in proper motion of the whole group of the Yale stars. Then we have—

$$\bar{\pi} = 0''\cdot 0460 \pm \cdot 0035, \qquad \sigma_\pi = 0''\cdot 0664 \pm \cdot 0025$$

$$\bar{p} = 0''\cdot 6763 \pm \cdot 0205, \qquad \sigma_p = 0''\cdot 3997 \pm \cdot 0205$$

we may take $r = \eta_{13} = \cdot 4006$.

Thus
$$\pi_m - \bar{\pi} = \frac{r\sigma_\pi}{\sigma_p}(p - \bar{p}),$$

or, in numbers,
$$\pi_m = 0''\cdot 0010 + 0\cdot 0666p,$$
$$= \tfrac{1}{1000} + \tfrac{1}{15}p.$$

If we neglect the $\frac{1}{1000}''$ as negligible in the case of parallax, we conclude that the mean parallax of an array of stars of given proper motion is one-fifteenth of that proper motion.* This is absolutely identical with the statement made by Newcomb, largely on theoretical grounds: †

* The 70 odd stars dealt with in the earlier memoir, if we compare total proper motion in a great circle and parallax, give the correlation of parallax and proper motion as ·58 and the regression coefficient $\frac{1}{17}$. This is as close to $\frac{1}{15}$ as we could expect from the material.

† *The Astronomical Journal*, vol. xxii. p. 169, 1902.

"That is, if we measure the parallaxes of all the stars having a given proper motion, we may expect the mean result to be about $\frac{1}{15}$ of the proper motion."

The interesting point of this result is that the value which was given on theoretical grounds by a brilliant astronomer, and appears then to have passed unquestioned, should have met with disapproval when actually found from observations by the statistician. As we have just indicated, Newcomb's $\frac{1}{15}$ is but little more or less than our statement that the correlation between proper motion and parallax equals ·4. Newcomb's

$$\text{mean } \pi = 0{\cdot}064\mu,$$

where μ = our p, is only a regression line without the constant term, which in this case, if it be not actually zero, is certainly very small. As far, then, as our reduction of the Yale data goes, it tends to confirm Newcomb's theory of stellar distribution; it also shows that our values of the parallax and proper motion correlation and of the parallax and proper motion standard deviations were reasonable values.

We find from the Yale data that the mean proper motion p_m for stars of a given parallax π is given by the regression line

$$p_m = 0{\cdot}5654 + 2{\cdot}4114\pi.$$

This does not agree with Newcomb's relation (*loc. cit.*, p. 168), and, of course, only applies to the range of stars in the Yale data. These, however, do not appear to satisfy Newcomb's equation.

(7) *Parallactic Motion.*—Still another method of approaching the parallax and proper motion correlation may be deduced from the Sun's motion. Let v = the velocity of the Sun, ρ the distance of a given star, and i = the index v/ρ. Then, if π be the parallax, p the proper motion of the star, and a the radius of the Earth's orbit,

$$\pi = \frac{a}{v} i,$$

where π and i may be measured in seconds of angle.

Since v/a is a constant, we have

$$r_{ip} = r_{\pi p},$$

or the correlation of parallax and proper motion is the same as the correlation of this index and proper motion. Suppose that σ_x represents the standard deviation of any variable x, then

$$\sigma_\pi = \sigma_i a/v,$$

and

$$r_{ip}\sigma_i/\sigma_p = \frac{v}{a} \times r_{\pi p}\sigma_\pi/\sigma_p.$$

Now $r_{ip}\sigma_i/\sigma_p = m$ is the slope of the best fitting line to the curve in which the mean value of i is plotted to a given proper motion class of stars.

The value of m has not been found at present because of the

labour of determining i for small groups of proper motions, but some idea of its value for large groupings of proper motions can be obtained from the papers by Oscar Stumpe* and by Messrs. Dyson and Thackeray.† These give us—

TABLE XVI.

Dyson and Thackeray.				*Stumpe.*			
No. of Stars.	Proper Motion. Group.	Proper Motion. Presumed Mean.	Value of i.	Proper Motion. Group.	Proper Motion. Actual Mean.	Value of i.	No. of Stars.
2885	0″·00 to 0″·05	0″·025	0″·017				
800	0″·05 to 0″·10	0″·075	0″·045				
316	0″·10 to 0″·20	0″·150	0″·084				
163	0″·20 and over	0″·312	0″·251	0″·16 to 0″·32	0″·23	0″·140	551
...	...	...	...	0″·32 to 0″·64	0″·43	0″·295	340
...	...	...	..	0″·64 to 1″·28	0″·85	0″·608	105
...	...	...	...	1″·28 and over	2″·39	2″·057	58

It will be seen that Stumpe worked with stars of far larger proper motion than Messrs. Dyson and Thackeray. We have only been able to give the presumed means in the case of the first set of stars as the actual values of the total proper motion in a great circle are not tabled. Assuming the means to approximate to the values given, we find for the first series

$$m = \cdot 7395$$

and for the second series

$$m = \cdot 8767$$

using the method of least squares to determine $m = r_{ip}\sigma_i/\sigma_p$,—*i.e.* the correlation process. It will be seen at once that the two sets of observations give no close agreement; the ranges of stars dealt with are very different.‡ If we take $m = \cdot 88$ we find

$$r_{\pi p}\frac{\sigma_\pi}{\sigma_p} = \cdot 88\frac{a}{v}.$$

* *Astronomische Nachrichten*, Bd. 125, ss. 385–426.

† *Monthly Notices*, vol. lxv. pp. 429.

‡ The graph of the Dyson-Thackeray and Stumpe data shows that their first three points form a more or less continuous curve with his four points, but that if our estimation of the mean proper motion of the stars above 0″·20 proper motion be at all correct, the fourth Dyson-Thackeray point does not lie on this continuous curve. This is possibly due to a considerable underestimation of the mean value of the "over 0″·20" group. If we omitted this fourth point, we should get an initial slope m_0 to the regression curve of less than ·5, a value considerably nearer that required to reconcile the parallax and Sun's motion observations discussed below. The actual regression lines

$$i_m = \cdot 7395\, p - 0'' \cdot 004 \text{ (Dyson-Thackeray)}$$

$$i_m = \cdot 8767\, p - 0'' \cdot 074 \text{ (Stumpe)}$$

differ significantly. This is due to the fact that the regression curve is far from linear.

Now if v be 16 miles per second $v/a = 5{\cdot}4$ about, and accordingly

$$r_{\pi p}\frac{\sigma_\pi}{\sigma_p} = {\cdot}16 = \frac{1}{6} \text{ about.}$$

This is very different from the $\frac{1}{15}$ which has been given in the present memoir and by Newcomb. It may be said that this is due to the value found for the correlation of parallax and proper motion being too small. We can examine this as follows:—

Every correlation coefficient must be less than unity. Hence

$$r_{\pi p} = \frac{1}{6}\frac{\sigma_p}{\sigma_\pi},$$

with the above values, must be less than unity. Now the variability of proper motion is fairly well known. We find it to be $0''{\cdot}6$ for Stumpe's 1054 stars. It is $0''{\cdot}4$ for the Yale stars. Let us put it $0''{\cdot}5$, then

$$r_{\pi p} = \frac{1}{12\sigma_\pi},$$

and accordingly σ_π must be $> \frac{1}{12}$ or $> {\cdot}083$. The Yale results give $\sigma_\pi = {\cdot}066$. We thus reach an impossible value for the correlation of parallax and proper motion. We need σ_π to be about $0''{\cdot}2$ to get this correlation a reasonable value. The curious part about this result is this: If e be the error in determining the true parallax π', the observed parallax $\pi = \pi' \pm e$, and accordingly

$$\sigma_\pi^2 = \sigma_{\pi'}^2 + \sigma_e^2.$$

There cannot be a doubt that σ_e is large; thus we should expect the true value of variability in parallax to be considerably less than the observed value, owing to the liability of parallax to error.

The only suggestions that we note for overcoming the difficulty are—

(i) That a much higher value must be attributed to the velocity of the Sun than is usually adopted. Spectroscopic determinations, however, indicate a lower value.

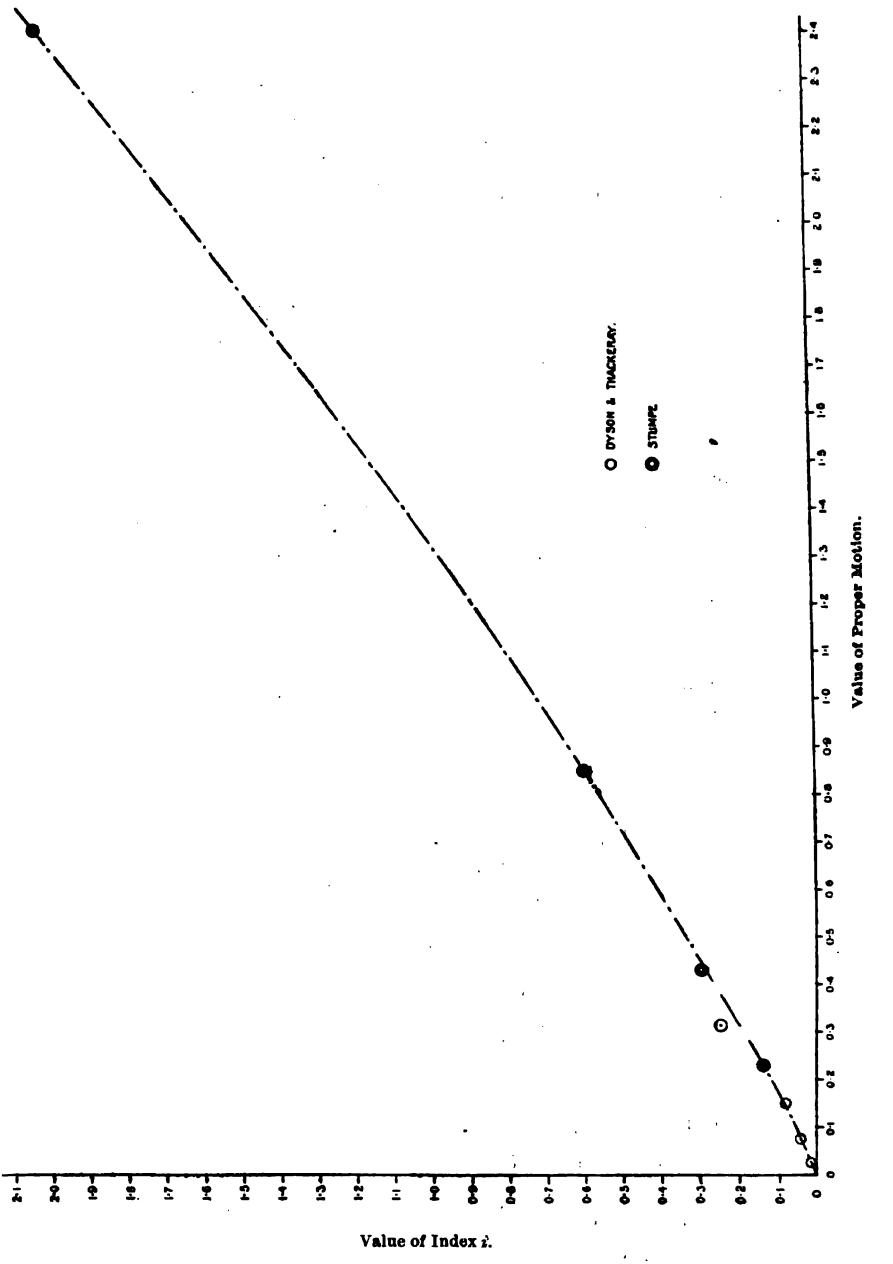
(ii) π observed is largely a differential parallax $= \pi_2 - \pi_1$, say. Hence

$$\sigma_\pi^2 = \sigma_{\pi_2}^2 + \sigma_{\pi_1}^2 - 2\sigma_{\pi_1}\sigma_{\pi_2}r_{12},$$

where r_{12} is the correlation between the component parallaxes. If r_{12} were zero, then again σ_π is sensibly larger than the true value σ_{π_2} or σ_{π_1}; but if the component parallaxes were highly correlated, *i.e.* if the two stars observed were about the same distance from the Sun, notwithstanding their differences of magnitude or proper motion, then we should have

$$\sigma_\pi^2 \text{ possibly nearer } (\sigma_{\pi_2} - \sigma_{\pi_1})^2 \text{ than to } \sigma_{\pi_1}^2 + \sigma_{\pi_2}^2;$$

in other words the observed variability of parallax might be much in defect. This is the only suggestion we can make to account for σ_π being sensibly less than the true value of the sample. It


DYSON & THACKERAY.
STUMPE.
Value of Proper Motion.
Value of Index i.

involves the assumption that near stars on which the differential parallax has been worked are really near each other.

(iii) If we study the graph (Diagram VI.) of the Dyson-Thackeray and Stumpe observations, we see that the three points of the former investigators and the first of the latter fall nearly on a straight line of slope about ·5. If we were to give m its initial value = ·45, say, an increase of speed in the Sun to about 20 miles per second would render the two results accordant. The assumption here made is that the stars with a large proper motion belong possibly to a different system to those with the smallest proper motions, and ought not to be used to determine m.

(iv) Lastly, it is possible that among the stars used by Dyson and Thackeray and by Oscar Stumpe, there are many with considerably higher parallaxes than appear in the small samples for which parallax has at present been determined, and that thus the variability of parallax will ultimately be found to exceed the values so far determined. This must denote the existence of a *considerable* number of really large parallaxes hitherto undiscovered, for we must not forget that the present values of parallax have, in all probability, a considerably exaggerated variability due to observational error.

From the statistical standpoint, therefore, the determination of considerable numbers of additional parallaxes seems to be a rather pressing problem. It is not unreasonable to suggest that there is something that needs reconciling in the values determined for the index i and the value usually assumed for the Sun's motion in space.

(8) *Correlations with Proper Motion.* — Sufficient material exists for finding the association, if any, between proper motion and colour, but we have not yet had the opportunity of dealing with it. The fact that the correlation ratio for spectral classes is as high as ·39 would indicate that colour and proper motion may well be sensibly related. Magnitude and parallax correlations with proper motion have been dealt with above. Accordingly, we have approximately the following system :—

Proper Motion and Parallax	·39
Proper Motion and Spectral Class . . .	·36
Proper Motion and Magnitude	·35

(9) *Conclusions.* — While we are fully aware how badly the mere statistician may stumble in dealing with astronomical data, we still think that the general relationships shown by the statistical correlation constants may be of value to astronomers. They serve to indicate the directions in which closer relationships may be found, and where, possibly, more effective classifications may be made. The values given in the accompanying general scheme are certainly not final, but we do not think that they give at all misleading values, or values really far from the truth. There remains much to be done, and the scheme indicates some of the values which

yet remain to be found. While parallax touches one element only of position, direction as indicated by the usual stellar co-ordinates is a second; this "position" in the narrower sense has not been discussed in the present paper. We have purposely included it in the table, to indicate that we have not overlooked those position correlation problems to which the astronomer is now turning, and to which approach is possible from more than one direction. We hope later to deal with the question of correlation of stellar characters and position. Taken as a whole, we are, we think, compelled to conclude that the associations between parallax, proper motion, and magnitude are considerably less than we should anticipate if we hypothecated any approach to a uniform distribution of stars with a system of random velocities throughout space. The existence of correlations between colour and spectral class, not only with magnitude, but with parallax and proper motion, suggests, if it does not demonstrate, that chemical constitution and luminosity are dependent in some manner not only on spatial distribution, but on velocity in space.

TABLE XVII.

Correlation of Stellar Characters.

	Colour.	Spectral Class.	Magnitude.	Parallax.	Proper Motion.	Position.
Colour	1·00	·71	·30	?	?	?
Spectral class	·71	1·00	·69 (·43, ·54*)	·36	·36	?
Magnitude	·30	·69 (·43, ·54*)	1·00	·30	·35	?
Parallax	?	·36	·30	1·00	·39	?
Proper motion	?	·36	·35	·39	1·00	?
Position	?	?	?	?	?	1·00

* According to Pickering's and Lockyer's classifications respectively.

Erratum in Annual Report.

Vol. lxviii. p. 298, line 5, *for* o″·1 *read* o″·01.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVIII. APRIL 10, 1908. No. 6

H. F. NEWALL, Esq., PRESIDENT, in the Chair.

John Bullock, M.A., 78 Airedale Avenue, Chiswick, W.;
Arthur Brunel Chatwood, B.Sc., Assoc.M.Inst.C.E., Astronomer to the Government of H.H. The Nizam, Hyderabad, Deccan, India;
F. R. Cripps, Esq., 22 Hornsey Rise Gardens, N.;
Henry Zouch Darrah, Esq., C.S.I., Allahabad, U.P., India;
Charles Gaskell Falkner, Esq., M.A., Ireton Bank, Rusholme, Manchester;
Harold Morris-Airey, Esq., M.Sc., Armstrong College, Newcastle-upon-Tyne; and
Henry William Moore, B.A., New University Club, St. James's Street, and 64 Curzon Street, Mayfair, W.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Hugh Cameron Campbell, Science Department, Surgeons' Hall, Edinburgh (proposed by Alex. D. Russell); and
Arthur Mackreth Deane, M.A., Canon of Chichester, Ferring Vicarage, Worthing, Sussex (proposed by F. J. W. Crowe).

The following were proposed by the Council as Associates of the Society:—

Benjamin Baillaud, Director of the Observatory, Paris;
C. V. L. Charlier, Director of the Observatory, Lund, Sweden;

E. B. Frost, Director of the Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.;
The Rev. J. G. Hagen, S.J., Director of the Vatican Observatory, Rome; and
Johannes Franz Hartmann, Astrophysical Observatory, Potsdam, Germany.

Seventy-one presents were announced as having been received since the last meeting, including, amongst others:—

J. G. Böhn, die Kunst-Uhren auf der K.-K. Sternwarte zu Prag, presented by the Prague Observatory; The History of the Geological Society of London, by H. B. Woodward, presented by the Society.

Sixteen charts of the Astrographic Chart of the heavens, presented by the Royal Observatory, Greenwich.

On the Lunar Inequalities due to the Motion of the Ecliptic and the Figure of the Earth. By Ernest W. Brown, Sc.D., F.R.S.

1. *The general disturbing function for the motion of the ecliptic.*—Let θ_1, θ_2, θ_3 be the angular velocities of a set of moving rectangular axes about themselves; x, y, z the co-ordinates; u, v, w the velocities of a particle with respect to these axes; and let F be the force function divided by the mass of the particle. Then the equations of motion are given by

$$\frac{du}{dt} - v\theta_3 + w\theta_2 = \frac{\partial F}{\partial x},$$

$$\frac{dv}{dt} - w\theta_1 + u\theta_3 = \frac{\partial F}{\partial y},$$

$$\frac{dw}{dt} - u\theta_2 + v\theta_1 = \frac{\partial F}{\partial z},$$

where

$$u = \frac{dx}{dt} - y\theta_3 + z\theta_2,$$

$$v = \frac{dy}{dt} - z\theta_1 + x\theta_3,$$

$$w = \frac{dz}{dt} - x\theta_2 + y\theta_1.$$

Put

$$H = \tfrac{1}{2}(u^2 + v^2 + w^2) - F - R,$$

where

$$R = vx\theta_3 - wx\theta_2 + wy\theta_1 - uy\theta_3 + uz\theta_2 - vz\theta_1;$$

then, if we assume that θ_1, θ_2, θ_3 are independent of x, y, z, u, v, w, we may write the equations of motion in the canonical form,

$$\frac{du}{dt} = -\frac{\partial H}{\partial x}, \quad \frac{dx}{dt} = \frac{\partial H}{\partial u},$$

$$\frac{dv}{dt} = -\frac{\partial H}{\partial y}, \quad \frac{dy}{dt} = \frac{\partial H}{\partial v},$$

$$\frac{dw}{dt} = -\frac{\partial H}{\partial z}, \quad \frac{dz}{dt} = \frac{\partial H}{\partial w}.$$

If R be neglected, the equations become of the same form as for fixed axes; hence R is the disturbing function for the motions of the axes. These motions are so small that we can neglect squares and higher products of the terms in R, and therefore can treat R as a disturbing function of the same nature as that used for ordinary planetary perturbations.

In order to obtain θ_1, θ_2, θ_3, we put i' for the inclination of the moving ecliptic (xy plane) to the fixed ecliptic (that of 1850·0), τ for the longitude of the node on the fixed ecliptic, L for the angle which the zx plane makes with the plane containing the poles of the fixed and moving ecliptics. Then, by Euler's equations,

$$\theta_1 = \frac{di'}{dt}\sin L - \sin i' \cos L \frac{d\tau}{dt},$$

$$\theta_2 = \frac{di'}{dt}\cos L + \sin i' \sin L \frac{d\tau}{dt},$$

$$\theta_3 = \frac{d\tau}{dt}\cos i' + \frac{dL}{dt}.$$

As L is at our disposal, we so take it that the distances of the origins of reckoning on the fixed and moving ecliptics from their common node are the same, that is, so that $L = 90^\circ - \tau$. The moving axis of x therefore passes through a "departure point."

When we are given the values of i', τ in terms of the time, we have the material necessary for the solution of the problem.

2. As a matter of fact, i', $\frac{di'}{dt}$, $\frac{d\tau}{dt}$ are so small that their squares and products may be neglected. We can therefore put

$$\sin i' = i', \quad \cos i' = 1, \quad \tau = \text{const.},$$

and obtain

$$\theta_1 = \frac{di'}{dt}\cos\tau, \quad \theta_2 = \frac{di'}{dt}\sin\tau, \quad \theta_3 = 0.$$

Whence

$$R = \frac{di'}{dt}\left\{(wy - vz)\cos\tau + (uz - wx)\sin\tau\right\},$$

or, since u differs from $\frac{dx}{dt}$ by a quantity of the same order as i', etc.,

$$R = \frac{di'}{dt}\left\{\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right)\cos\tau - \left(x\frac{dz}{dt} - z\frac{dx}{dt}\right)\sin\tau\right\}$$

$$= \frac{di'}{dt}Q, \text{ suppose.}$$

The method of the variation of arbitrary constants permits us to substitute in Q the values of x, y, z, and of their derivatives in terms of the arbitrary constants and the time as found in the solution of the main problem of the lunar theory. These values can be obtained directly from the results which I have given in my papers in the *Memoirs* of the Society on this subject.

3. The value of i' is of the form

$$i' = pt + P,$$

where p is a constant and P a sum of periodic terms whose arguments depend on those of the motions of the Earth and planets. The latter terms have generally been neglected, but I shall show that they give rise to a few terms which are not insensible. At this stage the question arises as to what moving ecliptic we shall refer the motion of the Moon. We may refer it to the *actual* ecliptic, in which case I have found by calculation that P gives rise to a number of primary terms of short period and a few of long period. Or we may refer it to the *mean* ecliptic, *i.e.* put $P = 0$ in the above formulæ; but if we do so, it will be necessary to introduce the latitude of the Sun above this mean ecliptic into F. With this latter method, I have found that all the primary short-period terms practically disappear and the long-period terms have rather smaller coefficients than with the former plane of reference.

I therefore adopt the mean ecliptic together with the one or two very minute terms which are of long period relative to that of the Moon's node, and which do not then give rise to any terms in the Moon's co-ordinates. Hence, for the computation, we may put

$$R_1 = pQ.$$

4. The principal part of the force function for the Moon's motion under the influence of the Earth and Sun is

$$F = \frac{\mu}{r} + \frac{m'}{r'^3}\left\{\frac{3}{2}\frac{(xx' + yy' + zz')^2}{r'^2} - \frac{1}{2}(x^2 + y^2 + z^2)\right\}$$

where x', y', z' are the co-ordinates of the Sun.

The main problem is solved with $z' = 0$. Hence the additional portion due to z' is, if we neglect squares of z',

$$R_2 = \frac{m'}{r'^3} \cdot \frac{3(xx' + yy')zz'}{r'^2}.$$

With the notation and limitations of § 2, 3 we have

$$z' = i'(y' \cos \tau - x' \sin \tau).$$

Also, with this value for z' and by means of the equations $\frac{d^2x}{dt^2} = \frac{\partial F}{\partial x}$, etc., it is easy to show that

$$\frac{m'}{r'^3} \frac{3(xx' + yy')zz'}{r'^2} = i' \left\{ \left(z\frac{d^2y}{dt^2} - y\frac{d^2z}{dt^2} \right) \cos \tau - \left(z\frac{d^2x}{dt^2} - x\frac{d^2z}{dt^2} \right) \sin \tau \right\}.$$

Hence, since τ is a constant, we obtain

$$R_2 = -i'\frac{dQ}{dt},$$

where for i' we substitute its periodic portions, omitting those mentioned at the end of § 3.

5. It is of some interest to compare the two disturbing functions R_1, R_2. In general we have

$$R_1 = Q\frac{di'}{dt}, \quad R_2 = -i'\frac{dQ}{dt} = Q\frac{di'}{dt} - \frac{d}{dt}(i'Q).$$

Thus the effect of the rotation of the axes alone, apart from the motion of the Sun out of the fixed plane, is to introduce a term $\frac{d}{dt}(i'Q)$. From these expressions we may also prove the statements made in § 3 with reference to the presence of long- and short-period terms, remembering the two forms which have been given for $\frac{dQ}{dt}$. In fact, the terms of long period in Q are relatively much larger than those in $\frac{dQ}{dt}$ owing to the presence of small divisors; for a term of *very* long period in R_2, the portion arising from $\frac{d}{dt}(i'Q)$ is small compared with that arising from R_1.

6. The computation from the two functions R_1, R_2, as defined in §§ 3, 4, give the results which follow. The values of Leverrier * have been used for i', τ, and the method is the same as that which I have used for planetary inequalities in general.

I put w_1, w_2, w_3 for the mean longitudes of the Moon, of its perigee and of its node; l, D, F, n, e, γ are the quantities as defined by Delaunay; T, V, J are the mean longitudes of the

* *Ann. Obs. Paris* (*Mém.*), vol. iv. pp. 13–21.

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Earth, Venus, and Jupiter, respectively; $\phi = w_3 + 96^\circ\cdot2$; $\phi' = w_1 + 96^\circ\cdot2$.

Then the variations of the elements for the principal inequality, argument ϕ, arising from R_1, are

$$\delta w_1 = -\ 0''\cdot289 \sin\phi, \quad \delta n = +0''\cdot0014n \cos\phi,$$
$$\delta w_2 = +\ 0''\cdot840 \sin\phi, \quad \delta e = \ \ \cdot000 \cos\phi,$$
$$\delta w_3 = -15''\cdot59 \sin\phi, \quad \delta\gamma = +0''\cdot698 \cos\phi.$$

These agree very closely with the values found by Professor Newcomb.* They differ a little more from those found by Dr. Hill;† he, however, used a method with a literal development, and the small differences are probably due to slow convergence.

The principal term in latitude, argument ϕ', has a coefficient of $+1''\cdot382$.

7. The new terms arising from R_2 are, if we omit terms with coefficients less than $0''\cdot010$,

In Longitude.	In Latitude.
$+0''\cdot019 \sin(\phi + 5T - 3V + 119^\circ)$	$+0''\cdot077 \sin(\phi' + 5T - 3V + 119^\circ\cdot4)$
$+0''\cdot003 \sin(\phi + 2w_2 - 2J + 90^\circ)$,	$+0''\cdot030 \sin(\phi' - 5T + 3V - 119^\circ\cdot4)$
	$+0''\cdot035 \sin(\phi' + 2J + 72^\circ)$
	$+0''\cdot018 \sin(\phi' - 2J - 72^\circ)$.

The two terms in longitude are "primary" terms of long period. The second one is of interest, since its period is that of the empirical term which appears to represent best the long-period difference between theory and observation, namely, about 280 years. Its coefficient is, however, too small to account for this difference.‡ There is a term with the same argument due to the direct effect of the planets, but its coefficient is also insensible. The terms in latitude are "secondary," and have periods approximating to a month.

8. *Inequalities arising from the figure of the Earth.*—These have been computed very fully by Dr. Hill,§ who used the method of Delaunay, and there would be no reason for the mention of them here if it were not that slight doubts have arisen concerning their degrees of accuracy on account of slow convergence. I have used the equations of variations and the results of my lunar theory which allow the coefficients to be obtained easily within $0''\cdot003$. This method is brief, and the computations occupied about eight

* Carnegie Institute, publ. 72, p. 132. The difference in the value of δn is due to a difference in the definition of this variation.

† *Annals of Math.*, vol. i. p. 57. Coll. Works, vol. ii. p. 77.

‡ In a note on p. 170 of the present volume, and in this paper as read before the Society, I gave $0''\cdot21$ as the value of this coefficient. A recalculation with the first form of R_2 revealed an error in the former computation.

§ *Amer. Eph. Papers*, vol. iii. pp. 201–344. Coll. Works, vol. ii. pp. 181–320.

days. I put $\psi = w_3 +$ the precession, and obtain for the variations of the elements

$$\begin{aligned} \delta w_1 &= +\ 7''\cdot317 \sin\psi\,, & \delta n &= -\ 0''\cdot009 \cos\psi\,, \\ \delta w_2 &= -\ 2''\cdot092 \sin\psi\,, & \delta e &= +0''\cdot002 \cos\psi\,, \\ \delta w_3 &= +96''\cdot69\ \sin\psi\,, & \delta\gamma &= -\ 4''\cdot351 \cos\psi\,. \end{aligned}$$

The short-period terms add $-0''\cdot017 \sin(\psi + \mathrm{F})$ to the latitude. The principal term in longitude is δw_1, and the principal elliptic terms are $+0''\cdot519 \sin(\psi + l) + 0''\cdot515 \sin(\psi - l)$. The principal term in latitude is $-8''\cdot355 \sin(\psi + \mathrm{F})$, and that with argument $\psi - \mathrm{F}$ is $+0''\cdot338 \sin(\psi - \mathrm{F})$. When Hill's results are reduced to the value of the ellipticity used above (1/296·3),* my coefficient of the principal term in latitude agrees with his, and my coefficient of the principal term in longitude is 0″·030 less. The coefficients for the term with argument $\psi - \mathrm{F}$ also agree; this fact furnishes a useful test, since this coefficient is the difference of two numbers each nearly thirteen times as large as the coefficient. The two principal elliptic terms are 0″·021, 0″·017, respectively, greater than those of Hill. An examination of Hill's literal developments for the coefficients shows that the small differences can all be explained by slow convergence.

The complete results for the classes of terms considered here will be given in the fifth (concluding) part of my *Theory of the Motion of the Moon*, with the terms due to the action of the planets and to perturbations not considered in the previous parts.

New Haven, Conn.:
1908 *February* 1.

Postscript (1908 *April* 4).—The terms of the second order have been examined and produce nothing sensible. However, in consequence of my attention being called by Dr. Hill to a doubt as to whether there was a portion of the secular acceleration due to the figure of the Earth terms being referred to a moving ecliptic, I made an actual computation of this second order perturbation and found that the greatest effect could be exhibited in the form of a term of period about 15,000 years and coefficient 0″·15. This, equivalent to a secular acceleration of 0″·0001 within historic times, is entirely insensible. The computation will be given in chapter xiv. of my memoir just mentioned.

* Corresponding to the result marked (β) in *M.N.*, vol. lxiv. p. 531.

Note on the newly discovered Eighth Satellite of Jupiter, photographed at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

Since the date of the last communication successful photographs have been obtained of this object on the following nights:—March 27, 31, and April 3. From these it would appear that the object discovered is a satellite of Jupiter very much more distant from the planet than the sixth or seventh satellites. The following are the places deduced from photographs taken with the 30-inch reflector.

Date and G.M.T.				Exposure.	Apparent R.A.			Apparent Dec.			Sat. VIII—Jupiter.		
											R.A.		Dec.
1908.	d	h	m	min.	h	m	s	°	′	″	′	″	′ ″
Jan.	27	12	41·4	64	8	45	51·86	+18	5	1·4	−0	8·29	−43 17·3
Feb.	1	11	52·1	111		43	20·67		17	36·3	+0	2·20	41 41·1
	3	10	27·3	80		42	21·90	18	22	25·7	0	6·37	41 4·2
	22	10	56·6	100		33	35·41	19	5	49·8	0	48·38	33 58·1
	23	8	33·8	42		33	13·98		7	37·6	0	50·31	33 35·7
	24	12	24·9	80		32	47·37		9	51·3	0	53·15	33 9·2
	27	10	59·4	70		31	42·89		15	18·1	0	59·94	31 59·7
	28	11	28·6	80		31	21·76		17	6·7	1	2·34	31 35·2
Mar.	27	8	54·3	43		26	32·33		47	53·5	2	11·35	20 59·9
	31	9	16·5	165		26	40·24		49	12·3	2	21·55	19 38·1
Apr.	3	9	51·7	90	8	26	54·38	+19	49	38·8	+2	28·99	−18 40·2

The G.M.T. is the arithmetical mean of the times of beginning and end of exposure.

These places are definitive, replacing those given in the previous number, p. 373, which are provisional only.

The new satellite has also been photographed by Dr. Max Wolf at Heidelberg on March 3, 23, and 24, and at the Lick Observatory on March 8.

[A photograph of Jupiter's Eighth Satellite, as well as of the Sixth and Seventh Satellites, has since been obtained on April 24.]

Note by Mr. Melotte.

Throughout this and previous oppositions, in examining the photographs of Jupiter's sixth and seventh satellites a good look-out has been kept, with a view to detecting other satellites, should any exist. In view of the large gap between the orbits of the inner satellites and the sixth and seventh, the existence of others would appear not unlikely. On several occasions previous to this, suspected images had been noticed on the plates, and in some

cases (notably on four plates in 1905) images had been found on plates taken a few days before or after, which might possibly correspond. But the difficulty of distinguishing between photographic defects and true images is naturally very considerable, and, until the present occasion, it had never been possible in any case to obtain sufficient evidence to decide whether the images were real. A record of the positions of all suspected images is made for future reference.

Royal Observatory, Greenwich:
1908 *April* 10.

First approximation to the orbit of J VIII = CJ.

By A. C. D. Crommelin, B.A.

The hypothesis of retrograde motion gives a much more reasonable value of the distance of this object from Jupiter than the hypothesis of direct motion; it has therefore been provisionally adopted. Assuming that the distance of the object from Jupiter has remained sensibly constant during the period of observation (this assumption being justified by the uniformity of the rate of motion, when corrected for the varying distance and direction of the planet from the Earth), it is possible to deduce this distance by the curvature of the apparent path (the greater part of this curvature being due to the Earth's motion).

The result of several successive approximations gives the following values:—

Distance from Jupiter at inferior geocentric conjunction, in astronomical units.	0·213
Date of this conjunction . . .	1908 Feb. 20·0
R.A. of N. pole of orbit plane . .	336° 25′
N. dec. „ „ „ . .	54 48
Inclination to Jupiter's orbit . . .	32°
Daily angular motion about Jupiter .	0°·288

This satisfies all the observations within some 20″; a closer approximation could doubtless be obtained, but it is hardly worth while to do so till enough of the orbit has been described to make a reliable estimate of the solar perturbations, which must be extremely large. The rate of linear motion appears to have diminished by 1 per cent. in the period March 3 – April 3 as compared with Jan. 27 – March 3. This would be sufficiently explained by the Variation causing the motion to be most rapid at New and Full Moon.

The daily angular motion in an undisturbed orbit at distance 0·213 is 0°·310, with which the value 0°·288 is in satisfactory accord,

especially as the effect of the Variation both on the distance and the velocity must be considerable. The sidereal period is probably between three and four years, the synodic period being about one year shorter if the motion be retrograde.

The mean distance of satellite VII from Jupiter is 0·0785, so that the value of the distance found for the new body is about three times as great.

Table giving approximate values of the perturbations of Halley's Comet by Jupiter and Saturn in the first and fourth quadrants of the orbit. By P. H. Cowell, M.A., F.R.S., and A. C. D. Crommelin, B.A.

A sufficient number of revolutions of the comet have now been computed to enable the perturbations to be tabulated as functions of g', the mean anomaly of the disturbing planet at the adjacent perihelion passage. The formula for determining g' was given in *M.N.*, lxviii. 3, p. 177. The table is not intended to supersede calculation, but it serves—

(1) To form a good first approximation to the date of any perihelion passage, as a preliminary to more exact calculation.
(2) To detect any serious errors of calculation.
(3) To readily correct computations made with an erroneous assumed date of perihelion, by means of the rate of change of the tabular quantities.

	Jupiter, 1st quad.			Jupiter, 4th quad.			Saturn, 1st quad.			Saturn, 4th quad.		
g'	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$
°	+″·85	−210″	+140″	+″·26	−460″	−60″	+″·19	0″	+20″	−″·03	+10″	+
10	·93	130	160	·32	450	90	·14	−30	20	·06	0	
20	·99	−30	180	·37	430	120	·09	50	20	·10	0	
30	1·05	+100	200	·43	410	140	·07	60	20	·15	+10	
40	1·10	220	200	·48	380	150	·05	50	10	·19	20	
50	1·13	340	190	·54	350	160	·04	30	10	·23	30	
60	1·14	450	170	·59	320	170	·03	20	10	·24	20	
70	1·13	560	140	·64	270	170	·02	−10	10	·23	+10	
80	1·10	660	90	·67	200	160	·01	+10	10	·21	−10	
90	·97	670	40	+·38	−100	130	+·01	20	+10	·18	30	
100	·71	610	+10	−·27	+130	110	·00	40	0	·15	50	
110	·39	470	−30	·90	420	70	−·01	50	0	·12	60	
120	+·03	330	70	1·20	620	−30	·01	60	0	·10	70	
130	−·18	210	110	1·28	710	+10	·02	60	0	·07	80	
140	·24	100	120	1·25	640	50	·03	60	0	·05	80	+

	Jupiter, 1st quad.			Jupiter, 4th quad.			Saturn, 1st quad.			Saturn, 4th quad.		
g'	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$
°	"	"	"	"	"	"	"	"	"	"	"	"
150	·27	+ 20	130	1·20	560	90	·04	50	− 10	·03	90	0
160	·27	− 50	130	1·16	470	120	·04	40	10	− ·02	80	0
170	·27	120	130	1·12	370	140	·05	30	10	·00	80	− 10
180	·27	170	130	1·08	270	160	·06	20	20	+ ·01	80	20
190	·26	220	120	1·03	180	170	·06	+ 10	20	·03	70	30
200	·24	280	110	·97	+ 70	180	·06	− 10	10	·04	50	20
210	·23	340	90	·90	− 30	190	·06	20	10	·05	40	20
220	·21	390	80	·83	130	200	·05	40	− 10	·06	20	10
230	·19	460	60	·76	230	200	·04	50	0	·06	− 10	10
240	·16	520	40	·68	320	190	·03	70	0	·06	+ 10	10
250	·12	580	− 20	·58	420	180	− ·02	80	0	·06	20	10
260	− ·06	620	0	·48	510	160	·00	90	+ 10	·06	30	10
270	·00	640	+ 20	·38	550	130	+ ·02	90	20	·05	40	10
280	+ ·07	640	40	·29	570	110	·04	100	20	·04	50	10
290	·16	630	60	·19	590	90	·07	90	30	·03	50	10
300	·25	610	80	·11	600	70	·09	90	30	·02	60	− 10
310	·34	580	90	− ·04	600	40	·12	90	30	+ ·01	60	0
320	·44	530	100	+ ·03	580	+ 10	·15	80	30	·00	50	0
330	·54	470	110	·09	560	− 10	·19	60	30	·00	30	0
340	·65	400	120	·14	540	30	·22	40	30	− ·01	+ 10	0
350	+ ·75	− 310	+ 130	+ ·20	− 510	− 40	+ ·22	− 20	+ 30	− ·02	0	+ 10

To $\int d\zeta$ in the first quadrant must be added the corresponding value of $\int dn \times$ period in days.

The argument g' is the value of the planet's mean anomaly at the preceding perihelion passage for the first quadrant, at the following passage for the fourth quadrant.

Note on the condition for the passage of the Earth through the plane of Saturn's Ring. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The interesting observations made recently on the ring seen edgewise have brought several inquiries as to the recurrence of this beautiful phenomenon; and the following note, originally made some years ago in consequence of an inquiry from Mr. C. T. Whitmell, may be of use to others. The late Mr. R. A. Proctor gives in his book *Saturn and his System* a general account of the manner in which the Earth may pass through the plane of the ring, either once or three times, at each favourable opportunity. The present note gives the explanation in more compact form.

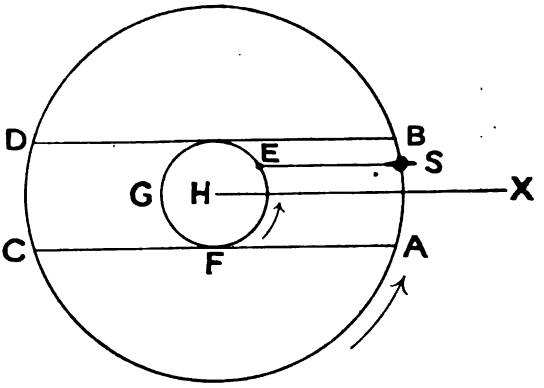

FIG. 1.

2. Let H be the Sun (fig. 1), EGF, ABDC, the orbits of the Earth and Saturn, supposed circular and in one plane. Let BD and AC be two tangents to the Earth's orbit parallel to the plane of Saturn's ring. Then if S be Saturn and E the Earth at a time of passage through the ring, ES must be parallel to BD or AC. Hence Saturn must be either in the portion AB or DC of his orbit. Since his orbit is ten times the size of the Earth's, these favourable opportunities are confined to limited periods which recur at long intervals. AB is about $\frac{1}{15}$ of the semi-orbit, and is described in about a year, so that the Earth meanwhile makes a complete revolution. If, when Saturn is near A, the Earth is near F, then there may be three passages through the ring.

3. To find the condition in exact terms, take HX parallel to AC or BD as axis of x. Let the radius of the Earth's orbit be unity, and that of Saturn's orbit n^2. The co-ordinates of the Earth may be written

$$x_1 = \cos t,\ y_1 = \sin t,$$

and of Saturn (remembering Kepler's Third Law)

$$x_2 = n^2 \cos(t-\alpha)/n^3,\ y_2 = n^2 \sin(t-\alpha)/n^3,$$

where the unit of time is $1/2\pi$ of a year and the origin of time is taken when the Earth lies on HX, Saturn then being an angle α/n^3 behind it.

The condition for passage of the Earth through the ring is thus

$$y_1 = y_2 \quad \text{or} \quad \sin t = n^2 \sin(t-\alpha)/n^3,$$

a transcendental equation, which we can only solve by approximation.

4. Let us draw the curves

$$\text{(a)} \quad y = \sin t,$$
$$\text{(b)} \quad y = n^2 \sin(t-\alpha)/n^3,$$

then the required values of t will be given by their intersections. Now the curves are both sine curves, differing only in period and amplitude.

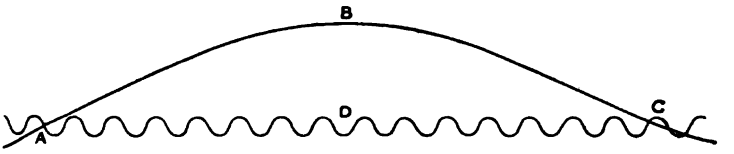

FIG. 2.

The Earth's curve (a) alternates much more rapidly, as in ADC (fig. 2). Saturn's curve (b) is a broad sweep, ABC, and we see at once how the crossings of the two curves correspond to the favourable opportunities of fig. 1; and further, that there may be a single intersection as at A, or a triple intersection as at C. There must always be an odd number of intersections; whether there can be five, seven, or more depends on the actual dimensions of the curves. If there is only one intersection, notice that the slopes of the two curves are in opposite directions, *i.e.* the planets are on opposite sides of the Sun; so that Saturn will be near conjunction with the Sun, and the phenomenon will not be so readily observable. If there are three intersections, the middle one is near opposition and the other two near quadratures.

5. In fig. 3, let ABCDE be two consecutive waves of the Earth's curve, and let FG, HK, LM represent three possible positions of a portion of Saturn's curve, each touching the Earth's curve in one point (F, K, L) and cutting it in another (G, H, M). It is clear that if Saturn's curve cuts the axis OX between T and V there will be only one intersection; if between V and W there will be three. The separating cases are thus defined by the three values of α, $\mathrm{OT} = \alpha_1$, $\mathrm{OV} = \alpha_2$, $\mathrm{OW} = \alpha_3$, and it is easily seen that $\alpha_2 = 2\pi - \alpha_1$, $\alpha_3 = \alpha_1 + 2\pi$.

6. To find the value of a_1, we have the conditions that both y and $\frac{dy}{dt}$ must be the same for both curves.

$$\left.\begin{aligned} \therefore \quad \sin t &= n^2 \sin (t-a)/n^3 \\ n^3 \cos t &= n^2 \cos (t-a)/n^3 \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

$$\therefore \quad \sin^2 t + n^6 \cos^2 t = n^4 \quad . \quad . \quad . \quad . \quad (3)$$

$$n^6 \sin^2 (t-a)/n^3 + \cos^2 (t-a)/n^3 = n^2 \quad . \quad . \quad (4)$$

whence $$\cos^2 t = (n^4 - 1)/(n^6 - 1)$$

so that $$\cos t = \frac{1}{n}\left\{1 + \frac{1}{n^4 + n^2}\right\}^{-\frac{1}{2}} \quad . \quad . \quad (5)$$

and further $$\sin (t-a)/n^3 = \frac{1}{n^2}\left\{1 + \frac{n^2+1}{n^4}\right\}^{-\frac{1}{2}} \quad . \quad . \quad (6)$$

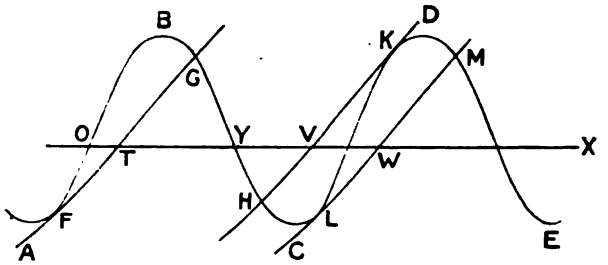

FIG. 3.

Since n^3 is 29·46, the number of years in which Saturn revolves round the Sun, $n = 3{\cdot}088$, $n^2 = 9{\cdot}54$, $n^4 = 91{\cdot}0$. Neglecting quantities of the order n^{-4} in t,

$$t = -\frac{\pi}{2} + \frac{1}{n} + \frac{1}{6n^3}, \quad (t-a) = -n + \frac{1}{2n} - \frac{1}{24n^3}$$

$$\therefore \quad a = -\frac{\pi}{2} + n + \frac{1}{2n} - \frac{5}{24n^3} \quad . \quad . \quad . \quad (7)$$

Thus for Saturn-Earth $a = 1{\cdot}683$, or 96° 25′. Or we can, of course, obtain the numerical values of t and $(t-a)/n^3$ directly from equations (5) and (6).

7. Hence $$a_1 = 96^\circ, \quad a_2 = 264^\circ, \quad a_3 = 456^\circ.$$

And in the long-run, since the period of Saturn is incommensurable with that of the Earth, the values of a will be distributed uniformly, so that in a long series of years the chance of three intersections is

$$(a_3 - a_2)/360^\circ = 192/360 = {\cdot}53,$$

and the chance of one intersection is

$$(a_2 - a_1)/360^\circ = 168/360 = {\cdot}47.$$

Starting from any crossing with a value α_0 for α, the next crossing *in the same direction* will be at a point

$$\frac{t-\alpha_0}{n^3}=2\pi$$

or $$t=\alpha_0+29{\cdot}46\times 2\pi$$

$$=\alpha_0+166^\circ \text{ omitting } 29\times 2\pi.$$

The intermediate crossing is in the opposite direction and at the opposite point of the orbit. The eccentricity of Saturn's orbit is so large that the intermediate value of α cannot be inferred by elementary considerations. There will be practically two separate series of values of α referring to the two nodes, each increasing by the difference 166°. But this value 166° is only approximate, and is modified by the motion of the ring plane. It appears from the subjoined table that at present the average value is about 162°.

8. The dates when Saturn crosses the line of nodes, or more properly when his ring-plane passes through the Sun, are given by Mr. Proctor on p. 223 (Table X.) of his book above mentioned (*Saturn and his System*). It will suffice here to give a few of them to the nearest day :—

Year.	Sun in plane of Ring.	Earth's Passage.	Fractions of Year. Sat.	Earth.	Diffce.		α.	
1789	Oct. 7	Sept. 13	·764	·698	·066	...	24°	...
1803	June 23	Mar. 12	·473	·192	...	·281	...	101°
1819	Mar. 23	Sept. 14	·221	·700	·521	...	187	...
1832	Dec. 4	Mar. 12	·923	·192	...	·731	....	263
1848	Sept. 3	Sept. 14	·671	·700	·971	...	349	...
1862	May 18	Mar. 12	·375	·192	...	·183	...	66
1878	Feb. 14	Sept. 15	·120	·703	·417	...	150	...
1891	Oct. 29	Mar. 13	·824	·195	...	·629	...	226
1907	July 29	Sept. 16	·572	·706	·866	...	312	...

Columns 1 and 2 above are from Mr. Proctor's book; column 3 is filled in from the *Nautical Almanac* by finding the date when the Earth had the same heliocentric longitude as Saturn had on the date given in the column before. The next two columns give these dates in fractions of a year, and the differences are the values of α in the unit adopted in § 3, viz. $1/2\pi$ of a year, more conveniently expressed in degrees in the next columns. The two series are kept separate, and it will be seen that the common difference is about 162° or 163°. If it were exactly $162^\circ = 180^\circ - 18^\circ$, the numbers would repeat after twenty terms, and we should have the series as follows :—

First Series.

Year	1789	1819	1848	1878	1907	1937	1966	1996	2025	2055	2084
α	25°	187°	349°	151°	313°	115°	277°	79°	241°	43°	205° etc.
No intersect^{ns}.	3	1	3	1	3	1	3	3	1	3	1

Second Series.

Year	1803	1832	1862	1891	1921	1950	1980	2009	
α	101°	263°	65°	227°	29°	191°	83°	245°	etc.
No intersect^{ns}.	1	3	3	1	3	1	3	1	

Under the actual conditions, the series will slowly diverge from these, just as the cycle of total solar eclipses on the Earth slowly changes.

9. If the slope of Saturn's curve were less relatively to that of the Earth, there might be five intersections. For observers on Venus or Mercury this might be the case. The limiting case when five intersections are just possible is when a Saturn curve through the point Y (fig. 3), just touches both the adjacent curves near F and K, and then equations (2) are satisfied by $\alpha = \pi$.

$$\therefore \text{ from equation (7) } \quad n = \frac{3\pi}{2} - \frac{1}{2n} - \frac{5}{24n^3} = 4{\cdot}54 \text{ approx.}$$

For values of n greater than 4·54 there will accordingly be three or five intersections, and five will occur with greater frequency as n increases.

Now Bode's Law gives for the successive relative values of n^2

4	7	10	100	[388]
Mer.	Ven.	Earth.	Saturn.	Neptune.

Thus for Saturn-Venus, $n^2 = 14{\cdot}3$, $n = 3.8$

for Saturn-Mercury, $n^2 = 25$, $n = 5{\cdot}0$

Hence Saturn's ring can disappear five times to Mercury, but not to Venus. The value of n for Saturn-Neptune is too small for any but three intersections. It will readily be seen that the above procedure is applicable to the outer planets, the curves interchanging character.

10. Although Saturn's ring is a special problem, there are cases somewhat similar, *e.g.* the changes in declination of another planet. Times when the declination vanishes are times when the planet passes through the Earth's equator, and thus would be in the plane of its ring if it had one. The three intersections can readily be traced in the tabulated declinations of (say) Venus.

The Variability of the Nucleus of the Planetary Nebula N.G.C. 7662. 1860·0, α $23^h\ 19^m\ 11^s$, $\delta + 41°\ 46'·0$.

By E. E. Barnard. (Plate 12.)

This remarkable nebula (N.G.C. 7662) is situated in Andromeda. In an ordinary telescope it looks like a small round disc of a bluish-green colour. In the 4-inch finder of the 40-inch telescope it is only distinguishable from an ordinary star of the 8^m or $8\frac{1}{2}^m$ by a slight fuzziness of the image. In the great telescope, however, under good conditions, it is a beautiful object —a slightly elliptical disc with quite sharply defined outlines. Unsymmetrically placed on this is a roughly elliptical broken ring of greater brightness. The interior of this ring is dark, but not black, and in this, approximately central, is ordinarily a faint stellar nucleus. The broken ring (broken on its n.p. side, and with a smaller break opposite this) is of a bright bluish-green colour, while the general disc of the nebula often has the appearance of a warm or faintly reddish tint. The northern part of this inner ring is the brightest part of the nebula. The nucleus, with which I am going to deal specially, is sometimes of a bright yellowish colour, though ordinarily it is faint and apparently colourless. There is a faint star of the 15·5 or 16 magnitude exactly on the following edge of the outer disc.

Shortly after beginning to observe this nebula in 1897 I suspected that the nucleus was variable (see *Astrophysical Journal*, xiv. p. 153). Familiarity with the changing aspect of a star involved in nebulosity, due to moonlight, bad seeing, etc., has made me extremely cautious in this matter. I have, however, for several years, established conclusively the fact that the nucleus of 7662 is actually variable to an extent of upwards of three magnitudes. At times it has appeared as a bright yellowish star of about the 12th magnitude; and at other times, equally favourable, it has been either entirely invisible or excessively faint. Though I have not definitely determined the period, the light changes seem to cover about 28 days. The light curve is still undetermined; the normal condition, however, seems to be faint—the nucleus remaining bright for a few days only.

I have collected here the principal observations of the brightness of the nucleus, etc., obtained with the 40-inch telescope in the past ten years.

On 1904 August 6, among other objects, I showed this nebula to Professor H. H. Turner. Though we both looked very carefully, under fairly good conditions, we could not see any trace of the nucleus. It must have been below the 16th magnitude at that time.

In all the observations of this nebula the eyepiece was adjusted for the nebula itself, the focus of which, as I have shown elsewhere (*M.N.*, lx. pp. 255–257, and *Astrophysical Journal*, xiv. pp. 151–157), is about a quarter of an inch outside that for a star.

Following are the observations of the nucleus, taken from my notes on the nebula with the large telescope.

Observations of the brightness of the Nucleus.

1897 Sept. 17. Julian day 4185, $10^h\ 0^m$. The nucleus is 15th or 16th magnitude.

Sept. 28. 4196, $10^h\ 30^m$. The nucleus is seldom seen on account of poor seeing.

Oct. 3. 4201, $8^h\ 0^m$. The nucleus is seen only once in a while. The seeing is bad.

1898 Sept. 20. 4553, $9^h\ 30^m$. The nucleus is either absent or excessively faint. I think I can occasionally see faint traces of it. From recollection alone, it seems this must be very much fainter than it was last year. The definition is fair.

Sept. 26. 4559, $12^h\ 35^m$. I think I can see the feeblest trace of the nucleus, but it must be excessively faint. The seeing is fair. $13^h\ 0^m$. Can see it faintly with 700 diameters.

1899 July 31. 4867, $14^h\ 0^m$. The nucleus is very difficult.

Aug. 1. 4868, $13^h\ 30^m$. The nucleus is very difficult. The inside of the inner ring is very slightly darker than the general surface of the nebula.

Aug. 14. 4881, $15^h\ 10^m$. Can only catch the feeblest glimpses of the nucleus. It seems to me that it was brighter when the former measures were made. The inner ring is broken in two opposite places.

Aug. 26. 4893, $11^h\ 40^m$. The nucleus is very hard to see. The seeing is very bad.

Sept. 3. 4901, $9^h\ 30^m$. Can once in a while see the nucleus. The seeing is very bad.

Sept. 25. 4923, $15^h\ 20^m$. The nucleus is seen only feebly.

1900 July 17. 5218, $14^h\ 30^m$. The nucleus is 12th magnitude, and is very easy. It is yellowish.

July 28. 5229, $14^h\ 0^m$. I cannot see the nucleus; possibly could catch feeble glimpses of it. It ought to be visible. I have thought several times that the nucleus of this nebula is variable. It looks strongly that way now.

Aug. 6. 5238, $14^h\ 0^m$. The nucleus is fairly distinct. There is a $15{\cdot}5^m$ or 16^m star exactly on the north edge of the nebula.

Sept. 20. 5283, $10^h\ 10^m$. The nucleus is extremely faint. The seeing is fairly good.

1901 Aug. 7. 5604, $12^h\ 30^m$. The nucleus is readily seen. It is yellowish, and of the 12th magnitude.

Aug. 24. 5621, $16^h\ 0^m$. Can feebly see the nucleus, but it seems very faint. The sky is poor, but the nucleus ought to be brighter; the sky is too poor to be certain. I have suspected the nucleus of being variable. Cannot see it well enough to measure its position.

Dec. 22. 5741, $10^h\ 0^m$. Can faintly see the nucleus, but not distinctly enough to measure.

1904 Aug. 6. 6699, $13^h\ 0^m$. Professor Turner and I looked for the nucleus. We could see no trace of it with 460 or 700 diameters. The seeing was fair, so that we could see the details of the nebula well. The nucleus must be variable.

Aug. 8. 6701, $14^h\ 20^m$. Have just tried the focus, etc. for the nucleus. Can see nothing of it. Tried eyepieces 460 and 700. The seeing part of the time was fair.

Aug. 13. 6706, $11^h\ 40^m$. Can see the nucleus quite distinctly. It is about the 14th magnitude and is quite stellar. I think we ought to have seen it on the 6th if it was as bright as now. Seeing 2 to 3.

Aug. 15. 6708, $12^h\ 0^m$. Can faintly see the nucleus, though the sky is thick.

Aug. 16. 6709, $14^h\ 0^m$. Can very faintly see the nucleus; eyepiece 700. I am not sure but that it may be fainter than last night, for the sky, though poor and thick, is better than then.

Aug. 22. 6715, $10^h\ 0^m$. Can see the nucleus very plainly; eyepiece 700. It is 13th or 14th magnitude. Very much plainer than the central star in M 57. It is quite bright. Seeing 3–4.

Aug. 27. 6720, $13^h\ 30^m$. Eyepiece 700. The nucleus is well seen, about $12\frac{1}{2}$ magnitude; the same brightness as star *b* (more carefully:—It is decidedly brighter than *b*). It is about midway between *a* and *b* in brightness and is very easy. Perhaps slightly yellowish.

Aug. 29. 6722, $14^h\ 10^m$. By glimpses can see the nucleus. Can just see *b* on account of poor seeing, so the nucleus is not fainter than on the 27th. Seeing very bad.

Sept. 3. 6727, $9^h\ 0^m$. Can see the nucleus, but it is not clear. It is about 14th magnitude. Decidedly less noticeable than the star *b*. Eyepiece 700.

Sept. 5. 6729, $13^h\ 40^m$. Cannot see the nucleus, or if it is seen it is excessively faint. I think in the moments of good seeing I ought to see it. At such times *b* is quite easy or rather conspicuous. Just in this direction the sky seems fairly clear.

Sept. 10. 6734, $9^h\ 30^m$. Can feebly catch glimpses of the nucleus, but the sky is thick. I can see the star *b*, but not very distinctly, on account of thick sky. Eyepiece 700.

Sept. 12. 6736, $11^h\ 45^m$. The nucleus is very faint, 15th magnitude. Seeing 3. The star *b* is conspicuous.

Sept. 16. 6740, $10^h\ 0^m \pm$ (no record of time). The nucleus is easy, say $13\frac{1}{2}$ magnitude. It is less than the star *b*.

Sept. 24. 6748, $12^h\ 30^m$. Eyepiece 700. Can see the nucleus steadily. When best seen it is brighter than the star *b*. It is not less than 13th magnitude. It is perhaps as bright as 12 or $12\frac{1}{2}$ magnitude. Full moonlight. Looking more carefully, it is decidedly brighter than *b*. It is not fainter than 12th magnitude. Seeing 3.

Oct. 1. 6755, $9^h\ 30^m$. The nucleus is seen occasionally.

It is much fainter than the star *b*. I should say it is 14th magnitude. Eyepiece 700. Seeing 2.

Oct. 11. 6765, 9^h 30^m. The sky is thick. Cannot be sure of the brightness of the nucleus. It is faint, but the magnitude uncertain, say 14th or less. Seeing poor = 2.

Oct. 15. 6769, 13^h 40^m. Cannot see the nucleus. The star *b* is easy and conspicuous. I do not think the seeing is such as to blot the nucleus out if as bright as it has sometimes been. Eyepiece 700.

Oct. 17. 6771, 7^h 35^m. Can see the nucleus quite easily, 13th or 14th magnitude. I am sure it is brighter than on the 15th. It seems nearly as conspicuous as the star *b*. Seeing 2 or 3.

Oct. 22. 6776, 14^h 5^m. I think I can see the nucleus occasionally, but cannot be certain. If it is seen it is about as bright as the star *b*, which is faint and not well seen, but the seeing is too poor to decide. The nebula is 5 hours west.

Oct. 29. 6783, 7^h 30^m. I can see the nucleus. It is about 14th magnitude. By glimpses it is fairly well seen but I do not think it is as bright as it has been. Seeing 2. Eyepiece 700.

Oct. 31. 6785, 7^h 30^m. The nucleus is fairly well seen, about $13\frac{1}{2}$ magnitude. It is not as bright as the star *b*. A little later :—I see it better now. It is 13th magnitude. Eyepiece 700.

Nov. 5. 6790, 10^h 50^m. Eyepiece 700. I can see the nucleus occasionally. It is faint, perhaps 14th magnitude. The star *b* is quite easy and bright. I think the conditions are such that the nucleus ought to show up distinctly.

Nov. 12. 6797, 9^h 20^m. Eyepiece 700. I can see the nucleus occasionally fairly well. Seeing poor, but by moments of steadiness it must be as bright as 13th magnitude. It is nearly as conspicuous as the star *b*.

Nov. 14. 6799, 6^h 0^m. Eyepiece 700. It is faint, say 14th magnitude. I think it ought to be brighter if it is not variable. Though I can see it, it does not appear as stellar as it has appeared at times. There is a half moon shining, which may have something to do with it. Seeing = 3.

Nov. 26. 6811, 9^h 15^m. Eyepiece 700. The nucleus is faint. It is not brighter than 14th magnitude. By moments the seeing is fairly good. It is far less bright than star *b*. It is not well seen.

Dec. 5. 6820, 5^h 30m. Eyepiece 700. I can see the nucleus, but it is faint. It is between 13th and 14th magnitude, and is much less conspicuous than the star *b*.

Dec. 10. 6825, 7^h 0^m. The sky is too thick and the nebula is dull. I can see the star *b* faintly, but cannot see the nucleus. The sky is too thick, however, for this observation to count.

1905 Oct 28. 7147, 7^h 30^m. The nucleus is steadily seen with both eyepieces, perhaps best with eyepiece 460. It is nearly as bright as the star *b*. Eyepieces 460–700. Seeing 3.

Nov. 26. 7176, 8^h 35^m. The nucleus is very faint, very much fainter than the star *b*. Seeing 3.

Nov. 28. 7178, $8^h\ 45^m$. Eyepiece 700. The nucleus is very faint, but I can see it in spite of the bad seeing. It is much fainter than the star *b*.

1906 Aug. 11. 7434, $14^h\ 40^m$. Eyepiece 460. The nucleus is of about the same brightness as star *b*. Its appearance is the same with eyepiece 700. It is not less bright than the star *b*. Seeing poor.

Aug. 14. 7437, $11^h\ 50^m$. The nucleus is very faint. I think it is one magnitude less than the star *b*.

Sept. 4. 7458, $13^h\ 50^m$. Eyepiece 700. The nucleus is bright and conspicuous, of about the 12th magnitude, and seems to be yellowish. The seeing is not especially good; it comes and goes.

Sept. 8. 7462. $13^h\ 45^m$. The nucleus is easy. It is $\frac{1}{2}$ magnitude less than the star *b*. I think it is a little less bright than at the last observation. It is 13th magnitude.

Sept. 11. 7465, $13^h\ 50^m$. Eyepiece 700. I can see the nucleus occasionally very distinctly. It is 13th magnitude. I do not think it is any different from the few last observations. It is much less bright than the star *b*.

Sept. 22. 7476, $13^h\ 20^m$. Eyepiece 700. I can feebly see the nucleus. I think it is faint to-night. It is 2 magnitudes or more less than the star *b*. I think there is no doubt of its variability. The seeing is fair, 2 or 3.

Oct. 13. 7497, $10^h\ 55^m$. I can scarcely see the nucleus, and then not distinctly. It is several times fainter than the star *b*. Eyepiece 700.

Oct. 30. 7514, $7^h\ 30^m$. The seeing is too poor to tell anything about the nucleus.

Nov. 13. 7528, $9^h\ 5^m$. The nucleus is very faint. It is $1\frac{1}{2}$ magnitude less than the star *b*. If as bright as I have seen it sometimes, it ought to be quite bright to-night. Seeing 3, fairly good.

1907 July 2. 7759, $14^h\ 40^m$. Eyepiece 700. I can see the nucleus fairly well, but the sky white from the moon. Nucleus is 13th magnitude. Seeing 3.

Aug. 11. 7799, $12^h\ 0^m$. Eyepieces 460 and 700. I can feebly see the nucleus. It is very faint and pale, and cannot be seen steadily. Cannot see it any better with eyepiece 460. Seeing 3.

Aug. 13. 7801, $11^h\ 35^m$. Eyepiece 700. I can possibly see traces of the nucleus. It is very faint. The seeing is poor, but the nucleus ought to be seen easily if as bright as it has appeared sometimes. Of course I always test for the best focus on this nebula. With eyepiece 460, with the best glimpses, the inside of the ring is dark, and I can perhaps feebly see the nucleus. Seeing 2.

Aug. 20. 7808, $11^h\ 0^m$. Eyepiece 700. Can faintly see the nucleus. Perhaps it is somewhat brighter than at the last observation (?).

Sept. 1. 7820, $10^h\ 55^m$. Eyepiece 700. Can see the

nucleus quite easily. It may be 13th magnitude. I think it is brighter than usual. Seeing 2.

Sept. 3. 7822, $13^h\ 50^m$. Eyepiece 700. Can see the nucleus distinctly, but it is faint, say $13\frac{1}{2}$ or 14 magnitude. Seeing 2 or 3.

Sept. 8. 7827, $10^h\ 30^m$. Eyepiece 700. Can see the nucleus once in a while, but it is very faint. Seeing 2.

Sept. 10. 7829, $13^h\ 0^m$. Eyepiece 700. Can see the nucleus but it is faint; perhaps not so faint as previously. It is hard to see, as the seeing varies so. The sky is thickish.

Oct. 1. 7850, $10^h\ 40^m$. Eyepiece 700. Can see the nucleus very faintly. Seeing 2.

Oct. 4. 7853, $8^h\ 30^m$. Eyepiece 700. Can see the nucleus faintly. The seeing is too bad to make much out of it.

Oct. 13. 7862, $10^h\ 10^m$. It is bright, 11th or 12th magnitude, and yellowish. I do not think the good seeing is responsible for the brightness of the nucleus. It is perhaps slightly brighter than the star *b*. With eyepiece 460 it is bright and yellow. It is $\frac{5}{10}$ magnitude less than the star *a*. It is almost as bright as that star, perhaps just as bright. It is a conspicuous object, and could be seen with almost the worst seeing.

Nov. 3. 7883, $9^h\ 0^m$. The sky is too thick to see the nucleus. I can, however, see the 3 stars that are north, following; the fainter ones are just visible. A little later; can see it faintly, but this does not mean that it is really faint. Eyepiece 460.

Nov. 12. 7892, $10^h\ 5^m$. With both eyepieces, 460 and 700, can see the nucleus. It is not faint, about as bright as star *c* through thin clouds. It would be easy on a good sky.

Nov. 25. 7905, $6^h\ 40^m$. Eyepiece 460. Can see the nucleus, but it is faint. Eyepiece 700. I can see it with this also, but it is faint. Sky clear. It must be $1\frac{1}{2}$ magnitude less than when I observed it on the 12th. Seeing 2.

Nov. 26. 7906, $8^h\ 0^m$. It is faint. It is just as it was last night.

Nov. 28. 7908, $5^h\ 50^m$. The nucleus is quite easy, though not bright. I think it is certainly brighter than on Nov. 26. The seeing is not any better than on that date. It is poor. Occasionally the nucleus comes out quite distinctly, though much fainter than I have seen it at times previously. Later:—I can see it quite distinct and steady at times, and think it is decidedly brighter than on the 26th. Eyepiece 460. With eyepiece 700 I can see it fairly well, but not so good. Seeing too poor.

Dec. 3. 7913, $8^h\ 50^m$. Can see the nucleus, but it is very faint. Eyepiece 460. I think it is fainter than at the last observation. It is fainter than any one of the stars following. Seeing 2.

Dec. 5. 7915, $7^h\ 0^m$. The nucleus is faint, but once in a while it is fairly distinct. It is much less than any of the stars north, following. Eyepiece 460. With eyepiece 700 it is very faint and difficult.

Dec. 12. 7922, 8^h 0^m. The disc of the nebula is sharply outlined all round. It is a beautiful full ellipse. The outline is faint on account of moonlight, but distinct. The faintest part of the disc to the edge is at the south end of the major axis. Seeing poor. Perhaps there is some haze.

The inner ring looks irregular, and is more square than elliptical. The inside of this ring is dark, but not black. It is decidedly darker than the general surface of the nebula outside the ring. This inner ring is greenish-blue, and about $1\frac{1}{2}''$ in thickness. There is a suggestion of warmth to the general disc.

The nucleus is faint. (Too much moonlight?) Seeing = 2. It is rather hard to see, and is about the brightness of *c* or less. The south preceding $\frac{1}{3}$ of the inner ring is faint. It is hard to tell just how this axis lies, because of the break in the ring and from bad seeing. Eyepiece 700. Seeing = 2.

Dec. 25. 7935, 6^h 20^m. Nucleus fairly well seen. Seeing = 3. It is less bright than either of the three stars north, following. There is a 14^m star in the direction of *b*, whose estimated position with reference to the nucleus is P.A. $48° \pm$, dist. $87'' \pm$. There are several faint stars about 1′ north, preceding the nebula.

The nucleus is not nearly so bright as I have seen it. The break in the following part of the inner ring is exactly opposite the break in the preceding side.

7^h 20^m the nucleus is quite bright and easy. Power 700. It is about as bright as *b* or *c*—perhaps less than these stars. It is slightly yellowish. The same results with eyepiece 460. Can feebly see the small star in the north, following edge of the nebula. I think the nucleus is less than either *b* or *c* by, say, $\frac{1}{2}$ magnitude. It is well seen occasionally. Carefully examined about *a*. There is no star nearer to it than *b*. The magnitudes of the three stars north, following, were independently estimated,—

$a = 12{\cdot}6$ magnitude.
$b = 13{\cdot}8$,,
$c = 13{\cdot}7$,,

The nebula was compared in the finder with several B.D. stars near it for brightness. The results were $8{\cdot}8^m$, $8{\cdot}6^m$, $7{\cdot}2^m$. Its ill-defined condition makes estimates of its magnitude rather uncertain. The mean of these, $8{\cdot}2^m$, will not be far out. This, of course, is on the B.D. scale of magnitudes.

1908 Jan. 2. 7943, 7^h 0^m. The nucleus fairly well seen. It is less than *b* by perhaps $\frac{1}{2}$ magnitude. Seeing = 3.

The notes sometimes seem to be conflicting as to the relative brightness of the nucleus—where an estimate of its magnitude would make it brighter than the comparison star, at the same time a statement is made that it is less than the star. This can only be explained by the difficulty of making any definite estimate of the magnitude of the nucleus, which is involved in a nebulous background, where the conditions of seeing, etc. affect its appearance far more than they do that of a star on the open sky.

Focal Measures of the Nebula and the Nucleus.

Following are the measures of the relative focus of the nebula and a star. These are readings from a scale of inches on the focussing tube.

1899 Aug. 1.	The nebula	2·54	(3)	inches
	A star	2.38	(3)	,,
	Nebula—star +	0·16		inch
Sept. 25.	The nebula	2·39	(5)	inches
	A star	2·20	(4)	,,
	Nebula—star +	0·19		inch
Oct. 7.	The nebula	2·49	(5)	inches
	A star	2·29	(5)	,,
	Nebula—star +	0·20		inch
Oct. 15.	The nebula	2·60	(4)	inches
	A star	2·34	(6)	,,
	Nebula—star +	0·26		inch
1907 Oct. 13.	The nebula	0·42	(6)	inch
	A star	0·20	(5)	,,
	Nebula—star +	0·22		inch

For a comparison of the nucleus with the nebula and the star we have—

1899 Oct. 7.	Nebula	2·49	(5)	inches
	Nucleus	2·43	(5)	,,
	Nebula—nucleus +	0·06		inch
	Nucleus	2·43	(5)	inches
	Star	2·29	(5)	,,
	Nucleus—star +	0·14		inch
1907 Oct. 13.	Nebula	0·42	(6)	inch
	Nucleus	0·30	(5)	,,
	Nebula—nucleus +	0·12		inch
	Nucleus	0·30	(5)	inch
	Star	0·20	(5)	,,
	Nucleus—star +	0·10		inch

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These give the following final values, weighted by the number of measures :—

Nebula—star
+0·21 inch

Nebula—nucleus
+0·09 inch

Nucleus—star
+0·12 inch

The plus sign indicates an increase of focus. From this we see that the focus of the nebula is 0·21 inch outside of that for the star, while it is only 0·09 inch outside of that for the nucleus. The focus for the nucleus is 0·12 inch outside of that for the star.

The readings on 1907 Oct. 13 were by a millimetre scale on the eyepiece. They have been converted into inches. The comparisons have been made with the star *a*.

The following measures were made of the dimensions of the nebula and of the positions of the axes of the larger and smaller ellipses.

For the Outer Ellipse.

	P.A. Maj. Axis.	Major Diameter.	Minor Diameter.
	°	″	″
1897 Sept. 17	34·9	30·64	26·08
20	36·4	31·55	25·94
21	32·9	30·08	25·41
1907 Dec. 12	35·4	29·75	25·83
	°	″	″
	34·9	30·50	25·81

For the Inner Ring.

	P.A. Maj. Axis.	Major Diameter.	Minor Diameter.
		″	″
1897 Sept. 21	...	13·58	11·16
28	43·0	...	...
1907 Dec. 12	54·4	15·10	11·32
	°	″	″
	48·7	14·34	11·24

These last results are discordant. It is difficult, from the nature of the inner ring, to measure its position angle.

Following are the measures of the three stars north, following the nebula.

The Nucleus and Star a.

	P.A.	Dist.	
1897 Sept. 17	62°·48	51″·31	from nucleus.
20	63·37	51·52	,, nucleus (?).
21	63·48	51·32	,, nucleus (?).
1900 July 17	63·07	51·24	,, nucleus. $a = 12^m \pm$
1901 Aug. 7	63·17	51·38	,, nucleus.
1904 Sept. 24	63·20	51·34	,, nucleus.
1907 Dec. 25	63·55	51·26	,, nucleus.
Mean	63°·19	51″·34	

The Nucleus and Star b.

	P.A.	Dist.	
1897 Sept. 28	45°·68	62″·76	from centre of nebula.
Oct. 3	46·33	63·01	,, centre of nebula (?). $b = 15^m \pm$.
1907 Dec. 25	46·92	62·12	,, nucleus.
Mean	46°·31	62″·63	

The Nucleus and Star c.

	P.A.	Dist.	
1897 Sept. 28	14°·81	68″·68	from centre of nebula.
Oct. 3	15·47	68·31	,, centre of nebula (?). $c = 14^m \pm$
1907 Dec. 25	14·94	67·38	,, nucleus.
Mean	15°·07	68″·12	

Photographs of the Nebula.

During the years 1899 and 1900 I made some experimental photographs of a few of the brighter planetary nebulæ, using a Cramer Crown plate, which was very sensitive, and a yellow colour-screen, the same one that had been used in the regular star photographs. The results, though not entirely satisfactory, were very gratifying. The great difficulty lay in the very long exposures that were required. The planetary nebula in Draco, N.G.C. 6543, could be secured in from twenty minutes to an hour, while the others tried by me required several hours, and were then under-exposed. In the case of several of these nebulæ remarkable details were shown that were not visible to the eye in the great

telescope. Especially astonishing were the photographs of the Draco Nebula, which showed it to have a delicate spiral structure.

Among the nebulæ thus photographed was N.G.C. 7662. In none of the photographs of this object was the exposure time long enough to bring out all the fainter parts of the nebula. The inner ring came out very strong, showing the breaks just as I had seen them in the telescope and drawn them. There were some thread-like lines (which I have not seen visually) running from the inside of the ring to the nucleus. On these photographs the nucleus does not seem to be central in this inner ring, but is perceptibly displaced to the following side. The main disc of the nebula is decidedly uneven in brightness. The north preceding portion is very faint, as is also the north following side, but less so. A section of the south following edge is very bright, while the opposite side of the nebula is somewhat more luminous than the general surface. A section of the north preceding outline of the nebula is lacking for want of longer exposure. There is evidence in the photographs of changes of brightness in the nucleus.

In all these photographs the plate was placed from $\frac{1}{4}$ to $\frac{1}{3}$ of an inch outside the stellar focus, in conformity with my previous observations, which showed that the focus for the nebulæ is outside that for a star. See *M.N.*, lx., January 1900, and *Astrophysical Journal*, xiv., for October 1901.

I hope soon to be able to take up this subject again, with the large telescope, with suitable screen and plates prepared by Mr. Wallace.

Following is a list of the photographs.

1900 Aug. 18. Exposure 0^h 35^m. Exposure stopped by storm. Shows feeble traces of nucleus, inner ring and part of disc.

Aug. 21. Exposure 1^h 18^m. Sky poor. Seeing poor. Faint traces of nucleus. Shows inner ring and bright spot on s.f. edge.

Aug. 28. Exposure 0^h 45^m. Bad sky ending in fog. Shows nucleus feebly. Only inner ring feebly shown.

Sept. 3. Exposure 1^h 5^m. Sky poor.

Sept. 3 (same date). Exposure 1^h 30^m. Seeing fine, sky good. The last one is best. They are both strong and show nearly all of the nebula. The nucleus is strong in both.

Sept. 20. Exposure 2^h 35^m. Sky very clear. Seeing poor. Shows inner ring and part of entire disc.

Sir John Herschel's Observations.

In a paper containing observations of the nebulæ made with the 20-feet reflector at Slough from 1825 to 1833, read by Sir John Herschel before the Royal Society, November 21, 1833, the following observations of this nebula are recorded :—

"Sweep 183. A fine planetary nebula. Diameter 12″; with 240 beautifully defined, light, rather mottled, and the edges the least in the world unshaped. It is not nebulous, but looks as if

it had a double outline, or like a star a little out of focus. A perfect circle. Has a star near; pos. 68°·1, well measured over the centre of the nebula. See fig. 45.

"Sweep 180. *v* B; R; bluish white; 8 or 10″ diam. It has no haze at the edges, but ? if it be not enveloped in an *e* F nebulosity (perhaps arising from glare), and also the star 13 m., whose pos. is 61·9 and Δ R A = 4ˢ·0. The light is a very little mottled, and not absolutely planetary. It is = a * 7 m. (*N.B.*—These *satellites* of planetary nebulæ ought to be especially attended to.)

"Sweep 204. Diameter in R A = 2ˢ·0 of time. The attendant star is double A = 12 m; B = 13 m., pos. of A = 58°·5; Δ R A from centre = 4ˢ·0. The nebula is not perfectly round, light equal, but a very little hazy at the edges.

"Sweep 189. Exactly R; 2ˢ·5 of time = diam. in R A. Has a * 13 m. pos. 69°·0; dist. 30″; the light of the disc is perfectly equable, and equals a star 8 m.

"Sweep 190. Exactly R; = a * 8 m; a * 13 m n f; pos. = 65°·6 by microm.; dist. = 30″. [*N.B.*—The great discordance in the measured positions of the satellite-star seems to arise from the star being double, and taken for a single one. Whenever this occurs, it is a source of error, and should be most carefully guarded against in all future observations of this interesting object.]"

I have carefully examined, with the 40-inch telescope, the immediate vicinity of the star *a*—the one referred to by Herschel as being double. There are no signs of duplicity, and there is no star nearer than *b*. This statement rests specially on observations made 1907 Dec. 25—a good night. From the recent measures there seems to be no certain motion in any of the stars *a*, *b*, and *c*, or the nucleus.

Observations with the Rosse Telescope.

In *Phil. Trans.* for 1850, p. 513, the Rosse observers, using the great reflector, give the following account of their observations of this nebula:—

"Oct. 31, 1848. Has a central spot, at moments very dark.

"Dec. 13, 1848. Nothing more, except perhaps that faint external annulus extends further than had been seen before.

"Dec. 14, 1848. Note by Mr Johnstone Stoney:—'Three stars near it, somewhat in this fashion; showed it to Sir James South.'

"Dec. 16, 1848. Sketches made by Lord Rosse and Mr Johnstone Stoney.

"Dec. 19. 1848. Drawing confirmed."

A drawing of the nebula is given, plate xxxviii. fig. 13. This does not show any central star.

In *Phil. Trans.* for 1861, plate xxx., another drawing appears, fig. 40, which differs very greatly from the first one. This shows

the central star conspicuously. The following additional notes are also given :—

"Since the publication in the *Transactions* for 1850. The outlying portions in the published sketch are parts of spiral branches. Fig. 40, plate XXX. represents it as seen on a very fine night (Sept. 16, 1852), with a freshly polished speculum which defined very sharply. Oct. 2, 1856. All the details in Mr Stoney's drawing very well seen. Oct. 16, 1857 [J.D. 2, 399, 604]. The spiral arms and the * in centre distinctly seen."

The drawing referred to shows a nearly round disc with a brighter spiral arrangement commencing near the edge and ending near the central star. This light spiral is outlined by a dark shading, or by a parallel dark spiral. I do not find any reference to previous observations of the nucleus by the Rosse observers, though, from the last sentence quoted, they seemed to be familiar with the star.

Other Observations.

Professor Burnham (*Pub. L.O.*, ii.) measured the central star on two nights with the 36-inch telescope :

1890·78 62°·9 51″·84

The central star was 15th magnitude and *a* 13^m. He also gives measures by O. Struve :

1847·86 61°·3 51″·85

Struve could not see the central star. His measures were, therefore, referred to the centre of the nebula.

Volume 13, Annals of the Harvard College Observatory, pp. 80–81, contains some observations of this nebula by Winlock and Searle about 1866. Combining their various measures of the dimensions, etc. of the nebula we get—

P.A. major axis of the nebula 47° ±
Diameter of the major axis 27″·4 ±
Diameter of the minor axis 24″·4 ±

Some of the notes in these observations are :—"Edges as definite as Jupiter's. Two other observers also considered the colour to be green, much deeper under illumination than without . . . . Two edges seen . . . . as of a brighter nebula placed over a dimmer one. The brighter and fainter ellipses have major axes slightly differing in direction; the brighter having the position angle of its major axis 10° to 15° the larger (J. W. obs.) 20° to 30° according to a less careful estimate by G. M. S., to whom the northern side appears brighter; appearance to J. W. that of a hollow cylinder with one end turned nearly towards the observer."

Searle and Winlock also measured the position angle and distance of the star *a* on two nights in the last part of 1866,

P.A. 62°·7 Dist. 52″·14

The central star was not seen at any time.

The Probable Period of Variation.

An investigation of the observations seems to give a period for the variability of the nucleus, or central star, of about 28 days. The observation of 1906 Sept. 4, however, does not seem to fit in with this period.

I give below some efforts to determine the period, utilising the early Rosse and Lassell observations in connection with my own. In general the results imply a period of 28 days. It is my intention to follow the variations of the nucleus more closely, and to determine accurately the period and the light curve. I think this is the first case in which the nucleus of a planetary or other nebula has been shown to certainly vary in its light. It is also my intention to follow up some of the other planetary nebulæ, for there are one or more cases where I have suspected variability in the nucleus.

If we take the observation of 1907 Oct. 13 and combine it as below we shall get the following results:—

Taking the observations of 1900 July 17, with an interval of 94 periods, we shall have the period 28·1 days.

If we use the observation of 1901 Aug. 7, with an interval of 80 periods, we shall have for the period 27·9 days.

If we combine the observation of 1904 Aug. 27, with an interval of 41 periods, the result is period, 27·9 days.

If we use the observation of 1904 Sept. 24, with an interval of 40 periods, we have period, 27·9 days.

The observation of 1906 Sept. 4, with an interval of 14 periods, gives a period of 28·9 days.

The observation of 1906 Aug. 11, gives, with an interval of 15 periods, 28·7 days.

In *Memoirs* of the R.A.S., vol. xxxvi. p. 51, Lassell, gives observations of this nebula with his 4-foot reflector at Malta on 1862 Oct. 23 (Julian day 2,401,437).

"Central point not stellar, but apparently a very minute planetary disk of a bluish colour. . . . . The central point is bright and conspicuous, not to be overlooked even in a cursory examination."

Assuming that the nucleus was at its maximum brightness at or near this observation, by comparison with the observation of 1904 Sept. 24 (J.D. 2,416,748) when it was at or near maximum, we have an interval of 547 periods, which gives a period of 28·0 days.

If we compare this observation with that of 1906 Sept. 4 (J.D.

2,417,458), 579 periods would have elapsed, which would give a period of 27·7 days.

Or taking the observation of 1907 Oct. 13 (J.D. 2,417,862), when it was evidently close to maximum, there will have elapsed 587 periods from which the period derived is 28·0 days.

For some reason all the comparisons with 1906 Sept. 4 give discordant results.

The Rosse observation of 1857 Oct. 16 (J.D. 2,399,604) was undoubtedly at or near maximum. Combining this with my observation of 1907 Oct. 13, assuming an interval of 652 periods, we shall get a period of 28·0 days.

The accompanying drawing of this planetary nebula (Plate 12) was made with the 40-inch refractor. I think it is a fair representation of the object.

In a paper read before the Royal Dublin Society (*Transactions*, vol. ii. (Series ii.)) on Feb. 18, 1878, the following additional notes from Lord Rosse are given :—

"1848 Oct. 31. Has a central spot, at moments very dark.

Dec. 13. Nothing more, except perhaps that F external annulus extends farther than had been seen before.

Dec. 15. 3 st. near it. Showed it to Sir J. South.

Dec. 16. Sketches made by Lord R. and myself.

Dec. 19. Drawing confirmed. [P.T., 1850, plate xxxvii. fig. 13.] This should be plate xxxviii.—(E. E. B.)

1852 Sept. 16. Very fine night, freshly polished speculum which defines very sharply. Drawing made [P.T. 1861, plate xxx. fig. 40]. In the figure in P.T., 1850, the outlying portions are parts of spiral branches.

1856 Oct. 2. All the details of Mr. B. Stoney's drawing very well seen by Lord R. and myself.

1857 Oct. 16. The spiral arms and the star in centre distinctly seen.

1872 Oct. 7. Moderately well defined, an outer F atmosphere; darker in the middle about the spiral.

1873 Aug. 20. Has a *13 m in Pos. 244°·8. Dist. 46″·6.

Sept. 28.	$a\beta$	Pos.	356°·1	Dist.	25″·7.
	$\beta\gamma$	,,	309·1	,,	44·3.
	N a	,,	61·2 (4).		

From a to limbs, mean 64·4 $\beta = \gamma = 16$ m.
Distances measured by Lord R. by C [Dr Copeland].

a to far limb	67″·2	64″·2
a to near limb	40·5	34·1
a to centre	51·3	48·9

Oct. 11. *8 m. Pos. 80°·0 (3). Dist. 498″·9 (2). Faintest of 3 st. near [γ]. Pos. 14°·2. Dist. from limbs 82″·1 and 51″·2. Certainly no conspicuous nucl. (xxxii. obs.)."

The position angle 244°·8 is 180° too great. It should be 64°·8, for the measures can only refer to the star *a*. The distance is too small.

A sketch accompanies the notes with the great reflector, p. 174. This shows the nebula and three stars α, β, γ, following it. I have identified these stars with mine, as below :—

$$\alpha = a$$
$$\beta = b$$
$$\gamma = c$$

I have computed, for comparison, the position angles and distances $a\beta$, $\beta\gamma$ and $a\gamma$ from my measures. They are :—

			Residuals R − B	
	°	″	°	″
$a\beta$ (ab)	358·7	20·20	−2·6	+5·5
$\beta\gamma$ (bc)	309·2	35·59	−0·1	+8·7
$a\gamma$ (ac)	326·7	51·30		

I do not think these large residuals indicate any real change in the relative positions of the stars. In this case, the position angles with the large reflector seem to be fair, but the distances must be affected with very serious errors.

It will be seen that the presence and absence of the central star, in the Rosse observations, are strongly attested by the notes of 1857 Oct. 16, and 1873 Oct. 11.

Yerkes Observatory,
Williams Bay, Wisconsin:
1908 *February* 6.

P.S., 1908 *April* 21.—In reference to the variability of the nucleus of the nebula N.G.C. 7662, I have carefully gone over all the literature of the subject, and in no case has there been any suggestion of variability (outside of my own suggestions). There does not seem to have been any suspicion, even, of variability.

S

N

The Planetary Nebula N.G.C. 7662.—Drawn by E. E. Barnard.

[The half-tone is too harsh. This causes the central darkness to be a little too great, relative to the general disc. It also makes the bright, inner ring too bright. The south portion of this ring should be shaded or slightly darkened, as the north portion is somewhat the brightest part of the ring.—E. E. B.]

so that at the same time of year they run together nearly for several succeeding years.

Hence the observations do not readily discriminate between them; but those at 2416820, 2417176, and 2417178, when the nucleus was faint, seem to favour the $27\frac{1}{3}$ day period, since they make the epochs 10 or 12 days away from maximum, while the $25\frac{1}{2}$ day period makes them close to maximum. Professor Barnard proposes to make further observations to settle the question.:

Corrections to Professor Turner's Paper "On the Classification of Long-period Variable Stars, and a possible Physical Interpretation." By the Rev. J. Stein, S.J., Sc.D.

(*Communicated by Professor H. H. Turner, D.Sc., F.R.S.*)

[My best thanks are due to Father Stein for pointing out the corrections noted below. According to the general permission given in the last sentence of his letter, I publish it at once.—H. H. T.]

Specola Vaticana, April 3, 1908.

Dear Professor Turner,—Lately I have studied your remarkable article on the "Long-period Variable Stars" (*M.N.*, lxvii. p. 332). My interest was the greater as this is the first serious attempt to bring the long-period light-curves in close relationship to the laws of sun-spot variation.

I take the opportunity for suggesting some ideas by which the number of possible cases in your hypothesis may be still more reduced.

1°. Is there not a little oversight in your paper on page 345? There it is said—

"The mean (foreshortening) factor for the spot may be taken as

$$\int \cos\zeta \, dZ = \ldots .$$"

Should this not be

$$\frac{\int \cos\zeta \, dZ}{\int dZ} \, ?$$

Then "the total effectiveness" would be (*top line of page* 346)—

$$s = \frac{1}{2\pi}(4\beta \cos\epsilon \sin\lambda + 4 \cos\beta \sin\epsilon \cos\lambda),$$

and in the "polar view"—

$s_1 = \sin\lambda$ (as correctly stated on page 342, line 6; but not as incorrectly on page 346).

In the "equatorial view"—

$$s_2 = \frac{2}{\pi} \cos \lambda \text{ (\textit{instead of} } 4 \cos \lambda \text{ \textit{on p.} } 346).$$

Thus $s_1 = s_2$ for $\tan \lambda = \frac{2}{\pi}$, or

$$\lambda_0 = 32^\circ{\cdot}5,$$

and λ_0 *is defined without any ambiguity*, such as is contemplated on p. 348. Of course, if we take a broad belt of sun-spots instead of an infinitesimal one, λ_0 will be slightly different from the given value. In the extreme case, when the whole sphere is all over covered with spots, the mean foreshortening factor will be $\frac{(\text{Disc})}{(\text{Hemisphere})} = \frac{1}{2}$, and $\lambda (= \lambda_0) = 30^\circ$. The variation of λ_0 being so slight, we can practically put $\lambda_0 = 30^\circ$ for all cases.

2°. The brightness at maximum being from 40 to 100 times that at minimum, it seems very probable that at maximum the whole surface of the star is in activity and covered with faculæ. This may be confirmed by what we know of the "polar type" of the Sun's corona at maximum, when the streamers are numerous in nearly all solar latitudes (*M.N.*, lxiii. p. 482), in opposition to the "equatorial" or "wind vane type" at minimum. In this hypothesis the maxima of polar and "equatorial view" must coincide, and this would seem a new argument for the more preferable "hypothesis of superposing maxima."

If these remarks are right, it follows that the latitudes given on page 348 for $\lambda_0 = 30^\circ$ are the most probable.

Now, starting from the equation $B = sF$, we can calculate F.

Subtracting the numbers of Table III. Group I. (the "polar view group") from unity, and multiplying by 1·6, we get

	Max.	2	3	4	5	6	7	8	9	10	11	12
log s_1 F	= 1·60	1·39	·99	·63	·21	·00	·10	·48	·96	1·12	1·33	1·44
log sin λ	= 9·70	9·62	9·45	9·28	9·13	9·19	9·52	9·91	9·99	·00	9·99	9·93
log F	= 1·90	1·77	1·54	1·35	1·08	0·81	0·58	0·57	0·97	1·12	1·34	1·51
F	= 79	59	35	22	12	7	4	4	9	13	22	32
λ	= 30°	25°	16°	11°	8°	9°	19°	54°	79°	82°	78°	58°

The hypothetical law suggested by these numerical results can hardly be better expressed than in the words used by the Astronomer Royal for characterising the sun-spot phenomena from 1874 to 1902 (*M.N.*, lxiii. pp. 452–3). We have only to omit the words between brackets :— . . . "The years of maximum showed spots in practically every latitude [between 30° N and 30° S]. In the years following the maximum a marked tendency was shown for the spots to appear in lower latitudes . . . . about one or two years before minimum no spots were seen outside the limit of 18° from the equator. But immediately minimum was reached the

spots became more widely extended in latitude, owing to the occurrence of outbreaks in high latitudes. Thus at minimum each hemisphere, considered separately, showed two clearly defined spot-zones marked off from each other by a broad belt in which there were no spots at all. Of these two spot-zones in each hemisphere, the lower appears to correspond to the series of spots of the expiring cycle. During the periods of increase the equatorial belt was almost wholly free from spots, indicating possibly the complete disappearance of the last members of the old cycle. At maximum, however, the spots of the new cycle were most widely spread, and were even seen in the near neighbourhood of the equator . . . ."

Finally, I wish to tender you my sincere congratulations on your original interpretation of the long-period variation, and to express the hope that my little remarks may contribute to enhance its evident plausibility.

It goes without saying, that you may dispose of these remarks at your own pleasure.—I am, very faithfully yours,

J. Stein, S.J.

Measures of Southern Double Stars in 1907. By James L. Scott.

The following measures of southern double stars were made with the same 5-in. refractor as those in *M.N.*, vol. lxiv. p. 52, bright wire illumination being used throughout. The weather during 1907 was, on the whole, distinctly poor, cloudy nights and bad definition being the rule rather than the exception; the number of stars measured was therefore smaller than usual.

Star's Name.	R.A. h m	S. Dec. ′	P.A. °	Distance. ″	Mags.		Nights.	Date. 1907.
Σ 3063	0 2	5 6	218·0	1·85	8½	10	3	·942
O. Stone 51	0 2	14 54	288·6	9·88	9	9	2	·942
β 391 (κ_1 Sculptoris)	0 3	28 32	270·8	1·15	6½	6½	3	·928
h 3375	0 29	35 32	166·3	6·05	6½	9	2	·868
O. Stone 3	0 47	23 9	267·2	2·10	8½	9½	3	·953
h 3407	0 48	25 31	127·3	13·20	7	7½	2	·871
Ll. 1662	0 53	16 13	214·8	6·50	8	8½	2	·006
h 2036	1 15	16 20	13·3	1·48	7	7½	3	·956
τ Sculptoris	1 32	30 25	97·2	1·90	6	7	3	·953
Σ 231 (66 Ceti)	2 08	2 52	232·0	16·05	5½	8½	2	·956
Hastings I	2 11	18 42	356·1	2·01	8	8½	3	·956
θ Eridani	2 54	40 43	86·1	8·46	3½	4½	2	·953
h 3750	5 16	21 20	279·8	3·50	4½	9½	2	·164
h 3752	5 18	24 52	102·7	3·25	5½	7½	2	·164

Star's Name.	R.A.	S. Dec.	P.A.	Distance.	Mags.		Nights.	Date.
	h m	° ′	°	″				1907.
Argelander (I)	6 02	25 1	295·0	4·74	8½	8½	2	·205
h 3858	6 22	34 59	316·6	3·75	7½	8½	2	·205
h 3938	7 9	22 42	249·2	19·50	7½	8½	2	·205
Σ 1104	7 24	14 47	333·6	2·30	7	9	2	·246
h 3973	7 26	20 40	38·4	8·90	8½	9	2	·249
O. Stone 18	8 5	26 50	261·2	3·32	8½	9	2	·249
Dunlop 63	8 6	42 21	80·9	5·50	7	8	2	·246
γ Argûs (A & B)	8 6	47 3	219·8	41·52	2	6	2	·249
(A & C)	...	...	148·7	62·11	2	8	2	·249
h 4093	8 23	38 43	122·7	8·03	7	7½	2	·260
Howe 23	9 3	31 11	307·2	3·30	8½	9	2	·260
h 4193	9 12	22 42	118·8	3·05	8	10	2	·249
H. C. Wilson	9 16	23 3	36·9	1·30	7¾	9¾	3	·249
ζ_1 Antliæ	9 26	31 27	210·2	8·20	5¾	6½	2	·260
h 4224	9 32	30 47	117·3	7·00	8	9	2	·249
C. G. C. 13,722	9 58	17 30	274·0	20·02	6½	7½	1	·260
Howe 15	10 58	26 58	338·2	2·10	7·5	9·3	2	·271
17 Crateris	11 27	28 43	209·3	8·88	5½	5½	2	·378
h 4455	11 32	33 1	244·0	3·80	6½	9½	2	·378
C. G. C. 15,942	11 35	37 30	95·8	16·50	7	9	1	·397
h 4481	11 52	21 59	197·1	3·30	8	8	2	·378
h 4495	12 0	32 23	316·8	6·60	6½	9	2	·378
C. G. C. 16,612	12 4	34 6	202·5	3·75	6½	9	2	·378
Jacob 8	12 5	34 9	20·3	3·10	6½	9	2	·394
S 634	12 6	16 14	285·5	6·67	7	9	2	·378
D Centauri	12 9	45 10	242·4	3·08	5½	7	2	·394
h 4518	12 19	40 50	207·6	10·10	6¾	9	2	·446
ε Corvi	12 25	15 58	214·3	24·45	3	8½	1	·430
Σ 1669	12 36	12 28	306·0	5·70	6½	6½	2	·446
γ Virginis	12 37	0 54	326·8	6·01	3	3	3	·430
h 4556	12 49	27 25	80·0	6·50	7	8½	2	·397
O. Stone 27	12 52	12 36	66·9	1·95	8½	8¾	2	·446
h 4563	12 55	33 5	236·0	6·82	7½	8¼	1	·430
h 4587	13 20	42 32	86·3	5·20	8	8½	2	·446
Ö. A. 12,867	13 21	22 43	356·2	1·61	8·8	9	2	·446
β 114	13 29	8 6	143·0	1·50	8	8	2	·446
h 4608	13 36	33 28	177·5	4·60	8	8¼	1	·430
h 4617	13 45	29 23	260·8	5·05	7½	9½	2	·446
β 343	13 46	31 7	118·0	1·03	6	7	3	·446
ε Centauri	13 46	32 30	109·5	8·17	4¾	6	1	·430

Star's Name.	R.A.	S. Dec.	P.A.	Distance.	Mags.		Nights.	Date.
	h m	° ′	°	″				1907.
Howe 26	13 49	31 37	115·9	6·40	7½	10	2	·485
h 4661	14 6	38 35	230·8	4·48	9	9	2	·482
Σ 1837	14 19	11 13	300·4	1·40	7	8·5	2	·482
Howe 29	14 31	37 6	213·4	4·25	8	8½	2	·485
h 4690	14 31	45 42	23·6	19·50	5½	8	1	·501
54 Hydræ	14 40	25 2	130·2	8·71	5¼	7¼	2	·482
β 106 (A & B)	14 44	13 44	341·4	1·60	5½	6½	2	·485
P XIV., 212	14 52	20 57	296·8	17·35	6	7½	2	·482
h 4722	14 53	30 18	340·4	8·65	7½	9	2	·501
β 239	14 53	27 16	316·8	0·90	6	6½	3	·485
h 4727	14 57	27 26	216·7	7·38	8¾	8¾	2	·482
β 119	15 0	6 37	300·4	1·60	8	8½	4	·501
h 4743	15 1	32 32	196·9	11·19	7¼	7¼	2	·485
Howe 31	15 7	36 52	46·8	6·50	7¼	7¼	2	·485
h 4776	15 23	41 34	228·6	5·52	7	8½	2	·501
h 4783	15 25	19 48	280·1	11·05	6¾	8½	2	·498
Howe 37	15 36	14 28	90·7	5·43	8	8	2	·490
η Lupi	15 53	38 7	20·4	15·30	4	8	1	·490
Lal 29,720	16 13	19 49	21·2	13·18	8	8¼	1	·501
σ Scorpi	16 15	25 21	272·7	20·60	3	7½	1	·583
h 4845	16 17	41 01	135·6	2·10	8	8½	2	·548
h 4848	16 17	32 58	153·2	5·90	7	7½	2	·501
h 4850	16 18	29 28	350·8	6·30	6	6½	2	·501
ρ Ophiuchi	16 20	23 13	355·2	3·40	5¼	6¼	2	·501
Skinner	16 39	17 8	86·0	3·50	8½	8½	2	·605
P XVI., 236	16 51	19 23	231·3	4·80	7¼	8½	1	·583
Σ 2119	17 01	13 48	192·2	2·20	8	8	2	·548
36 Ophiuchi	17 9	26 27	187·8	4·18	5¼	5½	2	·548
β 958	17 10	19 14	213·8	1·55	8½	9	3	·548
38 Ophiuchi	17 11	26 31	337·6	5·68	6½	9	2	·548
β 416	17 12	34 53	273·0	2·30	6	8	4	·559
β 126	17 14	17 39	262·5	2·00	6¼	8¼	2	·559
Σ 2204	17 40	13 16	24·3	14·10	7	7·2	1	·583
h 5003	17 53	30 15	104·8	5·25	6	7	2	·605
τ Ophiuchi	17 57	8 11	258·2	1·94	5½	6	3	·559
Howe 50	17 59	36 35	2·6	3·05	7¼	8¼	2	·583
70 Ophiuchi	18 0	2 33N	164·7	2·61	4½	6	4	·559
β 245	18 4	30 45	353·5	4·15	6	8	2	·643
H N 125	18 23	25 6	101·6	3·01	7½	7¾	2	·605
S 715	19 12	16 09	14·6	8·30	7¼	7½	2	·698

Star's Name.	R.A.	S. Dec.	P.A.	Distance.	Mags.		Nights.	Date.
	h m	° ′	°	″				1907.
h 1381	19 12	16 10	195·7	4·95	8·1	8·3	2	·698
β 142	19 22	12 21	342·6	1·60	8	8	3	·698
Ho. 462	20 59	11 29	218·5	2·55	9	9½	3	·832
h 5252	21 7	15 25	318·2	3·02	8	8½	2	·832
C. G. C. 29,658	21 35	18 53	66·0	5·00	8	9·4	2	·846
η Piscis Australis	21 55	28 56	116·3	1·74	6	6½	5	·868
See 470	22 5	24 5	36·8	1·66	8	9	4	·868
S 808	22 19	20 52	151·8	7·00	7	8·3	2	·797
53 Aquarii	22 21	17 15	310·8	6·80	6½	6¾	2	·797
ζ Aquarii	22 24	0 32	315·2	3·05	4·5	4·6	4	·832
h N 117	22 34	28 52	64·3	3·04	7¾	8½	2	·871
γ Piscis Australis	22 47	33 24	266·4	3·56	4½	8¾	2	·871
Σ 3008	23 19	9 1	235·8	3·60	7½	8½	2	·871
ι_2 Aquarii	23 41	19 14	137·8	5·90	5½	7	2	·928
B.A.C. 8308	23 49	27 36	269·3	6·90	6¾	7½	2	·950
Σ 3046	23 51	10 03	252·7	3·18	8¾	9¼	3	·950

Shanghai:
1907 *December* 31

Note on the adopted co-ordinates of the Bombay (Colaba) Observatory. By A. M. W. Downing, D.Sc., F.R.S.

It may be desirable to point out that the relatively large change in the position of the Colaba Observatory, as given in the *Nautical Almanac* for 1909 and following years, from that given in the *Nautical Almanac* for 1896–1908 inclusive, arises from the large difference existing in that part of India between the geodetic and astronomical co-ordinates. I am indebted to the courtesy of the Headquarters Staff of the Trigonometrical Branch of the Survey of India for the following particulars as to the position of the Colaba Observatory:—

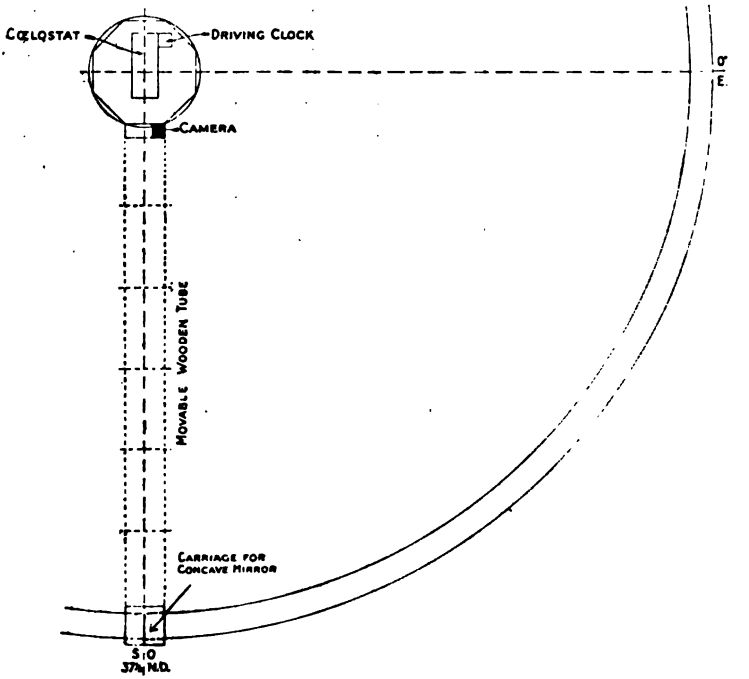
	Astronomical.	Geodetic.
Latitude,	18° 53′ 36″·18 N.	18° 53′ 46″·51 N.
Longitude,	$4^h\ 51^m\ 15^s{\cdot}72$ E.	$4^h\ 51^m\ 15^s{\cdot}15$ E.

It will thus be seen that the quantities given in the *Nautical Almanac* for 1909 and onwards are the astronomical latitude and the geodetic longitude. On the other hand, up to and including the current year, the quantities given are the geodetic latitude and the astronomical longitude; the values of these co-ordinates given above being later, and presumably more accurate, than those communicated to me in 1892, and which appear in the *Nautical Almanac* for the years specified.

Description of a 24-inch long-focus Cœlostat Reflector.

By J. H. Reynolds.

The idea of mounting a long-focus reflector in conjunction with a cœlostat was mentioned by Dr. Common in 1900, and it has been successfully carried out at Mount Wilson with the Snow telescope for spectroheliographic work. The present instrument is intended for use with a spectroheliograph which is in course of construction, but it is also arranged for photographing the Moon and planets, and



the brighter star clusters and nebulæ. For several reasons it was found advisable to keep the focal length within moderate limits; the concave mirror has therefore been made with a focal length of 38 ft., an enlarging lens being used when a greater scale is required.

The cœlostat is mounted in an octagonal wooden house, which revolves so as to face the various azimuths required; the roof revolves separately, and has a large opening covered with hinged shutters. The plane mirror, which is 28 in. in diameter, is furnished with a slow motion at right angles to the polar axis; this is very useful for getting the object in the centre of the field, and is practically indispensable for lunar work.

At the required distance from the cœlostat house a circular rail has been fixed for carrying the concave mirror. This rail runs from east to south, and gives a range in declination of $37\frac{1}{2}^{\circ}$. This, of course, is a limited range, and is the main defect of this form of mounting, especially for stellar and nebular work.

The concave mirror is slightly tilted, so as to bring the focal plane a little to one side of the plane mirror, thus forming a Herschelian reflector: with the long focal ratio this does not materially affect definition, and it would be quite possible to alter the form to the Newtonian if found necessary. For stellar work the following is done by means of an eyepiece with double cross-wires near the edge of the plate, and the plate-holder and screw motions can be rotated to bring the star trails parallel with the R.A. screw motion.

For the Moon and planets a negative enlarging lens is employed, and it has been found that with suitable air-conditions an equivalent focal length of 120 feet can be employed with advantage. Although the resulting photograph is not usually so crisply defined as one taken in the primary focus, there is an evident gain in detail, and the disadvantage of "grain" in the plate is not so obtrusive.

In order to eliminate as far as possible uneven motion in the instrument, it is intended, when the enlarging lens is used, to move the plate-holder by means of an oil cylinder, instead of driving the cœlostat. By this means a smooth, continuous motion is possible, and there is the additional advantage that the concave mirror need not be moved in azimuth for the different declinations required, as it is not necessary for the plane mirror to be parallel with the polar axis. The instrument is also well adapted for large-scale photographs of the solar surface.

It has been found desirable to cover in the space between the cœlostat house and the concave mirror with a movable wooden tube; unless this is used, the ground air-currents interfere with good definition.

On Dr. Roberts' method of determining the absolute dimensions of an Algol Variable Star. By the Rev. J. Stein, S.J., Sc.D.

(*Communicated by Professor H. H. Turner.*)

In an interesting article of the January number, 1906, of the *M.N.* (vol. lxvi. p. 123), Dr. Alex. Roberts starts from the hypothesis that the light-changes of the Algol variables U Pegasi and RR Centauri are fairly well explained by two eclipsing stars moving in a circular orbit, and tries to reduce the residuals (O—C) by applying some corrections to the theoretical light-curve. He supposes that the orbit is passing through the line of sight, and puts—

P = Period of variable in minutes.
ϵ (very small) = Eccentricity of Orbit.
λ = Longitude of periastron, reckoned from line of sight.
θ = True angular distance of eclipsing star from line of sight at time T.
Δm = Change in magnitude per minute at time T.
l = Time (in minutes) that light takes to cross the semi-orbit.
Δt = Correction to the assumed time of passing the principal minimum phase.
$p = \epsilon \sin \lambda$, $q = \epsilon \cos \lambda$.

Then, neglecting the second and higher powers of ϵ, he proceeds from the typical equation of condition—

$$\Delta m \cos \theta \frac{2}{n} p + \Delta m \sin \theta \frac{2}{n} q + \Delta m \, \Delta t + \Delta m |\cos \theta| l + \ldots = (O—C),$$

where p, q, Δt, l are unknown quantities, $n = \frac{2\pi}{P}$ and $|\cos \theta|$ denotes the absolute value of $\cos \theta$.

According to the author, the three first terms are corrections depending on eccentricity and variation of epoch; the fourth term is the correction due to the fact that light takes an appreciable interval of time to traverse the orbit. (The other corrections will here be left out of consideration.) Hence, resolving the equations of condition after the method of least squares, Dr. Roberts finds l, and consequently the absolute dimensions of the orbit.

It seems, however, that the point of departure of this method is open to criticism. Indeed, the first member of the fundamental equation is not the adequate expression of the corrections due to eccentricity, variation of epoch, and aberration of light.

To show this, let us take the most simple case—that the two components are both spherical, and that the nearly circular relative orbit is passing through the line of sight. In this case the apparent brightness of the binary system is only depending on the apparent distance ρ of the centres of the two spheres, as seen from our Sun.

Hence we may put—

Change in magnitude in theoretical light-curve deduced from circular orbit,

$$\Delta m = \frac{dm}{d\rho}\frac{d\rho}{dt} \quad . \quad . \quad . \quad . \quad (1)$$

For the purpose of determining the effect of eccentricity on ρ, we may neglect the aberration of light. Thus, in the circular orbit we have—

$$\rho^2 = r^2 \sin^2\theta \quad . \quad . \quad . \quad . \quad (2)$$

r denoting the constant radius-vector, and θ referring to the star S_1 eclipsing at principal minimum.

Consequently, from (1) and (2),

$$\frac{dm}{d\rho} = \Delta m \tan\theta \frac{1}{n\rho} \quad . \quad . \quad . \quad . \quad (3)$$

Putting $\Delta\rho$, Δr, $\Delta\theta$ = the variations of ρ, r, θ, at the time T by passing from the circular orbit to the improved, eccentrical one, we find—

$$\frac{\Delta\rho}{\rho} = \frac{\Delta r}{r} + \cot\theta\,\Delta\theta \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{\Delta r}{r} = -\epsilon\cos(\theta-\lambda) = -q\cos\theta - p\sin\theta \quad . \quad . \quad (5)$$

$$\Delta\theta = 2\epsilon\sin(\theta-\lambda) + \text{const.}$$

For $\theta = 0$ we shall have $(\Delta\theta)_0 = n\,\Delta t$; consequently

$$\Delta\theta = 2\epsilon\sin(\theta-\lambda) + 2\epsilon\sin\lambda + n\,\Delta t = 2q\sin\theta + 2p(1-\cos\theta) + n\,\Delta t \quad (6)$$

Substituting (5) and (6) in (4),

$$\frac{\Delta\rho}{\rho} = q\cos\theta - p\frac{(1-\cos\theta)^2}{\sin\theta} + n\cot\theta\,\Delta t \quad . \quad . \quad (7)$$

Finally, according to (3) and (7), the adequate variation of magnitude at the time T, due to eccentricity and variation of epoch, will be

$$\frac{dm}{d\rho}\Delta\rho = \Delta m\sin\theta\frac{1}{n}q - \Delta m\frac{(1-\cos\theta)^2}{\cos\theta}\frac{1}{n}p + \Delta m\,\Delta t \quad (8)$$

Vice versa, for determining the effect of aberration, we may neglect the eccentricity of the orbit, and put—

a_1, a_2 = the semi-axes of the circular orbits of the two components S_1 and S_2 around the common centre of gravity.

ρ_1, ρ_2 = the apparent projections of the radii vectores on the tangential plane of the celestial sphere at the time T, this being reckoned from the moment of principal minimum.

l_1, l_2, σ = Time (in minutes) that light takes to cross the semi-

axes of the orbits and the distance from the centre of gravity to the Sun.

Then the time taken by the light—

to pass from S_1 to the Sun will be $= \sigma - l_1 \cos n(T-\sigma)$

„ „ S_2 „ „ $= \sigma + l_2 \cos n(T-\sigma)$

Consequently, putting $n(T-\sigma) = \theta$:

$$\rho_1 = a_1 |\sin(\theta + nl_1 \cos\theta)|$$
$$\rho_2 = a_2 |\sin(\theta - nl_2 \cos\theta)|$$

where $\cos\theta$ is to be taken with its proper sign.

The quantities l_1 and l_2 being generally very slight, and since $a_1 l_2 = a_2 l_1$, we may put—

$$\rho = \rho_1 + \rho_2 = a|\sin\{\theta + n(l_1 - l_2)\cos\theta\}| \quad . \quad (9)$$

and consequently the correction for light-equation will be

$$\Delta m(l_1 - l_2)\cos\theta \quad . \quad . \quad . \quad (10)$$

Thus it would seem theoretically impossible to determine the absolute dimensions of the orbit in this manner. If this statement is correct, it follows that the numerical results of Dr. Roberts' paper cannot have the interpretation which is given to them.

Specola Vaticana: 1908 *March* 19.

Note to Father Stein's paper. By Professor H. H. Turner, D.Sc., F.R.S.

THE following elementary geometrical considerations indicate the reasons why the light-equation fails to give us expected information about the orbit. They were mentioned at the meeting of the Society in communicating Father Stein's paper, and are added here as a note at the request of one of the Secretaries.

Consider the three cases—(*a*) when the mass of the bright body is so small as to be negligible; (*b*) when the mass of the dark body is negligible; (*c*) when the two bodies are of equal mass. And in each case let us suppose the orbit circular and in a plane passing through the line of sight, and that light takes a time 2T to cross the orbit.

(*a*) In the first case (fig. 1) the dark body is stationary at the centre O, and the bright body revolves round it, and is eclipsed at A, the Earth being in the direction of the arrow. The light which suffers eclipse reaches O at a time T *after* the actual eclipse.

(*b*) In the second case (fig. 2) the bright body is stationary at the centre O, and the dark body revolves round it and eclipses it when at B. But the light which suffers eclipse must leave O at a time T *before* the actual eclipse.

Hence, referring events to the time at which light leaves O (or more generally the line XOY), we see that in case (*a*) the apparent eclipse is *retarded* and in case (*b*) it is *accelerated* by an equal amount. What happens in case (*c*) when the bodies are equal?

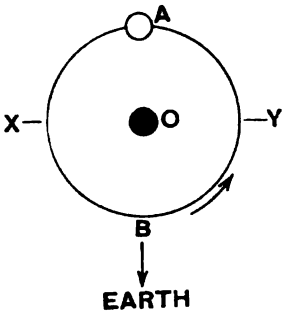


FIG. 1.

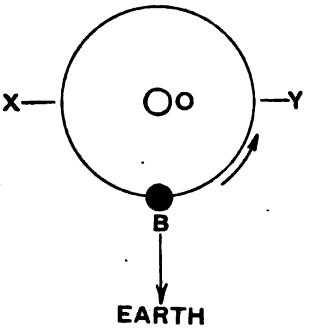


FIG. 2.

It is suggested that there is neither retardation nor acceleration, *i.e.* that the apparent eclipse takes place at the same time as the true eclipse, and this is actually the case. It was Professor Dyson who called attention to this case after reading Father Stein's paper at my request.

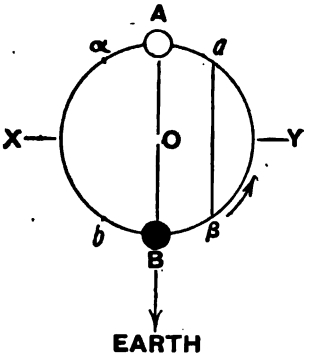


FIG. 3.

(*c*) When the masses are equal, they both revolve round O, their common C.G. The position for true eclipse is, bright body at A and dark at B; but the light which then leaves the bright body does not suffer eclipse, since when it reaches B the dark body will have moved away. Let aA, $A\alpha$, bB, $B\beta$ be four equal

small arcs described by either body in time T which light takes to cross the semi-orbit. Then the sequence of events is as follows:

	Bright Body.	Dark Body.	Eclipsed Light.
(1)	At a	At b	At a
(2)	At A	At B	In XOY
(3)	At a	At β	At β.

The eclipsed light must start before the bright body has reached A, and is eclipsed by the dark body after it has left B, so that the *position* for apparent eclipse is displaced from AB to $a\beta$. But we have no means of noting this displacement in space in the case of Algol variables. All we could note would be a change in epoch, and this is zero; for we see that the eclipsed light leaves XOY at the moment of true eclipse, when the bodies are at AB.

Hence, whatever the size of the orbit, when the masses are equal there will be no change in epoch of eclipse due to light-equation.

In other cases there will be a change of epoch,—a retardation if the dark body is larger, an acceleration if it is smaller. But the change will be small unless the masses are very unequal; and it will depend not only upon the size of the orbit, but on the ratio of the masses, which cannot be separately determined.

Errata.

In Professor Barnard's paper, page 357, line 27,

for Struve *read* Struve's.

In Mr. Baldwin's paper, page 369 (Table I.), the headings of the last four columns should be, respectively,

$d \log \sin^2 \mathrm{I}$; Zenith dist.; Extinction corr.; $\Delta \mathrm{M}$;

and the bracket over these columns omitted.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVIII. MAY 8, 1908. No. 7

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Captain Richard Algernon Craigie Daunt, D.S.O., Lynalta, Newtownards, Co. Down, Ireland,
and
Edgar Odell Lovett, Ph.D., Professor of Astronomy, Princeton University, New Jersey, U.S.A.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Warin Foster Bushell, The Hermitage, Harrow (proposed by Col. G. L. Tupman);
Charles P. Butler, A.R.C.Sc., F.R.P.S., Solar Physics Observatory, South Kensington, S.W. (proposed by William J. S. Lockyer);
William Doberck, Ph.D., late Director, Hong Kong Observatory, Kowloon, Elgin Road, Sutton, Surrey (proposed by Sir W. H. M. Christie);
James Nangle, Technical College, Sydney, N.S. Wales, Australia, and Private Observatory, Tupper Street, Stanmore, near Sydney (proposed by C. J. Merfield);
Charles W. Raffety, Wynnstay, Woodcote Valley Road, Purley, Surrey (proposed by Richard Kerr); and
Rev. T. J. Williams-Fisher, M.A., Rector of Norton, Atherstone (proposed by Rev. D. H. Sparling).

Seventy-one presents were announced as having been received since the last meeting, including amongst others:—Lieut. A. ff.

Garrett, The Jaipur Observatory and its builder, presented by the author; Greenwich Astrographic Catalogue, vol. ii., 16 charts of the Astrographic Chart of the heavens, and Observations of the planet Eros, 1900–1901, presented by the Royal Observatory, Greenwich; Professor E. S. Holden, Galileo, and other tracts, presented by the author; Oxford Astrographic Catalogue, vol. iv., presented by the University Observatory, Oxford; Pennsylvania University Publications, Catalogue of 648 double stars discovered by Professor Hough, presented by the University; three lantern slides of the Corona of 1908 January 3, taken by Professor Campbell, presented by the Lick Observatory.

An Empirical Law of Astronomical Refraction.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The following investigation was originally undertaken with the view of substituting a simple proof of the law of refraction for students who could not follow the more elaborate proof involving the differential equation. But it was a surprise to find how closely the observed refraction could be represented with so rough a supposition as that of three, or even two, homogeneous shells of atmosphere; and the question was suggested whether, in the present state of our knowledge, more elaborate hypotheses were really justified. If a rough supposition fits the facts, clearly it is no proof of the correctness of a more elaborate one that it also fits the facts.

2. Moreover, suspicion of the correctness of existing hypotheses was suggested from another direction. Meteorologists are finding that the temperature of the atmosphere does not follow a smooth gradient, as is generally assumed in refraction hypotheses: at a certain height a wholly unexpected state of things has been found to exist. According to M. Teisserenc de Bort there is above 10 or 12 km., an "isothermal layer" in which the temperature ceases to fall as we ascend; and the conditions are similar over parts of the world where the temperatures close to the surface differ widely. It is difficult to reconcile these results of observation with the hypotheses usually adopted in constructing tables of refraction.* It is, however, not intended to examine at present the consequences of taking M. Teisserenc de Bort's work into account;—merely to show that it may not be difficult to do so when we have fuller information, without, perhaps, dislocating existing refraction tables.

3. First let us consider an atmosphere of one homogeneous spherical layer. Let C be the Earth's centre; OM a section of its surface through C and the star; LB the section of the boundary of

* I am indebted to Mr. Saunder for a reference to a paper by Professor Bakhuyzen in *Konink. Akad. van Weten. te Amsterdam*, 1907 January 26; see *Nature*, 1907 April 4, p. 538; in which the discordance between observation and the assumption usually made is demonstrated by a concrete example.

the atmosphere; ABO the direction of the star's light, refracted once only at the boundary B. The star is thus seen in the direction OBD, instead of in the direction BA.

If $\angle ABE = \phi$, $\angle OBC = \psi$, $\angle BOL = \zeta$,

then by the law of refraction

$$\sin \phi = \mu \sin \psi \quad . \quad . \quad . \quad . \quad (1)$$

and from the triangle OBC

$$\sin \psi = \frac{OC}{BC} \sin \zeta = \frac{1}{1+z} \sin \zeta \quad . \quad . \quad . \quad (2)$$

if z be the ratio of the height of the atmosphere to the Earth's radius.

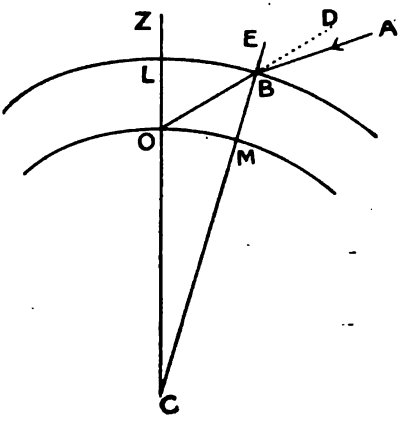


FIG. 1.

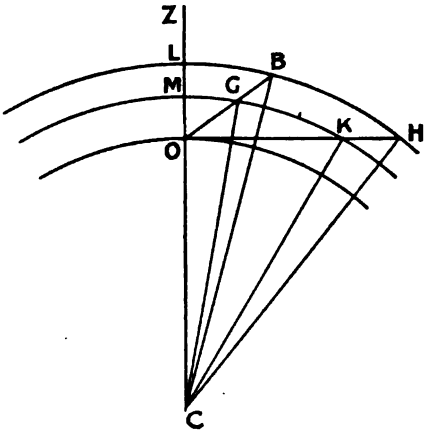


FIG. 2.

Hence $$\sin \phi = \frac{\mu}{1+z} \sin \zeta \quad . \quad . \quad . \quad . \quad (3)$$

4. The similarity of equations (2) and (3) suggests a geometrical construction for the refraction which is obvious enough, but which I do not remember to have seen in print.

With radius $\frac{1+z}{\mu}$ describe the circle MGK round C as centre (fig. 2).

Then $$\frac{\sin OGC}{\sin GOL} = \frac{1}{CG} = \frac{\mu}{1+z}.$$

Hence $OGC = \phi$, just as $OBC = \psi$, and the refraction is $\phi - \psi = GCB$.

5. If now we take the case of two homogeneous shells of atmosphere, we see that the refraction will be represented by the sum

of two angles G_1CB_1, G_2CB_2, with a gap between; the radii CG_1, CB_1, etc. being of constant length, and the angle ZOG being ζ, the apparent zenith distance, and generally, if there are n layers, expressing all the angles of incidence and refraction in terms of ζ, we have—

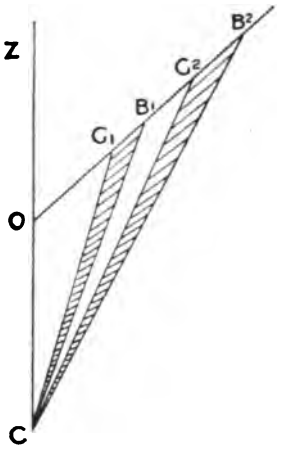


FIG. 3.

$$\left.\begin{aligned}
&\sin\psi_1 = \frac{1}{1+z_1}\ \sin\zeta = \frac{\mu_1}{\mu_1(1+z_1)}\sin\zeta \\
&\sin\phi_1 = \frac{\mu_1}{\mu_2}\ \sin\psi_1 = \frac{\mu_1}{\mu_2(1+z_1)}\sin\zeta \\
&\sin\psi_2 = \frac{1+z_1}{1+z_2}\sin\phi_1 = \frac{\mu_1}{\mu_2(1+z_2)}\sin\zeta \\
&\sin\phi_2 = \frac{\mu_2}{\mu_3}\ \sin\psi_2 = \frac{\mu_1}{\mu_3(1+z_2)}\sin\zeta \\
&\qquad\vdots \\
&\sin\psi_n = \frac{\mu_1}{\mu_n(1+z_n)}\sin\zeta \\
&\sin\phi_n = \frac{\mu_1}{\mu_{n+1}(1+z_n)}\sin\zeta
\end{aligned}\right\} \quad . \quad (4)$$

The notation is tolerably obvious: the radii of the successive surfaces of separation are $1+z_1$, $1+z_2$, $1+z_3 \cdots 1+z_n$; and the refractive indices of the successive homogeneous shells are μ_1, $\mu_2 \cdots \mu_n$. If the n^{th} be the outermost, $\mu_{n+1}=1$.

6. It will be seen that although B_1 (fig. 3) lies on the boundary of the first shell, B_2 does not lie on the boundary of the second. The radius CB_2 is not $1+z_2$ but $\frac{\mu_2}{\mu_1}(1+z_2)$; and if the refractive index decreases with the height, this is less than $1+z_2$.

Recent meteorological experiments, however, suggest that there may be anomalies in the upper strata. If the refractive index for the r^{th} layer should increase with height instead of decreasing, then CG_r would be greater than CB_r, and the angle G_rCB_r would be subtractive.

7. It is of interest to consider whether there are always gaps between the elements. If the angle B_1CG_2 vanishes, then $\phi_2=\psi_1$, or

$$\mu_1(1+z_1)=\mu_3(1+z_2) \quad . \quad . \quad . \quad . \quad (5)$$

In this case the refractions of the two assumed layers are exactly equivalent, at all zenith distances, to that of a single layer with outer radius $CB_2=\frac{\mu_2}{\mu_1}(1+z_2)$, and of refractive index $\frac{CB_2}{CG_1}=\frac{1+z_2}{1+z_1}$.

8. Condition (5) may be regarded as an equation determining μ_3, the refractive index of the shell surrounding the two inner ones; and hence we see that, *given any two homogeneous spherical shells of atmosphere with any refractive indices, they may be replaced for purposes of astronomical refraction by a single homogeneous shell, provided the refractive index of the medium immediately surrounding them has a certain value, and in that case only.*

This case must be carefully distinguished from the one in which a really homogeneous shell is arbitrarily treated as two by an imaginary surface of separation, at which there is no refraction. In this latter case one of the angles G_1CB_1 or G_2CB_2 would be zero; not necessarily the gap B_1CG_2.

9. Attention is drawn to the possibility above italicised, because it seemed at first that it might afford the explanation of the curious fact that the rough hypothesis of two or three homogeneous shells of air can give so good an approximation to our observed refractions as below. If it is possible to replace two elements by a single one, and to continue the process, there is nothing unreasonable in the exact representation of a large number of elements by a simple equivalent. But the numerical values obtained do not support this view, the gaps being wide compared with the thickness of the shells.

10. We now turn to numerical evaluation of the constants. The law of refraction is simple enough at moderate zenith distances, and we shall be chiefly concerned with its form near the horizon. In that case ϕ, ψ, and ζ are all nearly 90°, and it is more convenient to deal with their complements, which are small angles. Let us therefore write

$$\zeta=90^\circ-A, \quad \phi=90^\circ-\nu, \quad \psi=90^\circ-\lambda.$$

Let α and β be the values of λ and ν for $\zeta = 90°$ or $A = 0°$. Then from equations (2) and (3),

$$\cos\alpha = \frac{1}{1+z}, \quad \cos\beta = \frac{\mu}{1+z} \quad . \quad . \quad . \quad (6)$$

and equations (2) and (3) may now be written,

$$\cos\lambda = \cos\alpha \cos A, \quad \cos\nu = \cos\beta \cos A \quad . \quad . \quad (7)$$

In fig. 2, $\angle OCH = \alpha$ and $\angle OCK = \beta$: $\angle KCH = \alpha - \beta$. The refraction is $r = \phi - \psi = \lambda - \nu$.

11. In terms of the method usually adopted, we should find our constants from what we could learn of μ and z. From the point of view of this note, we are to determine α and β empirically from observed refractions at two different zenith distances; and the most convenient Z.D.'s to select are 45° and 90°. We shall confine our attention to mean refractions; and it is convenient to have the horizontal refraction exactly 33′, for a reason which will become clear when we deal with three separate layers. The mean Pulkowa refractions, as shown in the second column of Table I., have therefore been multiplied by a constant. Hence one equation between α and β is

$$\alpha - \beta = 33' = \cdot 00960 \quad . \quad . \quad . \quad (8)$$

12. At altitude 45° the equations may be combined to obtain the usual approximate law of tan Z.D. Subtracting one of equations (8) from the other, we have

$$(\cos\nu - \cos\lambda) = (\cos\beta - \cos\alpha)\cos A,$$

or

$$2\sin\frac{\lambda-\nu}{2} = (\cos\beta - \cos\alpha)\cos A \operatorname{cosec}\frac{\lambda+\nu}{2}.$$

As a first approximation, we can neglect the differences between λ, ν, and A on the right, and put $\frac{\lambda+\nu}{2} = A = 90° - \zeta$. If the refraction be denoted by r, we have for moderate zenith distances

$$r = 2\sin\frac{\lambda-\nu}{2}.$$

$$\therefore \quad r = (\cos\beta - \cos\alpha)\tan\zeta.$$

So that $(\cos\beta - \cos\alpha)$ is the well-known "coefficient of refraction" at moderate zenith distances: and when $\zeta = 45°$, $\tan\zeta = 1$, and

$$\cos\beta - \cos\alpha = 55''\cdot 70 = \cdot 0002700 \quad . \quad . \quad (9)$$

Hence

$$2\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2} = \cdot 0002700.$$

13. Substituting the value of $\alpha - \beta$ from equation (9), we find

$$\tfrac{1}{2}(\alpha+\beta) = 1^\circ\ 36'\ 43'',$$

and since

$$\tfrac{1}{2}(\alpha-\beta) = \quad 16\ 30\cdot$$

$$\alpha = 1^\circ\ 53'\ 13'' \qquad \beta = 1^\circ\ 20'\ 13'' \quad . \quad . \quad (10)$$

14. We can now estimate the error of the approximation above, where A is substituted for $\frac{\lambda+\nu}{2}$. At $A = 45^\circ$

$$\log\cos\nu = \log\cos 45^\circ - \cdot 000118$$
$$\log\cos\lambda = \log\cos 45^\circ - \cdot 000236$$
$$\therefore\quad \nu - 45^\circ = 56'', \quad \lambda - 45^\circ = 1'\ 52''.$$

These differences are found at once from a table of log cosines. Hence

$$\frac{\lambda+\nu}{2} = 45^\circ\ 1'\ 24'',$$

and the accurate equation to replace equation (9) is

$$\cos\beta - \cos\alpha = \cdot 0002700 \times 1\cdot 00041$$
$$\tfrac{1}{2}(\alpha+\beta) = 1^\circ\ 36'\ 43'' + 1''\cdot 9$$
$$= 1^\circ\ 36'\ 45'' \text{ say},$$
$$\therefore\quad \alpha = 1^\circ\ 53'\ 15'' \quad \beta = 1^\circ\ 20'\ 15'' \quad . \quad . \quad (11)$$

15. If a table of logarithms of cosines to seconds of arc is not available, we can throw the equation

$$\cos\lambda = \cos\alpha\ \cos A$$

into the form

$$\frac{\cos A - \cos\lambda}{\cos A + \cos\lambda} = \frac{1-\cos\alpha}{1+\cos\alpha},$$

or

$$\tan\frac{\lambda - A}{2}\cdot\tan\frac{\lambda + A}{2} = \tan^2\frac{\alpha}{2}.$$

Let

$$\tan\frac{\lambda - A}{2} = \frac{\rho_1}{2}: \text{ then since } \frac{\lambda + A}{2} = A + \frac{\lambda - A}{2}$$

$$\rho_1(\rho_1 + 2\tan A) = 2\tan^2\frac{\alpha}{2}(2 - \rho_1\tan A).$$

$$\therefore\quad \rho_1 = -a \pm \sqrt{a^2+4b}$$
$$= \frac{4b}{a + \sqrt{a^2+4b}}, \text{ the upper sign being the}$$

only one to take; where $a = \tan A\sec^2\frac{\alpha}{2}$, $b = \tan^2\frac{\alpha}{2}$

$$\therefore\quad \rho_1 = \frac{2(1-\cos\alpha)}{\tan A + \sqrt{\tan^2 A + \sin^2\alpha}} \quad . \quad . \quad (12)$$

When we neglect sin α in comparison with tan A, the denominator becomes 2 tan A, and

$$\rho_1 = (1 - \cos\alpha)\cot A.$$

Similarly,

$$\rho_2 = (1 - \cos\beta)\cot A,$$

and $$r = \rho_1 - \rho_2 = (\cos\beta - \cos\alpha)\cot A \text{ as before.}$$

When sin α is small compared with tan A we have

$$\rho_1 = (1 - \cos\alpha)\cot A\left[1 - \tfrac{1}{4}\frac{\sin^2\alpha}{\tan^2 A}\right] \quad . \quad . \quad (13)$$

and when $A = 45^\circ$

$$\rho_1 = (1 - \cos\alpha)(1 - \tfrac{1}{4}\sin^2\alpha)$$

$$= \frac{\alpha^2}{2} - \frac{\alpha^4}{6} \text{ to the second order in } \rho_1.$$

Similarly, $\rho_2 = \frac{\beta^2}{2} - \frac{\beta^4}{6}$; and to this order of approximation ρ_1 and ρ_2 are equal to their tangents. Hence the refraction is

$$\rho_1 - \rho_2 = (\alpha - \beta)\cdot\frac{\alpha+\beta}{2}\left[1 - \tfrac{1}{3}(\alpha^2 + \beta^2)\right] \quad . \quad . \quad (14)$$

Since $\alpha - \beta$ is the horizontal refraction we have

$$k = \frac{\text{Refraction at } 45^\circ}{\text{Refraction at Horizon}} = \frac{\alpha+\beta}{2}\left[1 - \tfrac{1}{3}(\alpha^2 + \beta^2)\right] \quad . \quad (15)$$

16. Let us now consider what happens if we have n concentric shells characterised by

$$(\alpha_1\ \beta_1),\ (\alpha_2\beta_2) \ldots (\alpha_n,\ \beta_n).$$

In each case the horizontal refraction is $(\alpha_r - \beta_r)$; so that the total horizontal refraction is

$$(\alpha_1 - \beta_1) + (\alpha_2 - \beta_2) + (\alpha_3 - \beta_3) + \ \ldots\ + (\alpha_n - \beta_n) = 33'.$$

Now the division of our shells is somewhat arbitrary. We can make them all of equal thickness, for instance; or we can choose their relative thicknesses so that they have equal refractions at some specified zenith distance. There is a convenience in making their *horizontal refractions all equal*, so that

$$\alpha_1 - \beta_1 = \alpha_2 - \beta_2 = \alpha_3 - \beta_3 = \ \ldots\ = \alpha_n - \beta_n = \frac{1}{n}(33') \quad . \quad (16)$$

Proceeding to the refraction at 45°, it is

$$\sum(\alpha_r - \beta_r)\left(\frac{\alpha_r + \beta_r}{2}\right)\left[1 - \tfrac{1}{3}(\alpha_r^2 + \beta_r^2)\right]$$

and hence substituting for $\alpha_r - \beta_r$, we have

$$nk = \sum\left(\frac{\alpha_r + \beta_r}{2}\right)\left[1 - \tfrac{1}{3}(\alpha_r^2 + \beta_r^2)\right] \quad . \quad . \quad (17)$$

where k denotes, as in equation (15), the ratio of refraction at 45° to that at the horizon.

One Shell.

17. The numerical results for one shell (with $\alpha = 1^\circ\ 53'\ 15''$, $\beta = 1^\circ\ 20'\ 15''$, as in equation (11)) are shown in the third column of Table I., and the discordances from the Pulkowa refractions in the fourth column. It is clear that, though the accordance is close as far as Z.D. 60°, the hypothesis does not fit the facts at all. The discordance at Z.D. 89° is nearly 5′, or 20 per cent.

TABLE I.

Z.D.	Pulkowa Refraction.	Calculated one Shell.	O—C.	Calculated Two Shells. Inner.	Outer.	Sum.	O—C.
45	55″·7	55″·7	0″·0	9″·5	46″·2	55″·7	0″·0
60	96·2	96·2	0·0	16·3	79·9	96·2	0·0
70	151·9	152·4	− 0·5	26·0	126·0	152·0	− 0·1
80	305·5	311·8	− 6·3	53·5	253·5	307·0	− 1·5
81	337·5	346·0	− 8·5	59·7	280	339·7	− 2·2
82	376·5	388	− 11·5	66	312	378	− 2
83	424·8	442	− 17	77	352	429	− 4
84	486·1	512	− 24	89	402	491	− 5
85	566·2	606	− 40	107	466	573	− 7
86	674·2	738	− 64	133	549	682	− 8
87	825·7	935	− 109	177	654	831	− 5
88	1049·0	1239	− 190	260	792	1052	− 3
89	1397·0	1678	− 281	470	925	1395	+ 2
90	1980·0	1980	0	990	990	1980	0

In column for one shell,

$\alpha = 1^\circ\ 53'\ 15''$ $\quad \log\cos\alpha = 9{\cdot}9997643$

$\beta = 1\ \ 20\ \ 15$ $\quad \log\cos\beta = 9{\cdot}9998817$

In columns for two shells,

$\alpha_1 = 0^\circ\ 41'\ 0''$ $\quad \log\cos\alpha_1 = 9{\cdot}9999691$

$\beta_1 = 0\ \ 24\ \ 30$ $\quad \log\cos\beta_1 = 9{\cdot}9999890$

$\alpha_2 = 2\ \ 48\ \ 50$ $\quad \log\cos\alpha_2 = 9{\cdot}9994760$

$\beta_2 = 2\ \ 32\ \ 20$ $\quad \log\cos\beta_2 = 9{\cdot}9995735$

Two Shells.

18. With two shells we have at first sight two more disposable constants; really only one. One of them is used up in the

assumption that the horizontal refractions of the two shells are equal, as in § 16. Our four constants $\alpha_1, \beta_1, \alpha_2, \beta_2$ are therefore to be determined by the equations

$$\alpha_1 - \beta_1 = \alpha_2 - \beta_2 = 16\tfrac{1}{2}' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$2k = \frac{\alpha_1 + \beta_1}{2}\left[1 - \tfrac{1}{3}(\alpha_1^2 + \beta_1^2)\right] + \frac{\alpha_2 + \beta_2}{2}\left[1 - \tfrac{1}{3}(\alpha_2^2 + \beta_2^2)\right]. \quad (19)$$

and one more condition still to be imposed. Inspection of the discordances for one shell shown in Table I. suggests that we should reduce the largest discordance to zero, *i.e.* should satisfy the observed refraction at 89°. This can only be done by "trial and error" methods. It was found useful to form tables of cos α cos A for values of A = 1°, 2°, 3° . . . 10°; and for every minute of arc of α. By their use, successive approximations gave the values

$$\alpha_1 = 0^\circ\ 41'\ 0'' \qquad \alpha_2 = 2^\circ\ 48'\ 30''$$
$$\beta_1 = 0\ \ 24\ \ 30 \qquad\ \ \beta_2 = 2\ \ 32\ \ 0$$

as a close approximation; the horizontal refraction of 33′ being divided into two of 16½′.

19. We must now examine how far these values satisfy the equation (19). They are close enough for substitution in the square terms. The following small table facilitates this substitution: the unit is ·00001.

TABLE II.

Value of ½α^2.

	10′.	20′.	30′.	40′.	50′.	60′.
0°	1	1	2	3	6	10
1°	14	18	23	28	34	41
2°	48	56	64	73	82	91
3°	101	112	124	136	149	162
4°	176	190	205	221	237	254
5°	271	289	307	326	346	366

Equation (19) runs

$$3^\circ\ 13'\ 25'' = 32'\ 45''\ (1 - \cdot00005) + 2^\circ\ 40'\ 15''\ (1 - \cdot00146)$$
$$= 3^\circ\ 13'\ 0'' - 0''\cdot1 - 14''\cdot0.$$

Hence our right-hand side is defective by 39″. Throwing this on to the second shell, which is less effective at small altitudes, we take as a trial

$$\alpha_1 = 0^\circ\ 41'\ 0'' \qquad \alpha_2 = 2^\circ\ 48'\ 50''$$
$$\beta_1 = 0\ \ 24\ \ 30 \qquad\ \ \beta_2 = 2\ \ 32\ \ 20$$

With these values of the constants the refractions in columns 5 and 6 of Table I. were calculated; and it will be seen from column 8 that the error of their sum is small.

20. The question left outstanding is the choice of slightly different constants which shall perhaps satisfy the observations at (say) Z.D. 80°–87° rather better, at the expense of Z.D.'s 88°–90°, where observations are rare and possibly affected by errors of several seconds. In Table III. are given the effects of altering α_1 and β_1 by 10′, and the similar effects of altering α_2 and β_2 by 10′. It will be seen that we can, by increasing the constants for one layer, and diminishing those for the other by an exactly equal amount, leave the moderate Z.D.'s almost unaffected, and at the same time introduce sensible modifications into the large Z.D.'s.

TABLE III.

Change for 10′ Alteration of Constants.

Z.D.	Inner Shell.	Outer Shell.	Difference.	Error from Table I.
45	(″3)	(″3)	″0·015	(″0·0)
60	(5)	(5)	0·065	(0·0)
70	(8)	(8)	0·25	(0·0)
80	(16)	(15)	2·2	−1·5
81	(17)	(16)	2·9	−2
82	20	18	2	−2
83	23	20	3	−2
84	26	22	4	−5
85	32	23	9	−7
86	40	24	16	−8
87	52	22	30	−5
88	74	18	56	−3
89	100	7	93	+2
90	0	0	0	0

21. The first five differences in the fourth column of Table III. were formed by a special method, as that used for the later ones is unsuitable. We may for moderate Z.D.'s simply differentiate the equation

$$\cos\lambda = \cos\alpha\cos A.$$

Thus

$$\frac{d\lambda}{d\alpha} = \frac{\sin\alpha\cos A}{(1-\cos^2\alpha\cos^2 A)^{\frac{1}{2}}}$$

$$= \frac{\sin\alpha}{(\tan^2 A + \sin^2\alpha)^{\frac{1}{2}}}.$$

The refraction due to a thin shell between α and $\alpha+\delta\alpha$ at altitude A

is $\frac{d\lambda}{da}.\delta a$. The change in this for a change Δa in a is $\frac{d^2\lambda}{da^2}.\delta a.\Delta a$: where

$$\frac{d^2\lambda}{da^2} = \frac{\tan^2 A \cos a}{(\sin^2 a + \tan^2 A)^{\frac{3}{2}}};$$

and when a is small compared with A

$$\frac{d^2\lambda}{da^2} = \cot A\,[1 - \tfrac{1}{2}(1 + 3\cot^2 A)\sin^2 a].$$

If we have two shells with characteristics a_1 and a_2, the difference in effect will be proportional to

$$\tfrac{1}{2}\cot A\,(1 + 3\cot^2 A)\,(\sin^2 a_2 - \sin^2 a_1).$$

The value of the coefficient $\frac{1}{2}\cot A + \frac{3}{2}\cot^3 A$ is given in the following small table.

TABLE IV.

Value of $C = (\frac{1}{2}\cot A + \frac{3}{2}\cot^3 A)$ and $D = 0''{\cdot}00756\,C$.

Z.D.	Alt.	C.	D.
°	°		″
45	45	2·00	0·015
60	30	8·6	0·065
70	20	32·6	0·25
80	10	286	2·2
81	9	380	2·9
82	8	545	4·1
83	7	813	6·1
84	6	1295	9·8
85	5	2240	16·9

The factor for the coefficient C in our special case is

$$10' \times \text{arc of } 16\tfrac{1}{2}' \times (\sin^2 2^\circ 40' - \sin^2 0^\circ 33') = 0''{\cdot}00756.$$

The values of D for Z.D. 82°–85° are larger than those found directly in Table III., probably because the approximation used in Table IV. is ceasing to be accurate; but the quantities are of the same order of magnitude.

22. Table III. makes it clear that we cannot sensibly improve the accordance at 85° without introducing a much larger error at 89°. Every 1″ of alteration at 85° gives 10″ at 89°. Hence we may accept the constants used in Table I. as giving as good a representation of the Pulkowa refractions as we can fairly get with two shells of equal horizontal refraction.

Three Shells.

23. We divide the horizontal refraction equally into three of 11′. It is an obvious convenience to have whole minutes of arc, and for this reason the horizontal refraction was taken as exactly 33′ in the first instance. We might have taken 36′, which divides by 2, 3, and 4; but some work had already been done with 33′ before it was found that two shells would give such a good approximation.

After some few trials, the following values were found to be near the truth :—

$\alpha_1 = 0^\circ\ 29'\ 0''$	$\alpha_2 = 1^\circ\ 24'\ 0''$	$\alpha_3 = 3^\circ\ 14'\ 0''$
$\beta_1 = 0\ 18\ 0$	$\beta_2 = 1\ 13\ 0$	$\beta_3 = 3\ 3\ 0$
$\frac{1}{2}(\alpha_1+\beta_1) = 0\ 23\ 30$	$\frac{1}{2}(\alpha_2+\beta_2) = 1\ 18\ 30$	$\frac{1}{2}(\alpha_3+\beta_3) = 3\ 8\ 30$

Hence equation (17) becomes in this case

$$4^\circ\ 50'\ 8'' = 23'\ 30''\ (1 + \cdot00003) + 1^\circ\ 18'\ 30''\ (1 - \cdot00034) + 3^\circ\ 8'\ 30''\ (1 - \cdot00191)$$
$$= 4^\circ\ 50'\ 30'' - 0''\cdot04 - 1''\cdot5 - 21''\cdot6$$

showing that the adjustment has been properly made.

TABLE V.

Three Homogeneous Shells.

Z.D.	Inmost Shell.	Middle Shell.	Outmost Shell.	Sum=C.	O—C.	Difference for 10′. Inmost.	Middle.	Outmost.
45°	4″·47	15″·09	36″·14	55″·70	0″·00	1″·95	1″·85	1″·85
60	7·67	26·16	62·42	96·25	0·00	3·56	3·18	3·18
70	12·21	41·34	98·27	151·82	+0·1	5·30	5·05	5·05
80	25·3	85·0	196·0	306·3	−0·8	11·3	10·2	9·1
81	28·3	94·4	215·8	338·5	−1·0	12·2	11·1	10·2
82	31·7	106·0	239·7	377·4	−0·9	13·9	12·5	10·9
83	36·2	120·7	269·0	425·9	−1·1	16·2	14·4	11·6
84	42·4	140·5	305·5	488·4	−2·3	18·5	16·1	11·8
85	51·0	167·0	350·5	568·5	−2·3	22·3	18·6	12·5
86	63·5	205·5	407·0	676·0	−1·8	27·7	21·8	13·2
87	85·0	264·0	477·5	826·5	−0·8	39·0	28·5	11·5
88	125·5	362·0	556·5	1044·0	+5·0	52·3	28·6	8·2
89	236·5	525·0	629·0	1390·5	+6·5	83·2	19·5	2·3
90	660·0	660·0	660·0	1980·0	0·0	0·0	0·0	0·0

24. The results of this hypothesis are shown in Table V., and from the seventh column, O—C, it will be seen that there is fair accordance with the Pulkowa refractions. This accordance might be improved; by use of the differences given in the last

three columns we could form linear equations to find small corrections to the mean radii of the three shells. But the improvement would be of the nature of a compromise, and would involve—

(*a*) A study of the accuracy of our knowledge of refractions at small altitudes. We should have to settle, for instance, what error at 89° Z.D. can be made to obtain an improvement of 1″ at Z.D. 80°.

(*b*) A more accurate computation of the differences for 10′. Those given above were found in the way made clear by the following example:—

		1st diff.	2nd diff.
log cos 18′ log cos 10° = 9·9933455 = log cos (10° + 16′·0)			
		25″·3	
log cos 29′ log cos 10° = 9·9933361 = log cos (10° + 41′·3)			12″·4
		37″·7	
log cos 40′ log cos 10° = 9·9933221 = log cos (10° + 79′·0)			

The first two lines are formed to find the refraction due to the inmost layer (with constants 18′ and 29′) for altitude 10°, the result being 25″·3 as given in the 2nd column. By adding the 3rd line we find that the refraction of a shell with constants 29′ and 40′ (each 11′ greater than the former) is 37″·7; so that the difference for 11′ is 12″·4; and for 10′ would be 11″·3, which is given in the 7th column. But in forming differences of differences slight inaccuracies are multiplied, and inspection of the figures near the top of the last three columns shows that (since the numbers are nearly equal for all three shells) we are really concerned with differences of these again.

For the present, no attempt will be made to improve the approximation, and we turn to some other points.

25. It is of some interest to deduce the heights and refractive indices of the shells above found empirically.

One Shell.

$$\log(1+z) = \log \sec \alpha = 0{\cdot}0002357 = \log 1{\cdot}000544$$

$$\log \mu = \log \cos \beta - \log \cos \alpha = 0{\cdot}0001174 = \log 1{\cdot}000270.$$

The "height of the homogeneous atmosphere" is thus indicated as $4000 \times {\cdot}000544$ miles, or 2·18 miles, which is a good deal smaller than that assigned by total pressure.

This is not surprising, for in replacing a series of successive bounding surfaces by a single surface, we should expect the equivalent single surface to lie in the midst of the constituents. We can imagine, for instance, a uniform graduation which would bring the equivalent surface to the mean height, *i.e.* would give us a "height of the homogeneous atmosphere" just half that assigned by calculating the total pressure; and it would not be surprising to find that the equivalent surface should be lower than this.

Two Shells.

Equations (4) of § 5 may be put in the form

	Log.	Number.
$\log(1+z_1) = \log\ \mu_1/\mu_1 + \log\sec\alpha_1 = 0{\cdot}0000309$		$1{\cdot}000071$
$\log\mu_1/\mu_2 = \log(1+z_1) + \log\cos\beta_1 = 0{\cdot}0000199$		$1{\cdot}000046$
$\log(1+z_2) = \log\ \mu_1/\mu_2 + \log\sec\alpha_2 = 0{\cdot}0005439$		$1{\cdot}001253$
$\log\mu_1/\mu_3 = \log(1+z_2) + \log\cos\beta_2 = 0{\cdot}0001174$		$1{\cdot}000270$

If we put $\mu_3 = 1$, then $\mu_1 = 1{\cdot}000270$, $\mu_2 = 1{\cdot}000224$. The heights of the bounding surfaces in miles are $z_1 = 0{\cdot}28$, $z_2 = 5{\cdot}00$.

Three Shells.

The method of formation of the quantities has been made clear in the last case.

	Log.	Number.
$\log(1+z_1) = 0 + \log\sec\alpha_1 =$	$0{\cdot}0000154$	$1{\cdot}000035$
For $\log\mu_1/\mu_2$, add $\log\cos\beta_1$,	$0{\cdot}0000094$	$1{\cdot}000022$
„ $\log(1+z_2)$ „ $\log\sec\alpha_2$,	$0{\cdot}0001391$	$1{\cdot}000321$
„ $\log\mu_1/\mu_3$ „ $\log\cos\beta_2$,	$0{\cdot}0000412$	$1{\cdot}000097$
„ $\log(1+z_3)$ „ $\log\sec\alpha_3$,	$0{\cdot}0007331$	$1{\cdot}001689$
„ $\log\mu_1/\mu_4$ „ $\log\cos\beta_3$,	$0{\cdot}0001175$	$1{\cdot}000270$

If we put $\mu_4 = 1$, we have $\mu_1 = 1{\cdot}000270$, as in equation (9). This gives a check on the work.

Further, $\mu_2 = 1{\cdot}000248$, $\mu_3 = 1{\cdot}000173$. The heights of the bounding surfaces are, in miles,

$$z_1 = 0{\cdot}14, \quad z_2 = 1{\cdot}28, \quad z_3 = 6{\cdot}76.$$

It does not seem likely that these figures can have any physical interpretation. They merely emphasise the fact that a serious part of the refraction takes place near the Earth, and not high in the atmosphere. By comparing the columns for the separate shells shown in Tables I. and V. it will be seen how rapid is the increase in importance of the inner shells as we approach the horizon. In Table V., for instance, the two inner shells together only give half the effect of the outmost at moderate Z.D.'s; and they do not produce so great an effect as the outmost until we reach Z.D. 88°, when they surpass and ultimately double it in the remaining 2° or 3°. This suggests that any influence of meteorological phenomena on astronomical refractions must be sought at large Z.D.'s, which is in accordance with experience. But discussion of such points in detail cannot be undertaken at present.

Summary.

§§ 1–2. Introductory.

§§ 3–6. A graphical representation of the refraction of a homogeneous spherical shell of atmosphere and of several such shells.

§§ 7–8. Condition that two or more such shells should be replaceable by an equivalent single shell.

§§ 10–15. Empirical determination of constants for a single shell from observed refractions at Z.D. 45° and at the horizon.

§ 16. Extension of formulæ to two or more shells.

§ 17. Numerical results for one shell. The errors at 80°, 85°, and 89° are respectively −6″, −40″, −281″. The hypothesis cannot be said to fit the facts at all beyond Z.D. 60° or 70° at most. See Table I.

§§ 18–22. Numerical results for two shells. See Table I. The errors at 80°, 85°, and 89° are reduced to −1″·5, −7″, and +2″—a very fair accordance for so obviously rough an assumption.

§§ 23–24. Numerical results for three shells: see Table V. The improvement is not great, much less marked than before; and it seems clear that the principal step was taken in passing from one shell to two.

§ 25. Some numerical values for heights, etc.

The Perturbations of Halley's Comet in the Past. Fourth Paper. The period 760 to 1066. By P. H. Cowell, M.A., F.R.S., and A. C. D. Crommelin, B.A.

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In the last paper, *M.N.*, lxviii. 5, p. 378, we found March 27 for the approximate date of perihelion passage in 1066, and 44″·686 for the value of n at that date. For the preceding passage we used the date given by Hind (989 September 12) for the purpose of computing the perturbations, and the following results indicate that this date is correct within a few days:—

Revolution 989–1066.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0°– 30°	+ ″·019	... ″	+ 532 ″
,,	330–360	− ·0035	...	...
Earth	0– 30	+ ·007	...	+ 196
,,	330–360	− ·013	...	...
Jupiter	0– 90	+ ·4535	−531	+12807
,,	90–270	− ·2780	−203	− 2905
,,	270–360	−1·2940	+708	+ 22
Saturn	0– 90	− ·0560	− 23	− 1578
,,	90–270	+ ·0754	− 6	+ 1979
,,	270–360	− ·2334	+ 7	+ 47
Uranus	0–360	+ ·0030	...	+ 55
	Sums	−1·3200	− 48	+11155

For this and earlier revolutions we neglect the Neptune perturbations as trifling, and take those for Venus, Earth, and Uranus from curves constructed from the results already calculated; in the case of Venus and the Earth these curves are for the combined results of the mechanical quadratures and the definite integral. Needless to say, we do not claim that the results from these curves are absolutely accurate, but such accuracy is uncalled for, since our calculated Jupiter and Saturn results are liable to sensible errors through the uncertainty attaching to our assumed position of the comet's orbit plane, etc. The resulting value of n at 989 is $44''{\cdot}686 + 1''{\cdot}320 = 46''{\cdot}006$. And calculated period in days $= \frac{1296000 - 11155}{46{\cdot}006} = 27928$. Now 1066 March 27 = J.D. 2110500; hence the calculated J.D. of previous passage is 2082572 = 989 October 9. This is 27 days later than Hind's date, a quantity sufficiently small to confirm his identification. We can reduce the discordance slightly by altering our assumed dates of perihelion passage, since the observations are not precise enough to fix the exact day. If we take them as 989 September 15 and 1066 March 25, we obtain from the revolution 989–1066 the values of n, 989 $45''{\cdot}969$, 1066 $44''{\cdot}649$; from the following revolution we obtain 1066 $44''{\cdot}688$, 1145 $44''{\cdot}920$.

The mean value for 1066 is $44''{\cdot}668$, to which corresponds $45''{\cdot}988$ in 989.

Proceeding to the revolution 912–989 we first took Hind's date, 912 April 1, and we give the perturbations deduced from this assumption:—

Revolution 912–989.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0– 30°	+″·021	...	+ 588″
,,	330–360	− ·016	...	...
Earth	0– 30	− ·0075	...	− 210
,,	330–360	+ ·0005	...	...
Jupiter	0– 90	− ·1905	+138	−5503
,,	90–270	+ ·2064	− 73	+7936
,,	270–360	+ ·0966	−598	+ 11
Saturn	0– 90	+ ·2251	− 19	+6402
,,	90–270	− ·0625	+ 15	+ 139
,,	270–360	+ ·0529	− 30	− 13
Uranus	0–360	+ ·0102	...	+ 148
	Sums	+ ·3362	−567	+9498

Taking $45''{\cdot}988$ as the value of n in 989, that in 912 is $45''{\cdot}652$, and calculated period in days $= \frac{1296000 - 9498}{45{\cdot}652} = 28181$. This gives J.D. 2054367 for the passage in 912 = 912 July 20.

This is nearly 4 months later than Hind's date, a larger quantity than is likely to arise from error in our calculation, and it is to be noticed that Hind's identification is antecedently very doubtful, being based only on observations on May 13, 15; we conclude that it is erroneous. There are vague references to other comets in 912, but nothing sufficiently precise to serve for identification. This is the first passage, reckoning back from the present time, that cannot be certainly identified with an observed comet.

Taking the calculated date 912 July 20, we obtain the following modified perturbations, 912 to 989. The value of $\int d\varpi$ may be taken the same as before.

Revolution 912–989 *revised.*

Planet.	Limits of u.	$\int dn$.	$\int d\zeta$.
Venus	0°– 30°	+″·0086	+ 240″
,,	330–360	– ·016	...
Earth	0– 30	+ ·003	+ 84
,,	330–360	+ ·0005	...
Jupiter	0– 90	– ·2445	–7026
,,	90–270	+ ·2284	+8576
,,	270–360	+ ·0966	+ 11
Saturn	0– 90	+ ·2321	+6602
,,	90–270	– ·0655	+ 69
,,	270–360	+ ·0529	– 13
Uranus	0–360	+ ·0102	+ 148
	Sums	+ ·306	+8691

This gives for the calculated period $\frac{1296000'' - 8691''}{45''\cdot682} = 28180$ days, practically the same as before.

Hence we take the date 912 July 19 as approximately correct for the perihelion passage, this result being from calculation alone, not from observation. As a check on its accuracy, we proceed to the revolution 837–912; a preliminary calculation indicates 837 February 28 for the preceding passage. It will be remembered that there is some question whether the numerous accounts of the brilliant object (or objects) that appeared in the spring of that year relate to one or to two comets. The date of perihelion passage of the earlier one, as investigated by Pingré, is indicated with tolerable precision as 837 March 1, the agreement with our calculated date being thus perfect. The other elements are also accordant except the position of the node, and it is well known that the indications of latitude in the Chinese accounts are somewhat vague, so that this is not a serious objection to the identification, especially as the comet was near the ecliptic throughout the

observations, so that a small error in the observed latitude would make a large error in the node. Hind took the second comet (if there were really two) as Halley's, finding 837 April 6 for the perihelion passage, but we think that the date March 1 is more likely to be correct.

Revolution 837–912.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$	$\int d\zeta$.
Venus	0° 30°	+ ″·020	...″	+ 550″
,,	330–360	+ ·0015	...	...
Earth	0– 30	+ ·023	...	+ 630
,,	330–360	− ·003	...	...
Jupiter	0– 90	+ ·975	− 84	+26892
,,	90–270	− ·226	−493	− 3554
,,	270–360	−1·283	+707	+ 48
Saturn	0– 90	− ·038	+ 50	− 1040
,,	90–270	+ ·044	−132	+ 633
,,	270–360	− ·018	+ 10	+ 8
Uranus	0–360	+ ·0105	...	+ 258
	Sums	− ·494	+ 58	+24425

Taking n in 912 as 45″·682 as found above, n in 837 = 46″·176, and period in days $= \frac{1296000 - 24425}{46{\cdot}176} = 27538$, which brings us to J.D. 2026828 = 837 Feb. 25. We adopt this result as the date of perihelion passage in 837.

For the previous passage we have a fairly firm standpoint, for the comet of 760 was identified with Halley's by Laugier, from the orbit alone, before the intermediate passages had been determined, his date of perihelion passage being 760 June 11 = J.D. 1998810.

Revolution 760–837.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0° 30°	+ ″·001	...″	+ 27″
,,	330–360	− ·015	...	...
Earth	0– 30	− ·006	...	− 165
,,	330–360	− ·013	...	...
Jupiter	0– 90	− ·250	−254	−7164
,,	90–270	+ ·017	+191	+7484
,,	270–360	+ ·2755	−274	− 45
Saturn	0– 90	+ ·071	− 90	+1995
,,	90–270	− ·011	+ 82	+1529
,,	270–360	− ·029	− 89	+ 1
Uranus	0–360	+ ·0145	...	+ 360
	Sums	+ ·055	−434	+4030

n 760 = 46″·176 − ″·055 = 46″·121.

Period in days $= \frac{1296000 - 4030}{46{\cdot}121} = 28013$, which brings us to J.D. $1998815 = 760$ June 15, which is 5 days later than Laugier's date.

This confirms the identity both of the comet of 760 and that of 837 with Halley's comet, and shows that we were right in taking the earlier date in 837 for the perihelion passage.

Now that the identification of the comet has been carried back to 760, it is of interest to quote the words of M. Laugier in the paper (*C.R.*, xv. p. 188) in which he identified the apparitions of 451 and 760 from the observations alone, and noticed that the average periodic time from 451 to 1378 was decidedly longer than in the subsequent seven returns:—"Le calcul des perturbations, s'il était faisable pendant cet énorme intervalle de temps, donnerait peut-être l'explication d'une variation aussi grande du temps periodique; mais, dans le cas où il ferait défaut, on pourrait se rappeler qu'une diminution analogue, quoique plus petite, a été observée dans les retours de la comète à courte periode."

He goes on to speak of the retardation of Encke's comet, and of the suggestions that were put forward to account for this, viz. resisting medium, and Bessel's suggestion of the effect of loss of matter from the comet when passing perihelion, to which we may add the more recent theory of the effect of light-pressure on finely divided matter.

The calculation of the perturbations to which M. Laugier refers has now been carried out, not with absolute rigour, but sufficiently to reconcile theory and observation within a few days in each revolution; and we have found that the large changes in the periodic time are explicable on gravitational grounds alone. Our work does not justify us in asserting that the effect of the various non-gravitational causes referred to above is absolutely insensible; but it suffices to show that their combined effect does not amount to more than a week per revolution, and it is probably still less than this.

Solar Prominences in 1907, *observed at the Kodaikánal Observatory.*
By John Evershed.

The year has been one of considerable activity as regards prominences, notwithstanding a noticeable reduction in the mean profile area, which amounts to about 10 per cent. for all classes of prominences.

At the Kodaikánal Observatory 78 prominences of 100″ or upwards have been recorded photographically and visually during 305 days of observation. The photographic records show also that large eruptive prominences have not been infrequent, seven of this class having been recorded; the greatest elevation measured was 6½ minutes of arc in a transient eruption, photographed on March 14 in solar latitude +52. A remarkable eruption was also photographed by Fox at the Yerkes Observatory on May 21 in solar latitude −68.

The general activity of the two hemispheres of the Sun compared with the previous year may be inferred from the following figures, deduced from the Kodaikánal results:—

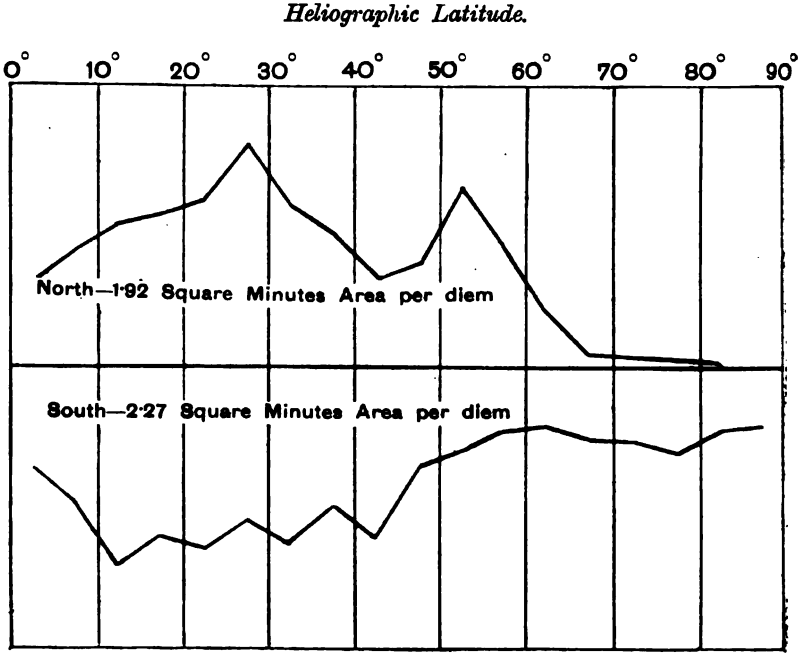
Mean Daily Profile Areas of Prominences.

	1906.		1907.	
North	2·51	square minutes.	1·92	square minutes.
South	2·17	,, ,,	2·27	,, ,,
Total	4·68	,, ,,	4·19	,, ,,

It is seen from the above that the general reduction of activity in 1907 is confined to the northern hemisphere, the southern showing a slight increase. In the latitude distribution a remarkable difference is shown between the two hemispheres, which are usually more or less symmetrical as regards the latitudes of the zones of maxima and minima. From the beginning of the year the northern polar prominences, which were strongly represented during 1906, practically ceased to exist, whilst the south polar region still continued active, the whole region between −45° and the south pole producing a very considerable number of large prominences. The region from lat. −10° to −45° has been the most prolific, however, in this hemisphere; but no clearly marked zones of maxima are shown. In the north, on the other hand, two well-defined maxima occur in the zones +25° to 30° and +50° to 55°.

Metallic prominences were of frequent occurrence, 111 having been recorded. Of these, 54 were confined to the northern spot zone, and had a mean latitude of +15·7°; 50 were confined to the southern spot zone, with a mean latitude of −15·6°; the remaining 7 were distributed in longitude in a narrow zone entirely outside the spot regions, the mean latitude being −72°. The only metallic

elements observed in these high-latitude prominences were Na, Mg, and Fe, whilst some of the prominences in spot-latitudes gave in addition the lines of Ba and Ca, together with a considerable number of unidentified lines, probably including Ni, Mn, Cr, and Ti.


Heliographic Latitude.
0° 10° 20° 30° 40° 50° 60° 70° 80° 90°
North—1·92 Square Minutes Area per diem
South—2·27 Square Minutes Area per diem

Distribution Curve of the Prominences for 1907.

The ordinates give the mean daily profile areas for each zone of 5°, obtained from observations on 296 days.

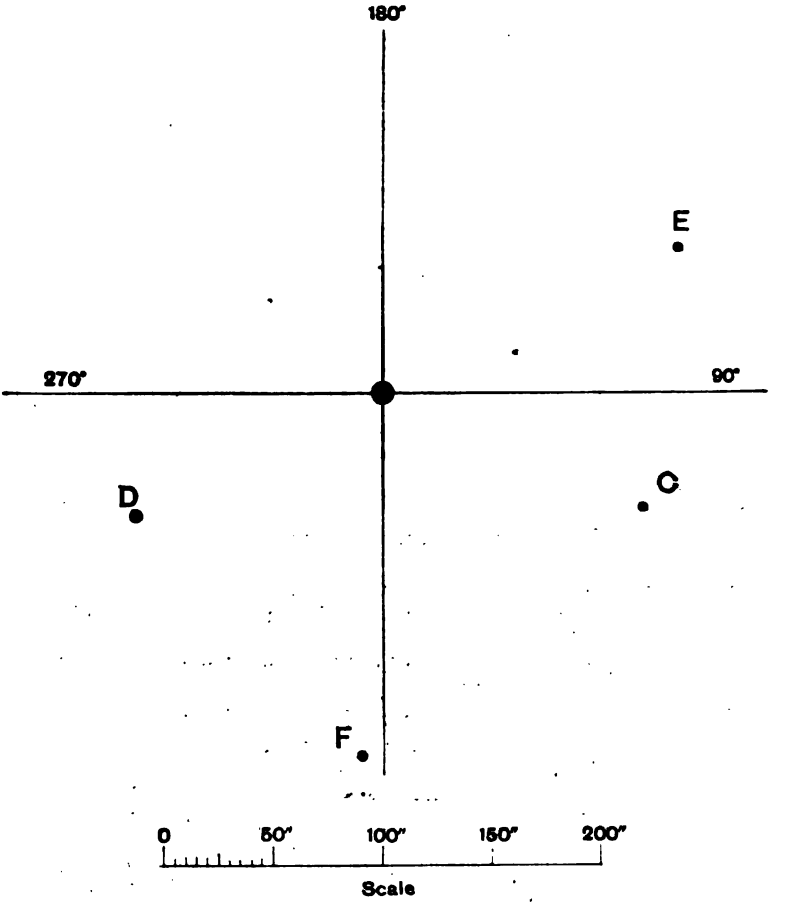
The Proper Motion of Small Stars. By S. W. Burnham.

Small stars having any sensible proper motion which can be detected by meridian or micrometric observations, so far placed on record, are much rarer than parallax stars taken from all magnitudes, bright and otherwise. Very few examples have been detected down to this time, although more or less searched for by all double-star observers and others using the micrometer for determining the relative positions of stars. The small stars referred to may be generally classed as below the limiting magnitude of the Durchmusterung, or from the tenth magnitude to the faintest stars which have been accurately measured by large instruments. Of course it is to be expected that this apparent fixity in space would be found in the great majority of instances; or that the motion would be so small that it could not possibly be separated from the unavoidable errors of observation in the most careful measures by all astronomical instruments. Therefore these small stars furnish by far the best means of determining any change in the position of brighter stars when the distance is within reach of the micrometer, and give the proper motion of the large stars more accurately than any number of positions with the meridian circle from the beginning of such observations, provided the interval of time covered by the micrometer measures is sufficient to practically eliminate the smaller errors of that instrument. So, when the prominent stars have been connected by measures with faint stars in the field by the Struves and other old observers, we have a value of the proper motions of the large stars which is a safe and certain correction to that given by meridian positions.

It is hardly necessary to say in this connection that the faint stars referred to have nothing to do with a large and distinct class of these objects which are moving in space with much brighter stars, and at exactly or nearly the same rate, and in substantially parallel directions. We have a large number of these attending stars, some of them of the smallest magnitudes visible in the largest refractors, and in brightness anywhere between that and the naked-eye star. In many instances the moving stars are separated by several minutes of arc. Whether these constitute binary systems in the ordinary sense of the term, or what common proper motion of stars so remote from each other implies, is, at this time, a matter of speculation only, and must necessarily remain so until careful observations have been made, extending over possibly several hundreds of years. That these stars have something in common seems to be at least highly probable. A large number of systems of this class will be found in my *General Catalogue of Double Stars*, indexed in Part I. under "Common Proper Motion," and "Stars of the 61 Cygni type."

One of the very few examples I have been able to find, and

perhaps the only one worth mentioning, of an isolated small star having a decided proper motion, is a faint star of about the twelfth magnitude in a low-power field with 17 *Lyræ* (Σ 2461). The observations of this star are sufficient in point of time to show that it has a remarkable movement, and one without any parallel in



17 *Lyræ.*

stars of the lower magnitudes, so far as appears from reliable observations of any kind.

The diagram shows the positions of this and other small stars which have been measured from 17 *Lyræ.*

The magnitudes of the three smaller stars have been measured photometrically by Mr. F. C. Jordan of the Yerkes Observatory.

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The magnitude of the large star D, which is DM 32°, 3325, is assumed to be 9·0.

C = 12·00 mag.
E 10·96
F 10·87

My measures of these stars with the 40-inch are:—

A and C.

1905·717	66°·4	127″·37
5·720	66·4	127·73
5·736	66·5	127·66
6·906	66·1	129·53
6·964	66·1	129·46
7·258	66·1	130·01
7·271	66·2	130·14
7·291	66·3	130·38

A and D.

1906·906	295°·5	126″·51
6·964	295·6	126·51
7·258	295·7	126·03
7·271	295·3	126·32
7·291	295·7	126·61

A and E.

1906·906	116°·5	150″·93
6·904	116·6	150·77
7·258	116·9	150·86
7·271	116·7	150·85

A and F.

1906·906	356°·7	167″·54
6·964	356·8	167·47
7·271	357·2	167·71

For the relation of A and C we have the following earlier positions by Engelhardt:—

1887·81	72°·02	102″·28	1n	Eng.
1894·85	69·23	112·31	2n	Eng.
1905·72	66·40	127·59	3n	*β*
1907·14	66·16	129·90	5n	*β*

These positions give for the most probable apparent annual motion of C:—

1″·542 in 45°·9.

The large star, 17 *Lyræ*, has a small proper motion which is given from meridian observations:—

0″·112 in 68°·5	Auwers.
0·121 in 74·8	Newcomb.

A more accurate value of this movement can be found from the measures of the other distant stars which it is certain have no sensible proper motion. These positions are:—

A and D.

1887·73	296°·40	124″·34	2n	Eng.
1894·85	296·00	125·32	2n	Eng.
1905·72	295·70	126·12	1n	β
1907·14	295·56	126·40	5n	β

A and E.

1887·81	116°·00	152″·65	1n	Eng.
1894·85	116·37	152·20	1n	Eng.
1905·72	116·50	150·75	1n	β
1907·10	116·67	150·85	5n	β

A and F.

1887·73	357°·77	168″·20	2n	Eng.
1894·85	357·50	168·11	2n	Eng.
1907·05	356·90	167·57	3n	β

These measures give for the proper motion of 17 *Lyræ*:—

	R.A.	Decl.
AD	+0″·1263	+0″·0342
AE	0·1260	0·0224
AF	0·1307	0·0402
Mean	0·1277	0·0323

Therefore the corrected proper motion is:—

0″·132 in 75°·8.

Applying this to the apparent movement of C, we have for the real proper motion of that star:—

1″·658 in 48°·2.

A photographic plate of this field was taken 1907·444 with the 40-inch. With a Wallace colour screen, and an exposure of about fifteen minutes, the resulting negative was beautifully sharp and well defined, and practically perfect for measuring the relative positions of the several stars.

As direct measurement of the position-angle cannot be made with any instrument here, the measures of this plate were confined to distances. The mean results of six sets of measures by four observers are as follows:—

AC =	130″·58
AD	126·40
AE	150·92
AF	167·78
CE	121·72

A study of the details of these measures seems to show that they have about the same accuracy as those made under the best conditions with the micrometer, and that the uncertainty of the result is probably less, or not more, than 0″·2.

Whether a proper motion star of this magnitude is more likely to have a parallax large enough for certain measurement, which should be at least 0″·3, by either or both methods, than a twelfth magnitude star taken at random, is a matter to be determined hereafter, but it would not be unreasonable to presume that such a star might be nearer our system.

There is one other example of proper motion in a small star, but nearly three magnitudes brighter than the one already referred to, which was discovered by Dr. Max Wolf in his photographic researches (*A.N.*, 4101) by the change shown on plates made in 1892 and 1906. This is a 9·2*m* star, not given in the *D.M.*, the place (1900) of which is—

R.A. = 11^h 23^m 20^s
Decl. = 8° 6′ 1″

There is a similar star, given by Wolf as 9·1*m*, closely preceding; and a 13*m* star more distant in the same quadrant. I have made two sets of measures of these stars as follows:—

A and B. (9·7—9·7.)

1906·906	252°·1	81″·63
7·006	252·0	81·68
7·102	252·1	81·42
7·120	252·2	81·44
1907·03	252·1	81·54
1908·118	253°·2	80″·92
·156	253·1	81·11
·159	253·1	80·68
·192	253·0	80·82
1908·15	253·1	80·88

B and C. (12·0.)

1906·906	275°·9	155″·17
7·102	275·5	154·48
7·120	275·7	154·17
1907·04	275·7	154·61
1908·118	275°·9	154″·90
·159	275·8	154·70
·192	275·7	154·50
1908·15	275·8	154·70

As Dr. Wolf gives no measures of the relations of these stars which can be utilised in this connection, I have measured as carefully as possible their positions on the paper print for comparison with the micrometer positions, with the following results :—

A and B.

1892·23	242°·1	91″·47		Wolf.
1907·03	252·1	81·54	4n	β
1908·15	253·1	80·88	4n	β

B and C.

1892·23	275·9	154″·94		Wolf.
1907·04	275·7	154·61	3n	β
1908·15	275·8	154·70	3n	β

It is evident that the comparison star B has no proper motion, and that the change shown by the measures of AB is due wholly to the proper motion of A. A comparison of the position in 1892 with a mean of the two later measures gives for the proper motion of A :—

1″·225 in 190°·2.

The minimum distance of A and B of 72″ will be reached in about thirty years, after which they will slowly separate.

The 7·5*m* star, DM 8°, 2512, which is 70° following and 185″·7 south, has a 13·2*m* companion not heretofore observed :—

1907·12	285°·0	18″·37
·14	286·3	18·37

I do not find any proper motion assigned to this star.

Yerkes Observatory :
April 10.

New Double Stars. By the Rev. T. E. Espin, M.A.

No.	B.D.	R.A. 1900. h m	Decl.	P.	D.	Mags.		Nights.	Date. 1908.	
	°		° ′	°	″					
552	+56,143	0 47·4	+56 41	85·9	12·60	7·0	13·0	2	·015	
553	+45,405	1 33·5	45 32	144·1	5·15	8·8	9·2	3	·014	
554	+42,549	2 27·4	42 32	24·9	5·80	9·1	9·3	1	·099	
555	+41,501	32·0	41 46	323·9	9·62	8·5	11·5	2	·083	
556	+41,543	42·2	42 0	352·7	6·40	8·4	10·7	2	·073	
557	+47,712	44·3	47 41	315·8	5·04	9·1	12·4	4	·075	
558	+45,710	3 0·0	45 22	357·8	8·47	7·5	9·4	2	·082	
559	...	6·8	43 54	250·6	3·17	9·3	11·2	2	·116	
560	+45,784	26·3	45 55	140·8	8·60	8·4	10·1	3	·091	
561	+45,787	26·6	45 25	38·8	7·40	8·9	9·1	2	·087	
562	+44,769	34·0	44 17	358·4	2·91	9·3	11·7	5	·066	
563	+44,809	47·6	44 35	23·3	6·14	9·0	12·6	4	·082	
564	+42,876	56·7	42 30	115·3	2·29	8·9	9·3	4	·117	
565	+42,890	4 0·0	42 35	62·2	4·75	9·3	10·6	2	·090	
566	+46,881	18·5	46 19	281·3	5·10	9·4	12·0	3	·106	CD
				150·2	11·10	8·9	10·8	2	·097	AB
				148·0	27·15			2	·097	AC
567	+44,945	18·7	45 1	128·9	7·97	8·8	9·4	3	·106	BC
				351·8	36·32	A = 8·8·		2	·097	AB
568	+42,969	20·7	42 58	304·3	5·07	7·7	12·0	2	·083	
569	+44,967	24·5	44 43	19·5	7·72	9·0	12·8	3	·122	
570	+41,898	26·8	41 15	178·7	3·90	9·1	13·0	1	·153	
571	+48,1146	39·9	49 3	59·9	3·35	9·1	9·2	2	·143	
572	+41,966	41·9	41 28	93·1	3·05	8·8	12·0	1	·153	
573	+42,1228	5 9·4	42 33	122·9	5·01	8·0	10·5	3	·040	
574	+47,1122	10·3	47 12	87·7	1±	9·7	10·3	3	·108	BC
				63·1	33·65	A = 9·4		3	·108	AB
575	+48,1264	16·9	48 16	353·6	14·35	8·1	10·7	3	·113	
576	+42,1274	17·2	42 31	342·8	8·45	8·0	13·7	2	·015	AB
				236·3	42·35		8·2	2	·015	AC
577	...	17·3	47 17	133·5	2·30	9·7	11·5	2	·101	
578	+49,1403	40·2	49 22	41·7	2·02	9·1	9·2	2	·131	
579	+47,1249	6 1·0	47 26	115·3	6·57	8·5	11·0	2	·119	BC
				341·3	57·00	A = 8·3		2	·119	AB
580	+44,1380	4·0	44 45	226·3	6·85	9·0	9·4	3	·078	AB
				119·2	22·30	C = 12·0		1	·082	AC
581	+49,1470	7·3	49 0	61·6	3·25	8·8	11·5	2	·121	
582	+44,1492	29·4	44 10	301·9	6·18	9·6	12·3	3	·107	
				79·0	33·92	A = 9·3		3	·107	
583	+44,1527	38·0	44 35	69·4	4·67	9·4	9·6	4	·082	

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.		Nights.	Date.	
	°	h m	° ′	°	″				1908.	
584	+47,1353	43·9	47 21	325·4	3·00	9 4	10·0	2	·045	
585	+45,1430	7 17·7	45 3	237·3	2·72	7·7	11·7	2	·138	
586	+41,1670	23·3	41 49	16·8	13·40	8·1	11·5	2	·189	
587	+46,1307	39·3	46 9	80·7	4·80	8·9	9·2	2	·083	
588	...	40·1	47 33	304·2	2·15	9·5	9·8	3	·078	
589	+48,1576	41·2	48 1	178·3	10·00	7·7	13·7	4	·132	
590	+43,1746	47·8	43 25	69·8	7·40	9·0	9·4	2	·095	
591	+45,1536	8 0·5	45 30	48·2	1·62	9·4	9·6	2	·083	
592	+41,1799	6·3	41 52	327·4	2·72	8·6	9·9	3	·308	
593	+41,1810	10·0	41 12	208·2	4·70	9·4	9·6	2	·235	BC
				230·2	19·82	A = 8·5		2	·235	AB
594	+43,1820	18·6	43 35	183·0	2·48	9·2	12·0	3	·112	
595	+48,1654	24·1	48 6	234·9	7·75	8·5	13·2	2	·082	
596	+46,1436	41·1	45 54	201·3	2·65	8·6	9·0	3	·095	
597	+45,1640	42·8	45 48	262·6	5·33	8·5	11·8	3	·091	
598	+47,1630	55·2	47 45	261·0	7·20	8·6	10·5	2	·119	
599	+41,1915	59·7	41 31	137·3	3·02	9·0	11·8	2	·152	
600	+50,1673	9 35·5	49 49	71·6	3·65	9·0	13·5	2	·152	
601	+46,1549	41·1	46 21	287·5	3·47	9·0	9·2	2	·119	
602	...	47·6	48 36	32·9	2·95	10·4	11·1	3	·261	
603	+48,1887	10 36·7	48 43	98·5	10·57	9·1	11·0	2	·260	
604	+45,1865	40·7	45 43	52·7	1·82	10·6	11·4	2	·249	
	+49,1900	49·2	48 40	158·4	47·00	8·0	8·7	2	·292	
605	+48,1953	11 27·0	48 11	64·5	4·17	8·9	13·7	2	·260	
	+43,2261	12 43·5	42 53	50·1	46·07	7·5	7·7	2	·323	
606	+43,2293	56·8	42 58	284·9	8·53	8·5	12·0	3	·304	
607	+43,2299	13 0·7	43 15	224·9	6·10	9·0	11·5	1	·334	
608	+48,2138	33·0	48 45	271·8	2·57	9·0	9·2	2	·289	
609	+48,2224	14 33·2	48 14	12·3	4·65	9·0	10·7	2	·289	

Notes.

559.—38^s f, 50″ S, OΣ 51.

574.—Measures of the close pair are unsatisfactory, and I had some doubts about the star being really double. Professor Burnham has, however, kindly looked at it with the 40 in., and confirms its duplicity.

583.—A 14 mag. south.

602.—Found and measured in looking for *h* 2510.

604.—The star is so faint that it is surprising that it is in the B.D.

605.—Angle mean of 61°·2, 67°·8. This is a very difficult pair to measure, from the faintness of the *comes*.

608.—In field south of *h* 2667.

Results of Micrometer Measures of Double Stars made with the 28-inch Refractor at the Royal Observatory, Greenwich, in the year 1907.

(*Communicated by the Astronomer Royal.*)

The measures were made with a bifilar position-micrometer on the 28-inch refractor, focal length 28 feet. The power generally employed was 670. When bright stars were observed a blue glass shade was usually employed to diminish the light and irradiation. The initials in the last column are those of the observers, viz.—

L.	Mr. Lewis.	W.B.	Mr. Bowyer.
B.	Mr. Bryant.	H.F.	Mr. Furner.

The stars have been observed from a working catalogue containing all the G. W. Hough stars within the working zone, and a number of miscellaneous stars showing motion, or for which observations are required.

Some of these being wide pairs of no immediate interest, the present list of measures is confined to stars of which the separation is under 4″, or which show orbital motion.

Stars observed but not included in this List.

Struve Stars.

Σ 183 AC	Σ 1158 AB	Σ 1384	Σ 1541	Σ 2185 AB	Σ 2524
1000	1158 AC	1426 AC	1555 AC	2185 AC	2690 AB
1100	1158 AD	1468	1585	2220	2704 AD
	1158 AE	1481 AB	1612	2434	2709
	1196 AC	1481 AC	2140	2458	2725
	1196 BC	1507			

G. W. Hough Stars.

Ho. 494	Ho. 337 AB	Ho. 387	Ho. 423	Ho. 274	Ho. 597
495	21	547	269	115	604
8	28	406 AB	433	116	475 AC
11	29	552	434	586	476
316	523 AB	553	91	117	478
219	348	409	446	588 AB	191 AC
501	349	557	445	588 AC	302 BC
325	38	416	448	593	204
327	259	419	450	131	
508	543	561	451	145	

Royal Observatory, Greenwich:
1908 *May* 5.

Micrometric Observations of Double Stars at the Royal Observatory, Greenwich.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.		Epoch 1907.	Obs.
Hu. 405 ...	0 12	65 57	275·5	1·18	4	9·3	9·5	·902	W.B.
β 1093 ...	0 16	79 31	69·5	0·33	1	7·3	8·2	·827	B.
β 779 ...	0 23	66 55	257·3	1·04	1	8·5	9·0	·827	B.
Ho. 212 ...	0 31	94 5	245·0	0·31	1	6·0	6·0	·827	B.
Hu. 411 ...	0 33	67 30	102·7	0·65	1	8·5	8·5	·925	W.B.
Hu. 413 ...	0 42	67 15	241·7	0·80	1	8·0	9·2	·925	W.B.
Ho. 306 ...	0 43	64 56	163·7	1·19	1	8·5	8·8	·044	W.B.
Ho. 4 ...	0 46	56 33	200·4	1·57	3	9·0	9·0	·592	W.B.
Σ 73 ...	0 50	66 52	28·2	1·13	1	6·2	6·8	·827	B.
			29·9	0·85	2	...		·864	W.B.
Ho. 307 ...	0 52	58 17	86·5	2·76	1	9·5	9·7	·044	W.B.
Σ 113 ...	1 15	91 0	355·2	1·33	1	6·2	7·2	·827	B.
Ho. 9 ...	1 24	68 45	94·8	2·44	3	9·0	10·0	·592	W.B.
Σ 138 ...	1 31	82 50	33·1	1·62	1	7·3	7·3	·827	B.
Σ 158 ...	1 42	57 18	257·7	1·90	2	8·3	8·8	·864	W.B.
Σ 183 AB ...	1 50	61 39	1·6	0·45	1	7·5	8·2	·827	B.
Hu. 1033 ...	1 50	54 6	234·4	0·85	1	8·5	8·8	·044	W.B.
Σ 186 ...	1 51	88 37	35·1	0·73	1	7·2	7·2	·827	B.
Σ 208 ...	1 59	64 31	91·2	0·48	1	6·2	8·4	·058	W.B.
			78·6	0·58	1	...		·162	H.F.
Ait. 959 ...	2 13	59 9	360·7	3·84	3	9·0	12·0	·342	W.B.
Ait. 962 ...	2 16	60 28	76·6	0·53	1	8·9	9·2	·162	H.F.
Σ 305 ...	2 42	71 1	316·3	3·22	1	7·3	8·2	·104	H.F.
			315·4	3·19	3	...		·673	W.B.
Ho. 317 ...	2 52	73 7	304·3	3·46	2	8·1	11·0	·867	W.B.
Ho. 500 ...	3 6	54 15	222·1	0·68	1	8·5	9·0	·162	H.F.
Ho. 502 ...	3 9	54 36	19·5	0·77	1	8·5	9·0	·162	H.F.
Hu. 1058 ...	3 19	50 6	114·7	0·75	2	7·8	8·5	·192	H.F.
Ait. 979 ...	3 20	59 35	270·9	1·48	2	9·2	10·0	·482	W.B.
Ho. 322 ...	3 20	44 43	124·1	1·90	1	9·2	9·5	·222	H.F.
Ait. 983 ...	3 26	60 40	304·1	0·49	1	8·5	9·2	·162	H.F.
Ait 987 ...	3 37	60 30	7·6	1·09	3	9·6	9·7	·347	W.B.
Hu. 103 ...	3 39	40 25	203·1	1·01	1	8·1	8·4	·222	H.F.
Ho. 504 ...	3 39	54 26	188·3	0·98	1	7·8	8·0	·893	W.B.
Hu. 815 ...	3 51	68 29	204·7	2·45	3	8·0	11·3	·324	W.B.
Ho. 505 ...	3 53	57 30	194·9	1·23	1	8·0	10·0	·044	W.B.
Σ 483 ...	3 58	50 43	223·6	0·53	2	8·0	9·5	·192	H.F.
β 1277 ...	4 0	61 51	300·7	0·75	1	8·0	12·2	·227	L.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle	Distance.	No. of Nights	Mags.	Epoch 1907.	Obs.
	h m	° ′	°	″				
β 1005 ...	4 1	61 18	64·2	2·40	1	8·5 11·7	·189	L.
β 1232 ...	4 3	61 3	188·2	0·43	2	8·4 9·3	·208	L.
Espin 278 ...	4 3	50 3	172·4	2·34	2	7·7 · 9·2	·192	H.F.
Ho. 326 ...	4 4	61 35	165·7	0·49	2	8·0 8·0	·208	L.
0Σ 77 ...	4 10	58 32	197·8	0·28	1	7·5 7·5	·227	L.
Ho. 15 ...	4 19	60 5	150·8	0·76	2	8·0 8·0	·492	W.B.
			151·9	0·70	1	...	·162	H.F.
β 1044 ...	4 35	73 39	219·8	0·66	1	9·0 · 11·0	·189	L.
Ho. 333 ...	4 38	69 55	164·9	2·75	1	9·2 9·3	·104	H.F.
Ho. 224 AB	4 59	61 24	281·5	1·70	1	9·0 10·7	·159	W.B.
Ait. 1024 ...	4 59	60 30	349·5	0·80	1	8·1 8·9	·227	L.
			355·9	0·56	2	...	·932	W.B.
Ait. 1026 ...	4 59	60 9	38·4	1·06	2	7·8 11·0	·492	W.B.
0Σ 95 ...	5 0	70 19	327·2	0·84	1	6·6 7·2	·189	L.
Ait. 1028 ...	5 1	60 3	238·6	0·45	1	8·5 9·1	·939	W.B.
Hu. 217 ...	5 24	54 42	259·0	0·61	2	7·0 8·5	·192	H.F.
Σ 749 ...	5 32	63 8	170·0	0·78	1	7·0 7·1	·175	B.
			172·7	0·97	1	...	·925	W.B.
Ait. 1040 ...	5 39	58 44	130·8	0·74	1	8·2 9·3	·222	H.F.
Ho. 337 BC	5 50	66 44	110·5	1·23	1	9·0 9·2	·159	W.B.
Ho. 23 ...	6 9	75 31	243·9	2·74	1	8·2 12·0	·159	W.B.
Ho. 239 ...	6 45	75 12	130·7	0·51	3	8·3 8·3	·177	W.B.
Ait. 1061 ...	6 54	64 49	321·3	0·40	2	8·2 8·5	·151	W.B.
Espin 289	6 57	50 54	101·8	2·04	1	9·4 9·7	·230	W.B.
Ho. 342 ...	6 58	76 46	83·7	1·39	1	8·0 8·8	·159	W.B.
Σ 1037 ...	7 7	62 37	299·2	0·52	1	7·1 7·1	·241	W.B.
Ho. 33 ...	7 15	67 40	326·4	2·95	1	9·0 12·0	·236	W.B.
Ho. 36 ...	7 42	64 19	120·6	0·57	1	8·5 8·5	·159	W.B.
Ho. 250 ...	7 52	68 48	153·1	0·54	3	7·0 9·0	·210	W.B.
Σ 1187 ...	8 4	57 31	41·9	2·23	2	7·1 8·0	·243	W.B.
Σ 1196 AB	8 7	72 4	340·3	1·21	2	5·0 5·7	·151	W.B.
			341·7	1·19	1	...	·222	H.F.
Hu. 1123 ...	8 9	53 14	155·7	0·36	2	8·5 8·8	·236	W.B.
Hu. 626 ...	8 14	57 23	153·6	3·39	1	8·0 12·0	·356	W.B.
Ho. 525 ...	8 18	69 42	150·7	0·55	3	8·5 8·5	·242	W.B.
Hu. 627 ...	8 25	55 10	264·3	1·01	1	9·0 10·0	·356	W.B.
Σ 1273 AC	8 42	83 12	241·8	3·07	1	3·8 7·8	·260	H.F.
Espin 294 ...	8 43	53 31	160·2	1·70	2	9·0 9·2	·276	W.B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Hu. 861 ...	8 53	75 25	20·9	0·28	1	8·5	9·1	·241	W.B.
Hu. 718 ...	8 55	57 13	192·0	0·41	2	8·7	8·9	·299	W.B.
Σ 3121 ...	9 13	61 3	35·0	0·43	3	7·5	7·8	·242	W.B.
Ho. 42 ...	9 9	56 4	7·9	1·62	1	9·5	9·5	·255	W.B.
Σ 1333 ...	9 13	54 15	47·4	1·60	1	6·6	6·9	·323	L.
Σ 1334 ...	9 13	52 48	239·1	2·91	1	4·0	6·7	·323	L.
Ho. 43 ...	9 14	68 48	298·0	0·31	1	8·0	8·5	·230	W.B.
Ho. 364 ...	9 14	66 42	332·9	3·87	1	8·2	11·2	·219	W.B.
Σ 1348 ...	9 20	83 12	318·5	1·81	2	7·5	7·6	·225	W.B.
Σ 1355 ...	9 22	83 19	329·2	2·24	2	7·2	7·2	·282	H.F.
Σ 1372 ...	9 32	73 19	56·4	0·69	2	8·2	8·3	·282	H.F.
Σ 1374 ...	9 35	50 35	286·6	3·27	1	7·0	8·3	·296	W.B.
			288·3	2·95	1	...		·334	H.F.
Ho. 369 ...	9 46	53 5	275·5	0·37	1	7·7	7·8	·323	L.
Σ 1389 ...	9 47	62 32	306·4	1·99	2	8·0	9·0	·277	W.B.
			308·2	2·65	1	...		·260	H.F.
			307·7	2·19	1	...		·323	L.
Σ 1406 ...	10 0	58 26	228·3	0·98	1	8·0	8·7	·334	H.F.
Σ 1413 ...	10 7	73 9	96·0	2·18	2	8·9	8·9	·282	H.F.
			97·6	1·90	1	...		·296	W.B.
Σ 1420 ...	10 13	50 24	327·5	2·24	1	8·2	9·9	·296	W.B.
Hu. 875 ...	10 13	52 2	64·1	1·05	1	7·0	9·8	·356	W.B.
Σ 1423 ...	10 14	68 56	57·7	1·23	1	8·6	9·3	·304	H.F.
Σ 1426 AB	10 16	83 4	283·0	0·72	2	7·5	8·0	·282	H.F.
0Σ 218 ...	10 23	85 56	79·1	0·78	2	8·2	9·7	·282	H.F.
Σ 1439 ...	10 25	68 44	113·0	1·94	4	8·0	8·5	·238	W.B.
			114·0	1·83	1	...		·260	H.F.
Σ 1457 ...	10 34	83 46	314·8	1·28	2	7·6	8·5	·282	H.F.
Σ 1476 ...	10 44	93 30	181·7	2·18	2	7·2	8·0	·319	H.F.
Σ 1500 ...	10 55	92 56	308·1	1·24	2	7·6	8·2	·319	H.F.
Σ 1517 ...	11 9	69 19	87·1	0·34	2	7·3	7·3	·301	B.
			84·0	0·45	2	...		·370	W.B.
Σ 1523 ...	11 13	57 54	131·6	2·86	1	4·0	5·0	·296	W.B.
			131·6	2·68	1	...		·334	H.F.
Σ 1525 ...	11 14	41 58	176·8	1·64	1	9·0	9·0	·392	H.F.
Σ 1536 ...	11 19	78 58	49·2	2·21	2	3·9	7·1	·301	B.
Σ 1537 ...	11 20	68 52	355·3	2·80	1	7·6	8·6	·219	W.B.
Σ 1555 AB	11 31	61 40	353·1	0·39	2	6·4	6·8	·301	B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Hu. 888 ...	11 38	68 25	320·1	0·69	3	8·4	8·9	·301	W.B.
Σ 1581 ...	11 51	43 53	172·0	2·04	1	8·3	9·5	·392	H.F.
Σ 1589 ...	11 55	45 50	161·3	1·94	1	9·0	9·5	·392	H.F.
Σ 1606 ...	12 6	49 33	328·7	1·06	1	6·3	7·0	·392	H.F.
Σ 1639 ...	12 20	63 55	351·3	0·36	2	6·7	7·9	·301	B.
Σ 1643 ...	12 23	62 28	34·7	2·15	1	8·7	9·2	·348	W.B.
			37·6	2·20	1	...		·438	L.
Σ 1647 ...	12 26	79 47	224·4	1·46	3	7·5	7·8	·308	W.B.
Hu. 892 ...	12 38	74 59	163·8	1·01	2	9·3	9·3	·301	B.
Hu. 893 ...	12 43	77 13	33·9	1·55	2	9·1	9·1	·301	B.
Hu. 894 ...	12 49	76 20	144·4	1·02	2	9·1	9·1	·301	B.
OΣ 256 ...	12 52	90 24	76·6	0·60	2	7·2	7·6	·301	B.
OΣ 261 ...	13 8	57 26	346·9	1·66	2	6·9	7·4	·301	B.
OΣ 269 ...	13 29	54 38	215·6	0·35	2	6·5	7·0	·319	B.
OΣ 278 ...	14 9	45 24	82·5	0·24	3	7·5	7·7	·514	B.
Ho. 541 ...	14 16	77 25	96·6	2·16	2	9·3	10·2	·388	W.B.
Σ 1834 ...	14 17	41 5	343·2	0·14	2	7·1	7·2	·319	B.
Ho. 542 ..	14 23	68 59	264·9	0·58	2	8·8	8·8	·301	B.
Σ 1865 ...	14 37	75 53	145·9	0·43	4	3·5	3·9	·339	B.
OΣ 285 ...	14 42	47 14	110·6	0·32	4	7·1	7·6	·463	B.
OΣ 287 ...	14 48	44 42	321·7	0·79	4	7·5	7·6	·463	B.
Ho. 389 ...	14 48	69 20	98·3	1·46	1	7·0	9·3	·427	W.B.
OΣ 288 ...	14 49	73 56	188·0	1·79	2	6·4	7·1	·340	W.B.
			185·9	1·61	1	...		·400	L.
Σ 1932 ...	15 14	62 50	164·5	0·72	1	5·6	6·1	·400	L.
Σ 1944 ...	15 23	83 36	324·0	1·25	1	7·5	8·1	·427	W.B.
Σ 1954 ...	15 31	79 9	185·5	3·68	2	3·2	4·1	·384	W.B.
OΣ 298 ...	15 33	49 54	191·5	1·11	2	7·0	7·3	·607	B.
Σ 1967 ...	15 39	63 25	113·3	0·63	1	4·0	7·0	·400	L.
OΣ 303 ...	15 57	76 28	145·2	0·84	1	7·4	7·9	·427	W.B.
Σ 2021 ...	16 9	76 14	336·6	3·84	2	6·7	6·9	·384	W.B
Σ 2026 ...	16 12	82 24	187·9	0·50	1	8·6	9·1	·496	L.
			189·4	0·49	1	...		·498	W.B.
OΣ 309 ...	16 16	48 7	64·3	0·42	3	7·5	7·8	·502	B.
Σ 2049 ...	16 24	63 49	204·0	1·17	1	6·5	7·5	·400	L.
Hu. 483 ...	16 24	68 54	295·0	1·59	2	8·2	13·0	·529	W.B.
Σ 2052 ...	16 25	71 24	271·0	1·14	1	7·5	7·5	496	L.
			269·8	1·27	2	...		·505	W.B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle.	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Σ 2055 ...	16 26	87 49	62·4	1·27	7	4·0	6·1	·524	W.B.
Hu. 487 ...	16 38	67 58	28·9	0·51	2	9·0	9·0	·559	B.
Σ 2084 ...	16 38	58 14	169·3	1·04	4	3·0	6·5	·554	B.
			163·4	1·08	2	...		·419	L.
			168·5	1·31	3	...		·558	W.B.
Σ 2091 ...	16 39	48 38	120·9	0·89	2	7·5	8·0	·604	W.B.
OΣ 315 ...	16 46	88 37	152·1	0·65	3	6·2	8·1	·544	W.B.
Σ 2106 ...	16 47	80 26	298·4	0·19	1	6·7	8·4	·578	B.
Ait. 1142	16 53	75 8	305·5	1·69	1	8·7	12·7	·630	L.
Σ 2114 ...	16 58	81 25	160·7	1·18	3	6·2	7·4	·495	W.B.
Hu. 1176 ...	17 5	53 57	111·7	0·15	3	6·0	6·0	·686	B.
Σ 2132 ...	17 7	93 56	112·0	1·43	1	8·3	9·0	·545	H.F.
Hu. 170 ...	17 8	80 8	278·4	1·88	1	8·5	10·8	·575	W.B.
Ho. 414 ...	17 19	63 50	88·5	0·60	2	8·4	8·8	·723	W.B.
Ho. 415 ...	17 19	64 10	332·9	0·96	2	8·0	8·7	·723	W.B.
Hu. 1179 ...	17 21	51 20	274·5	0·22	4	7·0	7·1	·655	B.
Σ 2171 ...	17 24	99 55	67·9	1·69	3	7·5	7·6	·559	B.
Σ 2173 ...	17 25	90 59	312·3	0·49	3	6·0	6·3	·559	B.
Ho. 417 ...	17 26	51 59	152·4	0·38	3	8·0	8·0	·635	B.
Hu. 179 ...	17 27	78 43	55·4	2·01	1	8·8	8·9	·471	W.B.
Ho. 420 ...	17 35	52 58	107·9	1·48	2	9·3	9·6	·626	B.
Σ 2205 ...	17 40	72 14	306·3	2·01	4	8·0	8·7	·501	W.B.
Σ 2215 ...	17 41	72 15	285 7	0·81	3	5·9	7·9	·511	W.B.
Σ 2220 BC	17 42	62 13	80·1	1·14	2	9·5	10·5	·526	H.F.
=AC 7			79·6	1·28	1	...		·630	L.
OΣ 338 ...	17 48	74 39	14·0	0·76	1	6·6	6·9	·712	B.
Ho. 442 ...	17 48	95 18	94·7	3·60	3	8·2	9·0	·566	W.B.
Ait. 235 ...	17 49	65 0	78·5	0·33	1	7·9	8·1	·712	B.
β 1299 ...	17 52	79 0	154·2	0·42	1	8·5	8·5	·712	B.
Σ 2262 ...	17 58	98 11	259·8	1·78	2	5·0	5·7	·526	H.F.
OΣ 534 ...	18 1	68 34	273·6	2·11	2	7·5	9·5	·549	B.
Σ 2272 ...	18 1	87 28	164·5	2·62	16	4·5	6·0	·595	W.B.
OΣ 341 ...	18 2	68 34	92·0	0·37	3	6·4	7·7	·565	B.
OΣ 524 ...	18 4	70 21	56·7	0·39	3	7·0	8·3	·600	W.B.
Σ 2283 ...	18 5	83 52	82·2	0·90	4	7·2	7·7	·568	B.
Hu. 314 ...	18 5	71 23	152·1	0·56	3	8·3	8·5	·600	W.B.
Hu. 315 ...	18 5	66 27	46·8	0·43	3	9·3.	9·3	·675	W.B.
Σ 2289 ...	18 6	73 32	229·3	1·15	4	6·0	7·1	·518	W.B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Σ 2294 ...	18 9	89 51	281·3	0·53	2	7·4	7·7	·526	H.F
Ho. 430 ...	18 18	69 32	194·7	2·93	1	8·5	9·0	·687	L.
Lewis 20 ...	18 18	69 30	332·1	2·32	1	9·0	9·5	·687	L.
* ...	18 18	70 2	138·0	3·77	1	10·5	11·0	·687	L.
Hu. 581 ...	18 19	75 5	130·2	0·30	2	8·4	9·0	·654	B.
			122·5	0·38	3	...		·706	W.B.
Ho. 83 ...	18 20	62 32	268·2	0·61	2	8·8	8·8	·583	W.B.
... ...			75·3	0·56	1	...		·687	L.
Σ 2320 ...	18 24	65 25	4·7	1·54	3	7·1	9·0	·495	W.B.
OΣ 354 ...	18 28	83 17	170·6	0·82	3	7·2	8·0	·580	W.B.
OΣ 358 ...	18 32	73 6	191·2	1·86	3	6·8	7·2	·533	W.B.
Hu. 198 ...	18 34	81 15	189·1	0·15	1	8·5	8·6	·657	W.B.
Ho. 437 AB	18 37	58 27	120·2	0·43	3	8·3	8·5	·739	W.B.
... CD			339·4	3·55	2	11·2	11·7	·731	W.B.
Ho. 88 ...	18 41	99 38	211·1	2·31	1	9·0	9·0	·578	B.
Σ 2375 ...	18 41	84 36	112·5	2·32	3	6·2	6·6	·517	W.B.
Σ 2422 ...	18 54	64 2	92·7	0·67	2	7·6	7·7	·529	W.B.
β 648 ...	18 54	57 13	179·0	0·42	1	6·0	9·5	·687	L.
			182·2	0·51	2	...		·731	W.B.
Hu. 330 ...	18 55	70 31	28·4	0·77	2	9·0	9·3	·742	W.B.
* ...	18 55	70 32	293·0	2·42	1	9·5	10·5	·668	L.
Hu. 678 ...	18 56	77 54	360·2	2·98	2	8·0	10·5	·507	W.B.
Ho. 92 ...	18 57	57 37	39·5	1·12	2	9·0	9·1	·762	W.B.
Σ 2437 ...	18 58	70 58	58·5	0·80	2	7·8	8·0	·507	W.B.
Ho. 93 ...	18 58	75 42	329·3	1·13	4	7·7	12·0	·711	W.B.
Σ 2455 ...	19 3	67 58	68·8	4·00	1	7·2	8·3	·668	L.
			72·4	3·73	3	...		·698	W.B.
* ...	19 3	67 8	296·2	3·73	1	9·5	10·0	·668	L.
			291·7	4·03	1	...		·715	W.B.
Ho. 444 ...	19 5	63 14	76·7	1·36	3	8·4	10·0	·620	W.B.
Ho. 98 ...	19 5	63 3	146·2	0·31	4	8·0	8·0	·719	W.B.
Ho. 442 ...	19 5	70 55	94·7	3·60	3	9·0	10·5	·566	W.B.
* ...	19 5	62 27	87·5	2·95	1	9·0	9·5	·668	L.
* ...	19 5	62 27	78·5	3·08	1	9·0	10·0	·668	L.
Ho. 447 ...	19 13	62 14	174·4	1·93	3	9·5	9·5	·593	W.B.
Ho. 103 ...	19 14	93 37	246·6	2·45	1	9·2	9·2	·578	B.
Ho. 105 ...	19 18	73 31	187·5	2·65	3	8·5	10·0	·517	W.B.
Hu. 339 ...	19 21	71 32	45 9	0·59	5	8·6	8·6	·596	W.B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Ho. 450 AB	19 23	51 22	267·6	0·97	1	8·0	8·7	·928	H.F.
Σ 2525 ...	19 23	64 52	305·2	0·67	5	7·5	7·7	·558	W.B.
Hu. 340 ...	19 24	71 43	119·9	0·87	4	9·0	9·3	·609	W.B.
Hu. 949 ...	19 30	57 5	103·0	0·69	1	8·6	9·0	·928	H.F.
Ho. 108 ...	19 30	56 43	47·9	0·54	1	8·0	8·0	·928	H.F.
Ho. 110 ...	19 35	71 31	80·5	2·67	3	9·5	9·5	·679	W.B.
Hu. 953 ...	19 36	54 58	177·0	1·08	2	8·6	9·0	·762	W.B.
Ho. 112 ...	19 37	71 35	70·2	2·72	3	9·0	9·0	·675	W.B.
Σ 2576 ..	19 42	56 38	287·9	2·45	2	7·8	7·8	·762	W.B.
AGC 11 ...	19 45	71 5	140·4	0·19	3	5·5	6·5	·705	B.
Σ 2596 ...	19 50	74 59	322·0	2·02	3	7·2	8·6	·692	W.B.
* ...	19 50	67 47	81·8	3·12	1	10·5	10·5	·657	W.B.
Σ 2600 ...	19 51	67 47	56·4	3·09	3	8·3	9·7	·673	W.B.
BD + 21° 3994 AB	19 54	68 6	275·7	1·32	3	9·0	10·4	·713	W.B.
			277·8	1·33	1	...		·928	H.F.
BD + 21°3996	19 55	68 11	298·9	1·94	3	9·0	9·5	·694	W.B.
Σ 2607 AB = 0Σ 392	19 55	48 1	292·1	0·65	1	7·2	9·0	·928	H.F.
Ho. 583 ...	19 55	68 8	256·0	1·21	1	9·0	10·7	·767	W.B.
Ho. 118 ...	19 59	56 35	22·7	2·74	2	9·1	11·0	·762	W B.
Ait. 1194 ...	20 0	77 54	305·1	0·62	1	8·9	9·0	·750	B.
			299·6	0·80	1	...		·767	W.B.
Ho. 122 ...	20 11	61 37	75·4	1·10	1	9·0	9·7	·756	W.B.
Ho. 591 ...	20 14	62 23	294·0	2·30	2	9·5	10·0	·655	W.B.
* ...	20 16	51 16	198·6	1·95	1	7·0	11·3	·928	H.F.
Ho. 126 ...	20 16	51 18	152·3	2·86	1	9·7	9·7	·928	H.F.
Hu. 1198 ...	20 20	77 17	32·8	0·65	1	8·4	9·2	·835	H.F.
Ho. 457 ...	20 21	60 55	60·0	2·13	3	8·1	8·1	·692	W.B.
Σ 2690 BC	20 27	79 5	217·2	0·41	2	7·5	7·5	·731	B.
Ho. 133 ...	20 28	103 51	184·3	0·87	1	8·0	8·0	·750	B.
Ho. 134 ...	20 32	55 9	203·2	3·36	2	8·5	8·7	·762	W.B.
β 151 ...	20 33	75 45	125·5	0·25	4	4·7	6·1	·730	B.
Ho. 458 ...	20 35	61 10	279·0	2·12	3	9·0	9·1	·673	W.B.
Σ 2716 ...	20 37	58 3	49·7	2·65	1	6·0	8·2	·835	H.F.
Hu. 690 ...	20 41	56 11	290·1	0·93	1	9·0	9·2	·928	H.F.
Σ 2723 ...	20 42	74 28	103·4	1·37	3	7·3	8·0	·692	W.B.
Ho. 141 ...	20 43	71 2	291·0	1·16	1	8·4	10·8	·619	W.B.

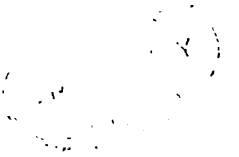
Micrometer Measures of Double Stars—continued.

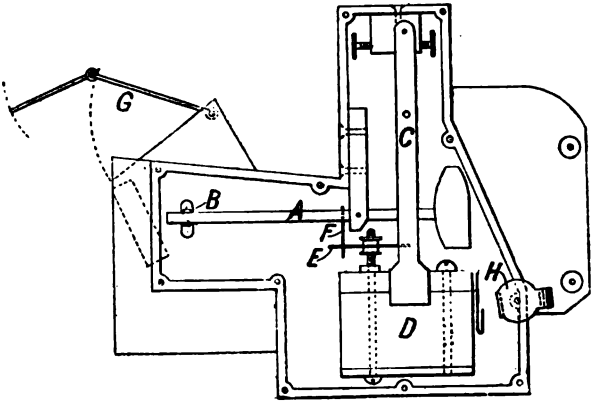
Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle.	Distance.	No. of Nights	Mags.		Epoch 1907.	Obs.
Ho. 144 ...	20 48	70 13	164·9	0″·43	1	7·0	7·0	·657	W.B.
Ho. 146 ...	20 51	55 7	51·9	0·56	1	8·0	8·1	·928	H.F.
Hu. 764 ...	20 56	55 55	190·2	0·48	1	7·5	8·7	·928	H.F.
Ho. 282 ...	21 0	44 30	359·0	0·28	1	7·0	7·0	·805	B.
Hu. 767 ...	21 9	74 24	143·7	0·18	2	7·0	7·0	·738	B.
0Σ 535 ...	21 10	80 22	171·9	0·16	1	4·5	5·0	·687	L.
			158·2	[0·15]	3	...		·706	W.B.†
			95·0	0·13	3	...		·718	B.
			76·5	0·14	3	...		·848	B.
			136·0	0·11	3	...		·862	W.B.
Ho. 284 ...	21 12	74 24	90·2	3·73	2	9·0	9·3	·636	W.B.
Ho. 153 ...	21 14	56 37	121·2	0·78	1	8·0	9·0	·928	H.F.
Ho. 286 ...	21 16	52 8	304·9	0·16	1	6·0	6·0	·764	B.
Ho. 155 ...	21 16	57 8	31·7	2·02	2	8·0	9·5	·783	W.B
Ho. 157 ...	21 19	58 22	23·1	3·69	2	7·7	7·7	·783	W.B.
Ait. 1219 ...	21 21	60 8	120·8	1·40	2	9·3	9·4	·762	W.B.
Ait. 1220 ...	21 21	58 54	145·5	1·73	2	8·5	9·0	·762	W.B.
Ho. 160 ...	21 25	47 29	178·9	1·72	1	8·3	9·0	·928	H.F.
Hu. 371 ...	21 31	65 57	162·1	0·22	3	7·0	7·5	·742	B.
Hu. 372 ...	21 34	66 48	149·6	0·30	3	9·0	9·0	·742	B.
Hu. 280 ...	21 38	84 30	134·6	0·18	3	7·7	8·1	·742	B.
Hu. 374 ...	21 39	66 32	32·8	0·33	3	9·0	9·0	·742	B.
Ho. 605 ...	21 40	55 32	344·0	2·09	1	9·2	9·9	·942	L.
β 989 ...	21 41	64 46	95·1	0·19	9	4·8	5·3	·806	W.B.
			93·1	0·08	1	...		·687	L.
			97·6	0·13	5	...		·735	B.
Ho. 166 ...	21 46	62 34	75·3	0·34	2	7·5	7·5	·782	W B.
Ho. 169 ...	21 46	54 19	137·1	3·18	1	8·0	12·0	·942	L.
Ho. 467 ...	21 46	68 10	182·6	1·15	2	8·0	10·2	·712	W.B.
Ho. 179 ...	22 4	60 14	261·5	0·42	1	8·0	9·0	·810	W.B.
Ho. 182 ...	22 19	72 54	128·9	1·46	3	8·5	8·5	·719	W.B.
Σ 2900 ...	22 19	69 39	175·8	1·41	1	6·0	9·2	·942	L.
Ho. 475 ...	22 29	64 3	315·9	0·74	2	8·0	8·2	·769	W.B.
Ho. 294 ...	22 33	63 2	58·1	2·03	2	8·0	10·0	·763	W.B.
Ho. 480 ...	22 34	60 45	228·7	0·69	1	8·0	9·1	·810	W.B.

† Distance estimated.

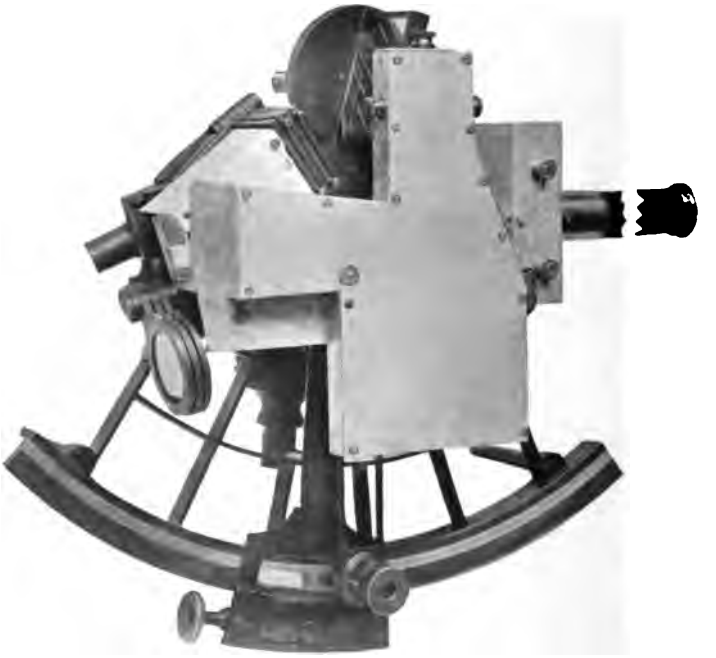
Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Ho. 296 ...	22 36	75 56	61·7	0·46	3	5·5	5·5	·742	B.
			64·0	0·33	2	...		·791	W.B.
Hu. 780 ...	22 37	75 56	13·9	0·33	3	9·2	9·4	·742	B.
			5·3	0·51	1	...		·942	L.
Σ 2934	22 37	69 5	134·3	0·63	1	8·2	9·2	·942	L.
Hu. 781 ...	22 38	75 16	311·9	0·30	1	8·5	9·5	·862	W.B.
Ho. 481 ...	22 41	61 6	114·1	0·31	3	8·3	8·3	·742	B.
Ho. 190 ...	22 45	62 28	153·9	1·86	1	9·2	9·2	·810	W.B.
Ho. 482 ...	22 47	64 5	56·5	0·22	5	6·8	6·8	·772	B.
Hu. 987 ...	22 51	74 42	243·8	0·54	3	8·6	8·8	·780	B.
			252·4	0·93	1	...		·942	L.
Ho. 192 ...	22 55	60 24	30·4	1·47	2	8·5	9·5	·763	W.B.
Ho. 193 ...	22 59	60 35	165·7	2·42	1	7·2	11·2	·871	W.B.
Ho. 196 ...	23 4	60 1	288·5	1·94	1	8·0	11·0	·871	W.B.
Ho. 198 ...	23 13	73 38	32·8	2·31	3	8·0	9·0	·860	W.B.
Hu. 400 ...	23 13	72 11	229·4	0·33	3	7·4	8·8	·780	B.
Ho. 489 ...	23 22	62 47	245·0	0·45	2	7·8	7·8	·832	W.B.
Σ 3012 ...	23 23	73 55	189·9	2·99	1	8·7	8·8	·942	L.
Σ 3013 ...	23 23	73 55	273·8	3·62	1	7·8	9·3	·942	L.
Hu. 298 ...	23 28	83 25	76·1	0·28	2	6·8	7·4	·757	B.
*	23 36	70 8	128·2	1·60	1	8·0	11·0	·942	L.
Hu. 300 ...	23 41	84 1	119·0	1·14	3	8·7	9·0	·780	B.
			118·8	1·15	1	...		·893	W.B.
Ho. 206 ...	23 55	56 14	189·0	1·69	1	8·0	10·0	·810	W.B.





Details of the Pendulum attachment.



Sextant with Artificial horizon attachment.

Note on "the Spanner," Captain Gadsden's Artificial Horizon for Sextants. By H. H. Turner, D.Sc., F.R.S., Savilian Professor. (Plate 13.)

It seems desirable that the existence of an instrument which has been found to work in practice, and which is useful to a sailor in fog or at night, should be made widely known. I venture to emphasise the fact that the instrument has been found practically useful; for this point is essential. There have been many promising suggestions for such an instrument, which have failed on actual trial.

The apparatus is attached to an ordinary sextant, and is the invention of Captain H. A. Gadsden, of 6 Binden Road, Ravenscourt Park W., London, to whom I must refer those interested for further details. The subjoined figures (Plate 13) will explain its general nature. The instrument was exhibited to the Society at the April meeting.

An artificial horizon is provided, which consists of two bars that exactly span the periphery of the Sun's image in the horizon glass, and are maintained in horizontal position by means of a pendulum forming part of the attachment. This is shown in detail in Plate 13. At *A* is a pivoted balance beam which carries the horizon bars *B*, the latter extending through a slot in the casing. The pendulum comprises a pair of plates *C*, pivoted to a block at their upper ends, while their lower ends carry a heavy weight *D*. A plate *E* is secured to the weight *D*, and is formed with an opening through which extends a rod *F*, projecting from the beam *A*. The pendulum maintains its vertical position when the sextant is held in the hand, and by means of the rod *F* the horizon bars are kept at a proper horizontal level.

As the horizon bars are near the eye of an observer, the telescope of a sextant cannot be used. Observations are therefore made with a simple tube, longer than that usually supplied, so that the eye can be properly focussed on the bars.

If it is desired to use the sextant in the ordinary way, the spanner can be "locked" out of sight.

Captain Gadsden has also added to the sextant a stock of simple form (resembling a gunstock), which is put against the shoulder and helps to steady the instrument.

Note on Father Stein's paper "On Dr. Roberts' method of determining the absolute dimensions of an Algol Variable Star." By Alex. W. Roberts, D.Sc.

Professor Turner has kindly sent me in advance the main conclusions Father Stein has arrived at in his paper in *M.N.* (vol. lxviii. p. 490).

Through unaccountable error, I regarded the value of the light ratio as a maximum when the difference between the masses of the component stars was a minimum. It then, however, is zero.

Although in the case where the masses of the component stars are equal it is impossible to arrive at the absolute magnitude of a binary system, this is not the case where there is disparity between the masses, and I still have the hope that through careful determination of the quadrant points of a close binary orbit we may advance our knowledge in this direction.

[*Note by H. H. Turner.*—It should perhaps be made clear that the above note was written before Dr. Roberts had actually seen Father Stein's paper. On referring to his notes, he found at once the unfortunate slip above stated, and hastened to admit the correction without awaiting the paper itself. His promptness makes it possible to include this note in the present number of the *Notices*.]

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVIII. JUNE 12, 1908. No. 8

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Hugh Cameron Campbell, Science Department, Surgeons' Hall, Edinburgh,

and

The Rev. Arthur Mackreth Deane, M.A., Canon of Chichester, Ferring Vicarage, Worthing, Sussex,

were balloted for and duly elected Fellows of the Society.

Benjamin Baillaud, Director of the Observatory, Paris;

Carl Ludwig Wilhelm Charlier, Director of the Observatory, Lund, Sweden;

Edwin Brant Frost, Director of the Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.;

Johann Georg Hagen, S.J., Director of the Vatican Observatory, Rome; and

Johannes Franz Hartmann, Astrophysical Observatory, Potsdam, Germany,

were balloted for and duly elected Associates of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

The Rev. Thomas Nicklin, M.A., Assistant Master, Rossall School, Bescot, Rossall Beach, Fleetwood, Lancashire (proposed by S. A. Saunder).

Eighty-one presents were announced as having been received since the last meeting, including amongst others:—Cape Catalogue of 1680 stars for equinox 1900·0, presented by the Royal Observa-

tory, Cape of Good Hope; G. E. Hale, The Study of Stellar Evolution, presented by the author; five spectroheliographs (enlargements), showing hydrogen and calcium flocculi, presented by the Mount Wilson Observatory; lithograph from a drawing of Donati's Comet, by Miss Charlotte S. Cooper, Markree, presented by Miss Cooper; 16 charts of the Astrographic Chart of the heavens, presented by the Royal Observatory, Greenwich.

Considerations on the Form and Arrangement of New Tables of the Moon. By Simon Newcomb.

Hansen's Tables of the Moon, with some patching up, have now been in use in the standard astronomical ephemerides for a full half century. With every passing year the necessity of replacing them by improved ones becomes more urgent. My own investigations on the Moon's motion have always been carried on with the view of ultimately constructing new tables, while experience in the use of Hansen's tables has from time to time formed a basis for a study of the relative merits of various forms of the lunar theory for the purpose of tabulation. But, as the years have passed, it has become increasingly apparent that I must leave to other hands the execution of the desired work. I therefore venture to summarise the suggestions which I have to make on the whole subject.

§ 1. *General Form of Tables.*

All my experience with Hansen's tables has led me to the conclusion that if the problem were only that of enabling an ephemeris of the Moon to be computed with the use of the fewest figures, it would scarcely be possible to improve on Hansen's arguments and system of tabulation. The Hansenian perturbations of the mean anomaly are more rapidly convergent than those of any co-ordinate that can be used, and therefore require fewest tables. The expression of the fundamental argument in terms of the time may also have a certain advantage, though this is a point on which I feel less confident, because the best unit is not a decimal of a day.

But the mental labour to be performed is not measured merely by the number of figures taken out and written down. A "straight-ahead" computation has a decided advantage over one in which it is necessary to form numbers from preceding tables, or to introduce tables of logarithms, as we must when we pass from Hansen's fundamental argument to the true longitude. This remark is especially applicable to the reduction to the ecliptic. The mental labour of forming arguments from numbers already used and of combining results from various tables is intensified by the care and attention which then have to be bestowed, and which it is desirable to render unnecessary.

The desired uniformity in proceeding would be attained in the highest degree by tabulating the perturbations of the ecliptic longitude and latitude in the form to which they are reduced by Brown, using a uniform system of arguments for the tables. But the number of tables then requisite would be so great that it is doubtful whether they could be put into a single volume of reasonable size, say that of Hansen's tables as they now are. The most troublesome and important terms would be those arising from the reduction to the ecliptic. It may be added that I have never found Hansen's system of constructing the double-entry tables for intervals of $0^d.25$ to offer a great advantage over the use of 12-hour intervals, except when an isolated place is to be computed.

The modification of the plan which I have to suggest will best be seen by starting with Hansen's fundamental argument. After it is formed, two steps are required—the reduction from the inequality of mean longitude to that of true longitude in orbit, and the reduction to the ecliptic. When the reductions are made in the general formulæ, tabulated, as they will be if ecliptic co-ordinates are tabulated, each of these steps adds a number of additional inequalities, but I think those arising from the second reduction exceed in number and magnitude those arising from the first. However this may be, it is certain that the tabulation of the true longitude will require much more voluminous tables than will that of the Hansenian mean longitude.

What I would suggest is, therefore, the tabulation of the longitude in orbit, that co-ordinate being defined as the distance from the mean node to the circle of latitude passing through the Moon perpendicular to the ecliptic. More geometric symmetry would be attained by taking as the fundamental plane the mean moving orbit of the Moon, and referring the longitude and latitude to that plane. But this would require a multiplication of the quantities by certain factors in order to find the true longitude. If the principal term of the latitude is computed with the true argument of latitude, the inequalities are fewer and smaller than if the mean argument is used.

§ 2. *Unit of Tabular Longitude.*

In the case of the longitude, the next question is that of the unit to be chosen. It is so important to avoid the use of larger numbers than is necessary that this choice requires careful consideration. The tabulated unit of Hansen's fundamental argument is approximately $0''.005$. The smallness of this quantity makes the interpolation laborious without really adding materially to the accuracy of the result. To devise the best unit, we must first consider the degree of precision to be reached by some arbitrarily chosen unit, say $0''.01$. Here some statements relating to the accumulation of small accidental errors become necessary.

§ 3. *Errors accumulating from omitted decimals.*

The theory of errors accumulating from omitted decimals is not usually developed in a way that seems to me entirely satisfactory. I therefore begin by stating some principles, definitions, and results pertaining to the subject.

1. I use the term *probable value* of a doubtful quantity in the sense introduced by Poincaré, which is—

If $$v_1, v_2, \ldots . v_n$$

be all the essentially different possible values of a quantity v, and if

$$p_1, p_2, \ldots . p_n$$

be the several probabilities of these values, then the probable value of v is that given by the equation

$$v = p_1 v_1 + p_2 v_2 + \ldots . + p_n v_n .$$

That is, the probable value is the mean of all the possible values, each weighted according to its probability.

2. I deal, in the case of each quantity, with the probable value of the square of its deviation from the truth. The square root of this square is generally designated in astronomy as the *mean error*. Pearson has introduced the somewhat more appropriate term of "standard deviation," which I abbreviate to S.D. We may therefore define this as the square root of the probable value of the square of the deviation.

3. Let us consider the sum S of any number n of independent quantities, $q_1, q_2, q_3, \ldots . q_n$.

Let these q's be uncertain by the several standard deviations

$$\epsilon_1, \epsilon_2, \epsilon_3 \ldots . \epsilon_n .$$

If we put ϵ for the S.D. of S we shall then have

$$\epsilon^2 = \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \ldots . + \epsilon_n^2 .$$

The advantage of this equation is that it is independent of the law of probability of an error as a function of its magnitude, which is not in all cases the normal law. For example, the error arising from the omission of a periodic term has a well-defined absolute maximum equal to the coefficient of the term. Moreover, its maximum value is more probable than any other. The standard deviation arising from the omission of a periodic term $a \sin mt$ is

$$\epsilon = \pm \frac{a}{\sqrt{2}} .$$

An examination of the coefficients of longitude found by Brown and Hill shows that there are about 300 periodic terms whose

coefficients lie between the limits 0″·01 and 0″·001. I take this lower limit as that beyond which it is unnecessary to go. In estimating the S.D. I take 0″·01 as the unit. An approximate estimate of the distribution of these coefficients in magnitude shows that the square of the S.D. produced by omitting them is, with sufficient exactness,

$$\epsilon^2 = 28 .$$

There remain, by a rough estimate, about 300 terms exceeding the limit 0″·01, nearly all of which I assume will be tabulated. I also assume that the individual numbers of these tables will be formed by carrying each number to 0″·001, and that the last decimal will be dropped in the final tabulated number, which will be given only to 0″·01. Making due allowance for all imperfections, I find that the standard deviation of a number interpolated from a single table thus formed will be ±0·26 if the number is written down to one decimal beyond that of the tables. But if the number is, as usual, only written to the tabular number of decimals, the deviation will be ±0·39.

I assume that not more than 120 tabular numbers will be added to form the longitude. The contribution to (S.D.)^2 arising from the summation of these numbers is

$$120 \times 0{\cdot}39^2 = 18{\cdot}2 .$$

The imperfections of the arguments will also have their influence. The deviation produced by them may be reduced by giving each argument to one decimal beyond that required by the condition that the error arising from a unit error of the argument shall always be less than that of the tabular unit. But, without going beyond this rule, the effect of the errors of the argument will not exceed that arising from the addition of thirty more tabular numbers. We may thus have—

$$(\text{S.D.})^2 \text{ from errors of arguments} = 4{\cdot}5.$$

In some cases the effect of the imperfections of the correction for the second differences may add to the S.D. I think that, with a little skill and attention on the part of the computer, this S.D. need not exceed ±0·2, giving 4·8 for its entire contribution to (S.D.)^2.

Summing up all the sources of accidental deviation of the tabular results from theory we have—

From the omission of small terms . .	(S.D.)^2 =	28
,, ,, tabular decimals .	,,	18
,, errors of arguments . . .	,,	5
,, ,, of second diff. . .	,,	5
In all	(S.D.)^2 =	56

And for the sum, S.D. = 7·5.

We shall therefore have, in the case of each computed longitude, a S.D. of $\pm 0''.075$ and a probable error of $0''.05$.

§ 4. *Degree of precision required.*

Let us compare this with the degree of precision required in a comparison with observation. My experience in the use and examination of lunar observations leads me to the conclusion that no single observation of any sort can be practically made without a mean error exceeding $\pm 0''.5$, corresponding to a probable error of $\pm 0''.34$. It is desirable that the mean error of a co-ordinate found from the tables should be less than this. But all the results obtained from lunar observations depend upon a great number of observations, which make the accidental errors unimportant in comparison with the systematic ones. The practical advantage of a degree of precision above that just mentioned in the tabular places is very small, and is practically evanescent if reduced below $\pm 0''.4$. This degree of precision will be surpassed by adopting as the unit to be tabulated the 10^{-8} part of the circumference, or $0''.01296$. Of course the unit $0''.01$ would answer our purpose if deemed more convenient in use. But the smaller the numbers used, the easier the computer will find it to avoid small errors, while the S.D. will still fall below $\pm 0''.09$.

There is, however, one point still to be considered in this connection which may modify our conclusions. Granting an S.D. of $\pm 0''.10$ in the individual longitudes, we must expect that in the course of a year there may be three or four of the 730 tabulated longitudes in error by three times the S.D., and possibly one of four times this amount. But every error approaching such a magnitude as this will be detected by differencing the 12-hour ephemeris. A legitimate proceeding will then be to smooth off the ephemeris by such small corrections as shall reduce the higher, say the fifth or sixth, differences to a sufficiently smooth series. Each corrected tabular result may then be regarded as the mean of two or more neighbouring quantities, and the maximum error of the ephemeris will be reduced nearly to the mean S.D. In a word, we may fairly count on having an ephemeris in which all the errors exceeding some limit between $0''.12$ and $0''.30$ will be eliminated. This limit is still within the errors of the best observations, and the cases in which it is approached will be rare.

§ 5. *Reduction to sexagesimal units.*

The proposed units will require the reduction of the final longitudes to degrees, minutes, and seconds. The tables necessary for this purpose will perhaps fill four pages, and the computation will be equivalent to the entry of three additional tables.

If the unit $0''.01$ is deemed preferable, its use will still require some study. It was adopted in Peirce's Tables of the Moon,

published in 1853. The tabular numbers were there expressed in degrees and seconds, minutes being ignored. I found the use of this system cumbrous, and should prefer to use seconds pure and simple, subtracting 1,296,000″ or its multiples when necessary. The proposed circumferential unit does away with this subtraction. Although it is a little easier to change the degrees into seconds and minutes than it is to change the circumferential unit, I still think the advantage to lie with the latter.

It may be of interest in this connection to note that if we should base the unit on the degree, tabulating to 0°·00001, the S.D. of the individual longitudes would still be only ±0″·18, and we might be fairly confident that no error exceeding 0″·4 would remain in a smoothed-off annual ephemeris as often as once a year.

Of course all this presupposes that the computer is always careful never to make a greater error than 0·5 in interpolating and writing down his number. The question may arise whether it is not well to allow him a margin of one or two units, by adopting smaller units. My answer is that the labour of handling large numbers involves more mental strain than that required in the accurate handling of small numbers, and that the assigned standard of precision will be more easily reached by the careful use of the smaller numbers than by the careless use of the larger ones.

§ 6. *Epochs and Arguments.*

For the practical work of computing places of the Moon for given dates, I do not think that any system more convenient than the usual one can be devised. The Hansenian form, in which the Gregorian and Julian calendars are used, is the most convenient of all. But it is always desirable to give the tables such a form that the relation between the tabular numbers and the original elements shall be easily examined, and corrections to the theory readily applied. This suggests a slight sacrifice of ease in computing an isolated position, or an ephemeris, to the requirements of the theoretical investigator.

Simplicity in the other direction is reached by the use of days of the Julian period. This was first employed, I believe, by Peirce, and is now extensively used in astronomy, especially in Oppolzer's works relating to eclipses. In using this system, a first and easy step is the reduction of the ordinary calendar date to days of the Julian period. Then absolute uniformity is reached in the construction and use of the tables.

The principal immediate drawback of this system is that, if used unmodified, the period of 1000 days must take the place of the year. The formation of the arguments of short period is then inconvenient. Many of the lunar arguments have periods not differing much from a month. From 12 to 15 multiples then suffice when the year is used, but with the period of 1000 days the number of multiples to be tabulated and subtracted will frequently be between 30 and 40, and some times more. Of course

this difficulty can be lessened by taking 500 days instead of 1000 as the second unit. But this will detract from simplicity of form.

A yet more serious drawback to the theoretical investigator is that the fundamental epochs usually adopted in astronomy, and for which the elements must be found, do not correspond to any power of 10 in the days of the Julian period. A complete transformation of the elements is therefore required to form the numbers on which the tables are based. If, therefore, multiples of 500 or 1000 days are used instead of years, I should prefer to count them back from 1900·0, thus gaining all the advantages of the Julian period without any other disadvantage than that of non-correspondence with the eclipse and other tables of Oppolzer.

The reduction of such a system to the ordinary calendar may be made a very simple matter. It seems to me, therefore, that the maximum of advantage will be reached by giving the fundamental arguments for cycles and periods based on multiples of 500 days before and after the fundamental epoch 1900 Jan. 0.

Probably the most convenient fundamental quantities to tabulate will be the longitude of the node, and the mean distances of the Moon and of its perigee from the node, all expressed in circumferential units. Then, whatever form the tables may be thrown into, we shall have the nearest approach to a simple, straightahead computation.

Finally, a serious problem is that of summing perhaps 100 periodic terms with coefficients not differing greatly from 0″·01. I have devised a machine for this purpose, the description of which must form the subject of another publication.

An Example of Professor Karl Pearson's Calculation of Correlation in the case of the Periodic Inequalities of Long-period Variables. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The following note is written with a twofold purpose. Firstly, it is hoped that an indication of some value has been obtained with regard to the features of "long-period" variability; and secondly, the opportunity is taken to write out in full a simple example of the calculation of "correlation" between quantities by the methods of Professor Karl Pearson.

In the *M.N.* for March last (p. 416) Professor Pearson himself gave an admirable summary of methods; but he naturally did not repeat the elementary working which has become so familiar to him, and has been given often before in other connections. There are doubtless many to whom this working is already familiar; but there are certainly many others who do not know it and who might use it if they had an astronomical example readily accessible. In these busy days many people have not the leisure to search for references in scientific literature outside their own subject.

2. In support of my view that there is need for an example, I may cite an illustration shown me by Professor E. C. Pickering (see *Observatory*, March 1905, p. 153).

Suppose we have the following observed values of A and B:—

A	.	.	.	5	6	7	6	7	8	7	8	9
B	.	.	.	6	6	6	7	7	7	8	8	8

First group according to B, in sets of three for which B has the same value.

Then when		B = 6 7 8
	Mean	A = 6 7 8
Therefore		A = B.

Secondly, group according to A.

Then when		A = 5 6 7 8 9
	Mean	B = 6 $6\frac{1}{2}$ 7 $7\frac{1}{2}$ 8
Therefore		(A − 7) = 2(B − 7).

Now, neither Professor Pickering nor myself was prepared at that time to deal with this situation by a definite process; and yet this is just an elementary case of the kind which Professor Pearson's methods were devised to meet.

This example is worked out below, and it is shown that the numerical measure of the correlation is

$$r = \cdot 08 \pm \cdot 22$$

or almost nothing at all: so that, in spite of appearances to the contrary, we are not entitled to assume any relationship between A and B. Putting it in another way, one proposed relation (A = B) is as good as another (A − 7) = 2(B − 7).

3. A point of detail may be mentioned here. In much statistical work, a large number of figures are used. Thus we get such statements* as

$$r = \frac{1262 \cdot 51}{2870 \times 1 \cdot 13637 \times 1 \cdot 52135} = \cdot 25445.$$

The probable error of r being about $\pm \cdot 01$, some of these figures are superfluous; and in what follows fewer figures are used. This, however, represents a personal view which is, I find, not generally approved by other workers.

4. The particular example selected for treatment is the discussion of the elements of maximum given by Chandler for long-period variables. The following particulars are taken from

* *Frequency Curves and Correlation*, by W. P. Elderton, p. 119: an excellent little book, from which much is to be learned.

his "Revision of Elements of Third Catalogue" (*Astronomical Journal*, No. 553):—

Star.	Epoch.	Elements of Maximum. Period.		Inequalities.
R Androm.	2400141	+ 410·3 E	+	30 sin (12° E + 90°)
S Cassiop.	2401603	+ 610·5 E	+	37 sin (15° E + 59°)

and so on. The mean period of S Cassiop. is 610·5 days; but there is an inequality which displaces the maximum 37 days one way, and 12 periods later (since 180°/15° = 12) the displacement is 37 days in the other direction. At these times the period is about the mean, but midway between such times it is longer and shorter. In the "Revision" above mentioned, Chandler gives such inequalities for 37 stars, as shown in Table I. We shall for the present only consider the period P, the coefficient C, and the argument A; thus for S Cassiop. P = 610·5, C = 37, and A = 15°. The epoch 59° does not concern us at present.

TABLE I.

Star's Name.	No.	P	C	A	Phase.	M − *m*	αP
R Androm.	112	410	30	12	218°	120	−170
S Cassiop.	432	610	37	15	148	280	− 50
S Piscium	434	404	18	10	57	163	− 78
R Arietis	782	187	7	5	292	92	− 3
o Ceti	806	332	18	5	...	125	− 82
R Persei	1222	210	15	8	224	96	− 18
R Aurigæ	1855	459	19	12	320	235	+ 11
R Lyncis	2478	379	14	15	249	186	− 7
R Gemin.	2528	370	35	6	121	121	−128
S Can. Min.	2684	330	20	12	120	164	− 2
R Cancri	2946	362	60	6	341	125	−112
R Carinæ	3418	310	25	9	289	136	− 38
R Leo Min.	3477	371	20	10	11	165	− 41
R Urs. Maj.	3825	302	11	8	63	110	− 82
T Urs. Maj.	4511	257	20	9	203	108	− 42
R Virginis	4521	145	20	2	103	68	− 9
S Urs. Maj.	4557	226	35	5	309	108	− 10
S Boötis	5157	270	60	4	87	132	− 6
R Camelop.	5190	270	65	4	244	142	+ 14
R Boötis	5237	223	9	9	251	102	− 20

TABLE I.—*continued.*

Star's Name.	No.	P	C	A	Phase.	M − m	aP
				°	°		
S Serpentis	5501	369	116	4	122	...	...
S Coronæ	5504	361	8	12	81	120	- 121
R Serpentis	5677	357	35	4	109	151	− 55
R Herculis	5770	318	17	10	29	...	...
W Herculis	5950	280	26	13	288	128	− 24
S Herculis	6044	308	35	9	239	152	− 4
T Herculis	6512	165	10	5	152	79	− 7
R Sagittarii	6905	269	18	10	18	138	+ 7
S Sagittarii	6921	231	15	10	167	102	− 27
R Delphini	7261	284	26	9	266	130	− 24
U Capric	7455	203	20	5	67	...	...
T Aquarii	7468	203	8	7	357	88	− 27
R Vulpec.	7560	137	18	4	127	62	− 13
X Capric.	7577	218	20	10	98	117	+ 16
R Pegasi	8290	378	60	8	71	172	− 34
R Aquarii	8512	387	35	10	97	...	...
R Cassiop.	8600	432	32	9	233	182	− 68

5. The first two columns of Table I. give the star's name and its number on Chandler's system. The third column, P, gives the period in days; C is the coefficient of the periodic inequality in days, and A the coefficient of E within the bracket. The explanation of the last three columns will be given later.

6. Now the example selected to be given in full is that of the relation between P and A. To determine whether there is such a relation, we could, of course, proceed in the ordinary way to group together stars with nearly the same P, and take the mean values of A for them. This would give us a fair indication, but no notion of its probable error. Professor Pearson's procedure is not much longer; it includes the ordinary procedure, and it gives us a definite measure of the probability of the relation being real.

7. We first form a "correlation table" as in Table II.

[TABLE

TABLE II.

Correlation Table for P *and* A.

P in days. \ A=	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	Sums
601 to 650														1 (36)	1
551 to 600														0 (30)	0
501 to 550														0 (24)	0
451 to 500								0	0 (3)	0 (6)	1 (9)	0	0	0 (18)	1
401 to 450								1	1 (2)	0 (4)	1 (6)	0 (8)	0 (10)	0 (12)	3
351 to 400			2 (5)	0 (4)	2 (3)	0 (2)	1 (1)	0	2 (1)	0 (2)	1 (3)	0 (4)	0 (5)	1 (6)	9
301 to 350			0	1	0	0	1	2	1	0	1	0	0	0	6
251 to 300			2 (5)	0 (4)	0 (3)	0 (2)	0 (1)	2	1 (1)	0 (2)	0 (3)	1 (4)	0 (5)	0 (6)	6
201 to 250			0 (10)	2 (8)	0 (6)	1 (4)	1 (2)	1	2 (2)	0 (4)	0 (6)	0 (8)	0 (10)	0 (12)	7
151 to 200			0 (15)	2 (12)	0 (9)	0 (6)	0 (3)								2
101 to 150	1 (28)	0 (24)	1 (20)	0 (16)	0 (12)	0 (8)	0 (4)								2
Sums	1	0	5	5	2	1	3	6	7	0	4	1	0	2	37

A	Sum.	Factor.	ν_1= Product.	ν_2= Product.
°				
2	1	−7	− 7	49
3	0	−6	0	0
4	5	−5	−25	125
5	5	−4	−20	80
6	2	−3	− 6	18
7	1	−2	− 2	4
8	3	−1	− 3	3
9	6	0	0	0
10	7	+1	+ 7	7
11	0	+2	0	0
12	4	+3	+12	36
13	1	+4	+ 4	16
14	0	+5	0	0
15	2	+6	+12	72
Sums	37	...	−28	410
Means (ν_1 and ν_2)			−·84	+11·08

$\sigma_A = \sqrt{\nu_2 - \nu_1^2} = \sqrt{10{\cdot}37} = 3{\cdot}22$

P	Sum.	Factor.	ν_1= Product.	ν_2= Product.
125	2	−4	− 8	32
175	2	−3	− 6	18
225	7	−2	−14	28
275	6	−1	− 6	6
325	6	0	0	0
375	9	+1	+ 9	9
425	3	+2	+ 6	12
475	1	+3	+ 3	9
525	0	+4	0	0
575	0	+5	0	0
625	1	+6	+ 6	36
Sums	37	...	−10	150
Means (ν_1 and ν_2)			−·27	4·05

$\sigma_P = \sqrt{\nu_2 - \nu_1^2} = \sqrt{3{\cdot}98} = 2{\cdot}00$

Products S′(ΔP

	− 17 +64
	+ 104 − 9
	= +142
37(−·84)(−·27)	= + 9
Difference =	133

$$r = \frac{133}{37 \times 3{\cdot}22 \times 2{\cdot}00} = 0{\cdot}56$$

8. This requires little explanation. We divide the periods into groups of 50 days as shown in the first column, and the number of periods in each group is shown in the final column. For the values of A no collection in groups is necessary, successive degrees being shown in the top line, and the number of corresponding cases in the bottom line. In the body of the table are given, in large figures, the number of cases corresponding to a definite degree for each group of periods. The small figures will be referred to presently.

9. The thick lines indicate the groups which contain approximately the mean of each co-ordinate. They are chiefly for convenience in working (to keep the numbers small), and if a wrong group is taken no real harm is done.

10. The figures below the table represent the whole working. We first form ν_1 and ν_2 for each co-ordinate by a process which is tolerably clear to inspection. For A write down the (top line for convenience only—not used in working; and the) bottom line of the table. Follow with a sequence of units, with 0 at the place of the adopted mean. Multiply the two columns; and again multiply the last two columns. We thus get $n\nu_1$ and $n\nu_2$, where n is the number of observations, and ν_1, ν_2 two quantities required. Dividing by n, we get ν_1 and ν_2, and finally

$$\sigma_A = (\nu_2 - \nu_1^2)^{\frac{1}{2}} = 3{\cdot}22 .$$

[If greater accuracy is required, we should subtract also "Sheppard's correction" of $\frac{1}{12}$ from ν_2 before taking the square root.]

11. The same procedure with the columns gives us $\sigma_P = 2{\cdot}00$.

12. We now form the product S(AP). The number in each box is to be multiplied by the little number shown in the corner of the box, which will be seen to be the product xy, where x represents the number of boxes to the right, and y the number up, from the adopted mean. The products in the four main divisions of the table are shown diagrammatically, and from them we get

$$S'(AP) = 142 .$$

From this we subtract $n\nu_1\nu_1^1$, where n is the number of observations, and ν_1, ν_1^1 have been found; and thus

$$S(AP) = S'(AP) - n\nu_1\nu_1^1 = 142 - 9 = 133 .$$

Then the "correlation" r is

$$r = \frac{S(AP)}{n \times \sigma_A \times \sigma_P} = 0{\cdot}56 .$$

13. This is quite a respectable value for r—anything over 0·5 is worth serious attention. But to complete our information we want the probable error of r, which is given by the formula

$$p.e = {\cdot}67(1 - r^2)/\sqrt{n} ,$$

which can be taken from the following small table (Table III.).

For the present case we have $n = 37$, $r = 0{\cdot}56$, and thus probable error $= \pm 0{\cdot}08$.

TABLE III.

Probable error of r.

n \ r	·1	·2	·3	·4	·5	·6	·7	·8	·9
10	·208	·202	·193	·177	·158	·136	·107	·076	·041
20	·148	·143	·137	·125	·112	·096	·076	·054	·029
30	·121	·117	·112	·102	·091	·079	·062	·044	·024
40	·104	·101	·096	·088	·079	·068	·054	·038	·021
50	·093	·090	·086	·079	·070	·061	·048	·034	·018
60	·085	·083	·079	·072	·064	·056	·044	·031	·017
70	·079	·076	·073	·067	·059	·051	·040	·029	·015
80	·074	·072	·068	·063	·056	·048	·038	·027	·015
90	·069	·067	·064	·059	·053	·045	·036	·025	·014
100	·066	·064	·061	·056	·050	·043	·034	·024	·013

14. Coming now to the relation between P and A, the mean value of A is got by adding ν_1 to our adopted mean, *i.e.* it is $9^\circ - 0^\circ{\cdot}84 = 8^\circ{\cdot}16$; and the mean value of P is $325 - 0{\cdot}27 \times 50 = 311{\cdot}5$ days, the unit adopted in the table being 50 days, by which therefore we multiply ν_1.

Assuming a linear relation between A and P, there are two appropriate forms for it, viz.

$$(A - 8^\circ{\cdot}16) = r \cdot \frac{\sigma_A}{\sigma_P}\left(\frac{P - 311}{50}\right) \text{ and } \left(\frac{P - 311}{50}\right) = r \cdot \frac{\sigma_P}{\sigma_A}(A - 8^\circ{\cdot}16),$$

and those new to correlation methods will notice with surprise that these are not convertible. To save repeating explanations already given in detail elsewhere, we need only remark that to find the most probable value of A when P is given is not the same as to find the most probable value of P when A is given. There is the same *kind* of difference (the analogy must not be taken too seriously) as between finding from the fact that a man is 25 the date of his probable death, and finding from the date of his death the probable date of his being 25: the latter case is clearly complicated by the possibility that he may never have reached 25 at all.

15. In astronomical examples it will frequently happen that one of the variables is better determined than the other, and hence one of the two alternative forms is indicated as preferable. Thus in our case P is well known; most of the periods are known within a day or two, some perhaps to fractions of a day; and since our unit in Table II. was 50 days, the precision is less than one-tenth of the unit. But A may be several degrees wrong, *i.e.* several

whole units. Hence it is natural to determine A from P, and the appropriate equation is

$$(A - 8^{\circ}\cdot16) = 0\cdot56 \times \frac{3\cdot22}{2\cdot00}\left(\frac{P - 311}{50}\right)$$

or

$$A = 2^{\circ}\cdot6 + \cdot018P,$$

P being now measured in *days*, and not in units of 50 days. The most probable values of A corresponding to different periods are thus:

For P =	100^{d}	200^{d}	300^{d}	400^{d}	500^{d}	600^{d}
A =	4°·4	6°·2	8°·0	9°·8	11°·6	13°·4

16. This single example will suffice to show the method of finding r in such cases. There are many other points to be considered, *e.g.* how to deal with cases where the relation is not linear, or where one of the quantities is not measured but only described. But it seems probable that in astronomy the evaluation of r by the above simple process will be the point most often occurring.

17. But before leaving the more general topic of correlation for the particular application of it we have in view, it is a matter of some interest to return now to the simple example given in § 2, and work it out as below.

TABLE IV.

The example given in § 2.

B \ A	5	6	7	8	9	
6	1 (2)	1 (1)	1			3
7		1	1	1		3
8			1	1 (1)	1 (2)	3
	1	2	3	2	1	9

A	Sum.	Factor.	ν_1	ν_2
5	1	−2	−2	4
6	2	−1	−2	2
7	3	0	...	...
8	2	+1	+2	2
9	1	+2	+2	4
Sums	9	...	0	12

B	Sum.	Factor.	ν_1	ν_2
6	3	−1	−3	3
7	3	0	...	...
8	3	+1	+3	3
Sums	9	...	0	6

Product S′(AB)

+3	0
0	+3

=6

$$\sigma_A = \sqrt{12} = 3\cdot46 \qquad \sigma_B = \sqrt{6} = 2\cdot45$$

$$r = \frac{6}{9 \times \sqrt{12} \times \sqrt{6}} = \cdot08.$$

This is a very low value, and on referring to Table III. we see that the probable error is greater than 0·2: so that the correlation of the quantities A and B may be regarded as unsupported. And yet at first sight there is a fair appearance of relationship. This shows the advantage of having a numerical test.

18. We proceed now to discuss the interpretation of this and similar results. And in the first place it is an obvious suggestion to inquire whether C is also related to P. In a precisely similar manner, which need not be given in detail, we find the correlation for C and P to be

$$r = \cdot 30 \pm \cdot 10,$$

and for the probable relationship

$$(C - 27\cdot 2) = \cdot 064(P - 311)$$

or

$$C = 7\cdot 3 + \cdot 064P,$$

where C and P are measured in days. But the value of r and its probable error show that the probability of a relationship is here more doubtful. This is quite possibly due to the much greater uncertainty of the values of C, which is more difficult to determine than A, for reasons which those who have discussed variable star observations will readily appreciate.

With this formula the values of C would be:

P =	100^d	200^d	300^d	400^d	500^d	600^d
C =	14	20	26	33	39	46

19. Now, having obtained the suggestion of correlation and two formulæ for A and C, I examined in detail some individual cases which had come under notice and which are given below. It will be seen that there is good confirmatory evidence, so far as it goes, of these relationships.

S Cassiopeiæ (Ch. No. 432).

20. In *Mem. R.A.S.*, lv., p. lxix, the Rousdon observations are compared with Chandler's formula in his 3rd Catalogue:

$$610\cdot 5E + 50 \sin (10^\circ E + 50^\circ).$$

From observations E = 17 to 21 (about) a period of 630·7 days is derived, differing sensibly from 606·2 given by the formula. It is remarked: "To obtain [630·7 days] from the formula we must alter the periodic term in some way, and especially we must increase the coefficient, for with the coefficient 50 the greatest interval between two consecutive maxima is the greatest value of

$$610\cdot 5 + 50[\sin \theta - \sin (\theta + 10^\circ)],$$

which is 610·5 + 100 sin 5° or 619·2 days. But we cannot discuss this point without reference to other observations. Chandler used maxima in 1843 and 1863–93, and any new formula must thus

agree with the old one at these periods. Tentatively we may suggest some such modification as

$$2401590 + 615E + 75 \sin (12^\circ E + 90^\circ).''$$

Chandler's revision gives

$$610{\cdot}5E + 37 \sin (15^\circ E + 59^\circ).$$

The value of the periodic term deduced from the period 610 days by the formulæ found is

$$47 \sin (13^\circ{\cdot}6\ E + ?).$$

The coefficient given by the formula is thus in accord with Chandler's first thought, and lies between his revised value and the value suggested very tentatively by the Rousdon observations. The mean of the three different suggested values, 75, 50, and 37, is 54; and the fact that the smallest of these, 37, was used in deducing the formula accounts for part of the difference between the formula value 47 and this mean.

It is perhaps worth noting that if the suggestion of $C = 75$ (made quite independently of the present investigation) is adopted instead of Chandler's $C = 37$, the value of r is raised from ·30 to ·43.

The argument 13°·6 given by the formula is between the value suggested by the Rousdon observations and Chandler's revision.

R Ursæ Majoris (Ch. No. 3825).

21. In *Mem. R.A.S.*, lv., p. lxxvi, the Rousdon observations are compared with Chandler's 3rd Catalogue formula

$$302{\cdot}1E + 15 \sin (10^\circ E + 190^\circ).$$

It is shown that the Rousdon observations indicate an error of 150° in the periodic term: so that the coefficient of E should be 8° instead of 10°. Chandler's revision gives

$$302{\cdot}1E + 11 \sin (8^\circ E + 238^\circ).$$

Pogson's observations of this star have since been very carefully discussed by Miss Blagg; and the outcome of this entirely independent discussion (which will shortly be published in *Mem. R.A.S.*, lviii., in the introduction to Pogson's observations) was to indicate a periodic term

$$40 \sin (8^\circ E + ?).$$

The formula gives for 302 days a term

$$27 \sin (8^\circ E + ?),$$

in which the coefficient is midway between Chandler's revision and the Pogson indication; and the argument in good accordance with both.

T Ursæ Majoris (Ch. No. 4511).

22. In *Mem. R.A.S.*, lv., p. lxxviii, the Rousdon observations are compared with Chandler's 3rd Catalogue formula

$$257{\cdot}2E + 20 \sin (9^\circ E + 90^\circ),$$

with the conclusion that the agreement is good. Chandler's revision gives no change. The correlation formulæ give for a period of 257 days a periodic term 24 sin (7° E + ?).

S Ursæ Majoris (Ch. No. 4557).

23. On p. lxxx the Rousdon observations are compared with Chandler's 3rd Catalogue formula

$$226{\cdot}1E + 43 \sin (5^\circ{\cdot}76\ E + 181^\circ{\cdot}5),$$

with the conclusion that "there is a fairly satisfactory accordance, though some correction to the formula would improve it."

Chandler's revision gives

$$226{\cdot}5E + 35 \sin (5^\circ{\cdot}4\ E + 194^\circ).$$

The correlation formulæ give for a period of 226·5 days a periodic term 22 sin (6°·5 E + ?), which suggests that Chandler's diminution of the coefficient has not been carried far enough, though in the right direction.

S Cygni (Ch. No. 7220).

24. On p. lxxxix the Rousdon observations are compared with Chandler's formula

$$322{\cdot}8E + 15 \sin (12^\circ E + 66^\circ).$$

It is remarked that the "periods at (the Rousdon) epoch would agree better if the coefficient of E in the periodic term were smaller, say 9° instead of 12°."

In Chandler's revision the periodic term is replaced by a secular term

$$323E + 0{\cdot}015E^2.$$

The correlation formulæ give for a period of 323 days

$$28 \sin (8^\circ E + ?),$$

so that the only suggestion made at the time of discussing the Rousdon observations is in the direction of better accordance with the formulæ.

R Sagittæ (Ch. No. 7257).

25. This star might have been included in Table I., but it was decided to draw the line as regards "long-period variables" at 100 days. The elements given in the revision are

$$70{\cdot}56E + 6{\cdot}5 \sin (2^\circ{\cdot}25 + 47^\circ).$$

Extrapolating our formulæ, we get C = 12 and A = 3°·9. But if we decide that this star may be regarded as a long-period variable, we should include it in our table, and then the values of C and A would be found closer to those observed, since the star would have great weight. Indeed, the correlation for A and P is raised to $r = 0{\cdot}64 \pm {\cdot}07$, and the formula becomes

$$(A - 8^{\circ}{\cdot}1) = 1{\cdot}01\left(\frac{P - 305}{50}\right)$$

or

$$A = 1^{\circ}{\cdot}9 + {\cdot}020\,P,$$

which gives A = 3°·3 for P = 70.

The value of r for C and P is, however, not much improved, being raised from ·30 to ·32.

R Cassiopeiæ (Ch. No. 8600).

26. On p. xcii of the Rousdon Memoir, the observations are compared with Chandler's 3rd Catalogue formula,

$$429{\cdot}5E + 25 \sin (15^{\circ}\,E + 0^{\circ}),$$

and it is remarked :—

"The corrections to period shown by the different columns of the Rousdon observations are so consistent that it is difficult to believe that the mean result can be so erroneous as the formula would make it. If the formula were altered to

$$431E + 30 \sin (12^{\circ}\,E + 9^{\circ})$$

[there would be a certain improvement]; but it is of course impossible to alter the formula definitively without discussing other observations."

Chandler's revision, which appeared after the above words were in type, gives

$$431{\cdot}6E + 32 \sin (9^{\circ}\,E + 60^{\circ}).$$

The correlation formulæ give for the period 431 days a periodic term

$$35 \sin (10^{\circ}{\cdot}7\,E + ?),$$

showing that the improvements suggested by the Rousdon observations were in the right direction, though not sufficient in magnitude; and that Chandler's revision is in good accord with the formula.

S Delphini (Ch. No. 7431).

27. Chandler gives no indication of a periodic term in his "revision," printing the period as 277·5 days. In discussing Baxendell's observations (before the present correlation work had been undertaken at all), Miss Blagg found clear indications of a periodic term at which a preliminary guess of 9 sin ($7\frac{1}{2}^{\circ}$ E + ?) was made, the 9 being mere guess-work, but the $7\frac{1}{2}^{\circ}$ being indicated with

fair precision. The correlation formulæ give for a period 277·5 days $A = 7^{\circ}\cdot6$, which is close to the value guessed. For the coefficient C the formula gives 25 days, which is a good deal larger than the guessed 9; but on *calculating* the coefficient from the separate corrections to epoch the coefficient came out 28·3, which accords well with the formula. Although the calculation is of a rough kind, it seems worth while giving it here to show the kind of accordance that is at present obtainable.

TABLE IV.

Baxendell's *S Delphini*.

E	Observed Correction to Epoch.	Argument ($7\frac{1}{2}^{\circ}$ E+const.)	Sin (Arg).	Product.
I.	−12^{d}	− 26°	−·44	+ 5·3
II.	− 5	− 18½	−·32	+ 1·6
III.	+ 3	− 11	−·19	− 0·6
IV.	− 5	− 3½	−·09	+ 0·5
V.	+11	+ 4	+·07	+ 0·8
VI.	+20	+ 11½	+·20	+ 0·4
VII.	+21	+ 19	+·33	+ 6·9
VIII.	+ 6	+ 26½	+·45	+ 2·7
IX.	0	+ 34	+·56	0·0
X.	+23	+ 41½	+·66	+15·2
XI.	+38	+ 49	+·76	+28·9
XXIV.	+ 5	+146½	+·55	+ 2·8
XXV.	+ 3	+154	+·44	+ 1·3
XXVI.	+ 1	+161½	+·32	+ 0·3
XXVII.	+ 9	+169	+·19	+ 1·7
XXVIII.	+ 6	+176½	+·09	+ 0·5
XXIX.	−19	+184	−·07	+ 1·3
XXXII.	− 8	+206½	−·45	+ 3·6
XXXIII.	−27	+214	−·56	+15·1
XXXIV.	−13	+221½	−·66	+ 8·6
XXXV.	−29	+229	−·76	+22·0
			Sum	118·9

28. The first column gives the sequence of periods observed by Baxendell. There is a wide gap between XI. and XXIV. with no observations, and it will be seen from columns 3 and 4 that it was unfortunately just in this gap that the corrections to epoch would have been largest, and given us an accurate value. Still, the periods when the correction vanishes are fairly well determined, which is something. They have been assumed to lie midway

between IV. and V., and between XXVIII. and XXIX. This is only a rough assumption, but the material does not warrant refinements. The "observed correction to epoch" given in the second column had been deduced from the observations by Miss Blagg, using a purely numerical process,* before there was any idea of applying them in this way. In the next three columns the product by sin (arg) is deduced, and is almost uniformly positive. Dividing the sum 118·3 by $\Sigma \sin^2(\text{arg})$, which comes out 4·18, we get as above mentioned

$$C = 118{\cdot}3/4{\cdot}18 = 28{\cdot}3.$$

S Serpentis (Ch. No. 5501).

29. For a different reason it is necessary to call attention to the case of S Serpentis, period 369 days. Chandler gives C = 116, A = 4°, while our formulæ indicate C = 30, A = 9°. But it is apparently possible to satisfy the observations in a different way, which assigns to C and A nearly the values of the formula. This investigation is given in a separate note following this paper.

30. What are now wanted are better determinations of C and A for stars with long and short periods. Too much depends on S Cassiopeiæ (610 days) and R Virginis (145 days), in the present state of our knowledge of them. "The dog is wagged by the tail." But it is the special value of an investigation of this kind that attention is directed to special needs which may guide observers in selection. The following stars, for instance, seem worth special attention from the northern observers:—

Short Periods.			Long Periods.		
Name.	No.	P	Name.	No.	P
Z Aquilæ	7260	127^d	S Cassiop.	432	610^d
W Cygni	7754	131	V Delphini	7458	540
R Vulpec.	7560	137	S Cephei	7779	486
R Virginis	4521	145	W Aquilæ	6900	480
S Aquilæ	7242	147	U Cygni	7299	461
V Capric.	7571	157	Z Sagittarii	6923	452
T Herculis	6512	165	R Leporis	1771	436
R Ceti	845	167	T Draco	6449	426
V Tauri	1717	170	R Cygni	7045	426
R Arietis	782	187	R Hydræ	4826	425

31. Returning now to Table I., in the seventh column is given $M - m$, the interval between maximum and the preceding minimum according to Chandler's revision. For most variables this is less than half the period, so that the quantity

$$aP = 2(M - m) - P$$

* The process can readily be inferred from the description on pp. 66 to 68 of *Mem. R.A.S.*, vol. lv., though this description is adapted to a period, *i.e.* a difference of epochs, instead of to the epoch itself.

shown in the last column is negative. The quantity α was used in *M.N.*, lxvii. p. 350, in connection with the classification of these variables according to their light curves, and it is interesting to see whether the type of light curve is related to the quantities C and A we have been discussing. The value of r for C and αP comes out ·011, showing that there is complete independence. In considering the type of light curve it was early noticed that it seemed to be independent of the period; and it is not surprising therefore that C and A, which seem to depend on the period, should be independent of the type of curve. The relation between A and αP, and those between α alone and C and A, might be actually worked out, but it seems improbable that they will yield any result of interest.

32. Column 6 of Table I., headed "Phase," represents a much more speculative investigation, but one which it seemed just worth undertaking. It seemed quite possible that C and A, or at any rate A, might be *directly proportional to P*. The limits of possible error are such that there is nothing extravagant in this supposition, which would be represented by the equation

$$(A - 8^\circ{\cdot}2) = k\left(\frac{P - 311}{50}\right),$$

where
$$k = \frac{8{\cdot}2 \times 50}{311} = 1{\cdot}32,$$

so that
$$A = {\cdot}0264P.$$

We may compare the resulting values of A with those of § 15.

P =	100^d	200^d	300^d	400^d	500^d	600^d
A by old equation =	4°·4	6°·2	8°·0	9°·8	11°·6	13°·4
A by new equation =	2·6	5·2	7·8	10·4	13·0	15·6

33. Now what makes this hypothesis of special interest is the fact that the cycle in which the term $C \sin(A^\circ E + \text{const.})$ is completed is 360/A periods or 360P/A days, and if P/A is constant we are confronted with the idea of a universal cycle controlling all these stars! The notion seems impossible, and yet there might be a simple explanation of it—the cause might be rooted in us, and not in the stars. It is difficult to see how any feature of the Earth or of the solar system could affect differential comparisons of distant stars, but though difficult it may not be impossible. The period indicated is 360/·0264 days or 37·4 years. If there is a phenomenon of this kind originating with the observer, it might have the same phase for all stars, or at least the phase might depend on the star's position. Hence the phases given by Chandler were reduced to the common epoch 240 5000, and are shown in the sixth column of Table I. They certainly tend to no one value more than another, and a few experiments on their being related to a star's position gave no encouragement.

34. Another point may be mentioned. Whether there is

actually a single periodicity or not, the cycle for the periodic term is about of the same order for different stars. The value of 360P/A in years given by the equation of § 15 is

P =	100^d	200^d	300^d	400^d	500^d	600^d
Cycle =	22^y	32^y	37^y	40^y	43^y	46^y

Now if its coefficient also remains of the same order of magnitude, the maximum gradient of the disturbing cause (whatever it may be) may be nearly constant. Suppose it constant for simplicity. Then the displacement of maximum of a variable is due to the addition to the term

$$y_1 = f \cos 2\pi t/\mathrm{P}$$

(which represents the ordinary variation near maximum) of a term

$$y_2 = kt,$$

where k may be nearly the same for all stars. The time of maximum will now be when

$$0 = \frac{dy_1}{dt} + \frac{dy_2}{dt} = -\frac{2\pi}{\mathrm{P}} f . \sin \frac{2\pi t}{\mathrm{P}} + k,$$

or, since t is small,

$$t = \frac{k\mathrm{P}^2}{4\pi^2 . f}$$

Now f represents the range of variation, and is not very different for the stars in Table I., being usually about 5 magnitudes. For some stars the range is as low as 3, and for others it is as great as 7; and in a complete investigation the influence of this range must undoubtedly be considered. But for the present we will consider f constant. It follows that C, which is the maximum alteration of epoch, would be proportional to P^2 rather than to P. Hence it is worth inquiring whether the correlation of $\sqrt{\mathrm{C}}$ with P is better than that of C with P. On working it out, the result came $r = 0{\cdot}34$, which is indeed greater than the $r = 0{\cdot}30$ previously found, but not much. It would seem that for the present our best work can be done in getting improved values of C and A for individual stars, especially those above given, both by making new observations and by collecting and carefully discussing those already made.

SUMMARY.

§§ 1– 3. Introductory.

§§ 4– 5. Tabulation of Chandler's periodic "inequalities."

§§ 6–17. An example of working out the "correlation" r between two quantities A and P. The result found is $r = 0{\cdot}56 \pm 0{\cdot}08$, which is worth serious attention. The inference is that the argument A° in Chandler's periodic inequalities can be deduced from the period P days by the following probable formula:—

$$\mathrm{A} = 8^\circ{\cdot}2 + {\cdot}018\mathrm{P}.$$

§ 13 includes a table (III.) for the probable error of r.

§ 18. For C (the coefficient of the periodic inequalities) and P we find $r = 0{\cdot}30 \pm 0{\cdot}10$, which is not so conclusive. The relation would be

$$C = 7{\cdot}3 + {\cdot}064P.$$

§§ 19–29. Examination of 8 stars in detail, for which special information was available. The result is favourable. Remarks on S Serpentis, which seems to be exceptional.

§ 30. List of stars needing special attention.

§ 31. The type of light curve seems to be independent of C and A.

§§ 32–34. Possibility of a single periodic cause affecting all stars considered, but not supported.

Note on the Period of S Serpentis. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The formula given for the maxima of S Serpentis (No. 5501) by Chandler in his Revision of the 3rd Catalogue (*A.J.*, No. 553) is

$$2388724 + 368{\cdot}5\ E + 116.\sin(4^\circ E + 62^\circ).$$

The periodic term in this formula attracted attention by considerable divergence from the value suggested by the preceding paper. The coefficient 116 days is much too large, and the argument 4° E is too small. With a view to seeing whether it was well established, or whether perhaps some other formula would fit the observations, inquiry was made of Professor Müller, of Potsdam, who very kindly sent a complete list of observed maxima, with full references; adding a comparison with Chandler's formula, the remark that it did not fit more recent observations, and a suggestion of his own for improving it, which modified both the coefficient 116 and the argument 4° E in the right direction. But this suggestion does not fit the observation of Lalande, and reasons will be given below why it is probably too early to suggest a completely satisfactory formula, in spite of the fact that Lalande's observation was made in 1794. Hence it seems unnecessary to reproduce here the full details, which will doubtless appear in the great work of reference for variable stars now being prepared by the German Committee of which Dr. Müller is a member.

2. For our present purpose the observations are sufficiently represented by the dates for every fifth maximum shown in the second column of Table I. The numeration in the first column is that of Chandler. In the third column are given the intervals, and we have to decide how to interpret Lalande's observation, which may belong to any one of the epochs −35, −34, or −33. Chandler takes −33; and this gives an average interval of 1876 days for 5 periods, extending over 7 × 5 periods in all. This was quite a possible interpretation before the modern observations (represented

by epoch 76) were made. Up to epoch 71 the interval was increasing, and might have gone on doing so. But later observations have made this interpretation impossible. The maximum interval is clearly past at epoch 71. We have only a single entry in the table to prove this, but the entry stands for 5 well-determined maxima in which 5 separate observers were concerned: 3 of them observed epoch 74, 2 of them epoch 75, and epochs 76, 77, 78 were all observed. The mean errors of Chandler's formula are −20, −31, −27, −44, −43 days; and it seems clear that the formula no longer holds.

TABLE I.

E.	Max.	Int.	A+365·1 E.	·080 × (E − 36)².	O − C₁.	−40 sin. (7°·2 E + 281°).	O − C₂.
−35	2376442		2376018	403	+21	−24	− 3
or −34		7(1772)	383	392			
or −33		or 7(1824)	748	381			
		or 7(1876)					
1	2389210		2389161	98	−49	+38	−11
6	—	3(1835)	—	—	—	—	
11	—		—	—	—	—	
16	2394714		2394638	32	+44	−24	+20
		1813					
21	6527		6463	18	+46	−38	+ 8
		1807					
26	8334		8289	8	+37	−38	− 1
		1798					
31	2400132		2400114	2	+16	−24	− 8
		1802					
36	1934		1940	0	− 6	0	− 6
		1809					
41	3743		3765	2	−24	+24	0
		1818					
46	5561		5591	8	−38	+38	0
		1834					
51	7395		7416	18	−39	+38	− 1
		1853					
56	9248		9242	32	−26	+24	− 2
		1874					
61	2411122		2411067	50	+ 5	0	+ 5
		1860					
66	2982		2893	72	+17	−24	− 7
		1873					
71	4855		4718	98	+39	−38	+ 1
		1854					
76	6709		6544	128	+37	−38	− 1
81	(to come)		8369	162		−24	
86	(,,)		2420195	200		0	

On the other hand, if we alter Lalande's observation by one whole period, we get an average interval of 1824 days for 5 periods. This means that a maximum interval occurs between epoch 1 and 16, where the average value is 1835 days. But this maximum is much less than that between epochs 61 and 76, where the average value is 1869 days.

3. The conclusion seems to be that we cannot satisfy the observations by a *single* inequality, however we choose the coefficient and argument, if Lalande's observation is to be accepted. But this is, after all, no new thing. Chandler gives several cases of the kind, and others may be found in the future. For instance, he gives for R Hydræ (No. 4826),

$$2411931{\cdot}0 + 425{\cdot}15\ E - 0{\cdot}36\ E^2 + 15 \sin (7^\circ{\cdot}5\ E + 202^\circ).$$

If we admit an additional term in E^2 (which means that the period is changing steadily), we can fairly satisfy the observations. The approximate value of the coefficient of E^2 is indicated by the fact just quoted, viz. that the average interval near the maximum at epoch 9 is 1835 days (average period 367 days), while that at epoch 69 is 1869 days (average period 374 days). Thus the period changes 7 days in 60 periods, or the coefficient of E^2 is about ·06.

4. But anyone who has had experience of such work knows that such rough approximations are followed by a series of "trials and errors"; and by experiment a better value for the coefficient was found to be ·080, as indicated in the fifth column of Table I. The accompanying value of the period (for $E=36$) is 365·1 days; and starting with an arbitrary epoch, this is represented in the fourth column.

5. The values of O—C are shown in the sixth column, from which it is seen that an inequality of much shorter period than Chandler's will satisfy them approximately if Lalande's observation be referred, not to $E=-33$ or even $E=-34$, but to $E=-35$. The numerical values assumed are

$$+40 \sin (7^\circ{\cdot}2\ E + 281^\circ),$$

which are in much better agreement with the values assigned by the correlation formulæ of the preceding paper, viz.

$$+30 \sin (9^\circ{\cdot}0\ E + ?),$$

than Chandler's expression,

$$116 \sin (4^\circ\ E + 62^\circ).$$

6. The final residuals are shown in the column $O—C_2$. We must wait for confirmation of the considerable assumption already made; and to this end the maxima ensuing in the next few years are predicted as—

	J.D.	
79	241 7817	= 1907 Aug. 29
80	241 8186	1908 Sept. 1
81	241 8555	1909 Sept. 5
82	241 8923	1910 Sept. 8
83	241 9291	1911 Sept. 11
84	241 9659	1912 Sept. 13
85	242 0027	1913 Sept. 16
86	242 0395	1914 Sept. 19

7. The complete formula is thus—

$$2388796 + 365{\cdot}1\,E + {\cdot}080\,(E-36)^2 + 40\sin(7^\circ{\cdot}2E + 281^\circ)$$
$$= 2388900 + 359{\cdot}3\,E + {\cdot}080\,E^2 + 40\sin(7^\circ{\cdot}2\,E + 281^\circ);$$

but the term ·080 E^2 is of a provisional character, and probably represents the present value of a term of long period.

8. The main interest of this inquiry is the result that even in such an exceptional case as that of S Serpentis it is found possible to obtain fair accordance with the suggested formula by a simple supposition of a kind already familiar in other cases; while it seems certain that the discrepant formula is not correct. Possibly in other cases of discordance similar explanations may be found.

On the Orbit of β 416. By J. Voûte.

(*Communicated by W. Bowyer.*)

The periods calculated for this interesting binary differ very much. The last, that of Doberck, with a period of forty-six years, was based on the measures up to 1903, and his ephemeris gives, with the recent observations, nearly constant negative differences. In the present instance the measures published up to 1906 have been used. In the first place, all the position-angles and distances are brought up to the epoch 1910·0; then 6 normal-places are formed with position-angles and distances. These normal-places are formed out of the groups marked by square brackets. First a normal-place is formed for every year with weights equal to the number of nights observed by the different observers, and then the normal-place of a group with weights equal to the number of years.

Normal-places.

1877·23	228°·77	″1·79	1898·14	305°·37	″1·85
1889·89	129·19	0·98	1901·77	291·35	2·11
1895·80	322·14	1·34	1905·60	280·08	2·21

The apparent orbit drawn through these normal-places gives, following the graphical method of Dr. Zwiers, the following elements:—

P = 41·47 years	a = 1″·86
T = 1891·45	Ω = 131°·0
n = −8·682	i = 49°·0
e = 0·552	ϖ = 64°·0

Motion retrograde.

Comparison of computed with observed places.

		θ_{1910}.	θ_{O-C}.	ρ_{O-C}.
		°	°	″
Burnham	1876·52	±239	± 5·8	±0·05
Cincinnati	77·53	222·4	− 5·9	0·00
Russell	77·64	224·2	− 4·0	−0·03
Burnham	88·72	147·4	+ 6·4	+0·69
,,	89·42	134·0	+ 3·7	+0·12
Pollock	89·63	131·8	+ 1·8	−0·08
Sellars	90·60	121·9	+12·8	+0·01
Burnham	91·53	81·2	+ 4·5	−0·05
,,	92·38	24·3	−10·3	+0·05
Sellars	93·55	348·2	− 6·9	−0·06
Barnard	94·56	334·4	− 1·1	+0·25
Sellars	94·57	331·7	− 3·7	−0·11
Comstock	95·60	321·6	− 0·9	−0·45
Sellars	95·62	322·1	− 0·3	+0·07
See	95·74	321·3	+ 0·1	−0·08
Aitken	96·44	314·7	− 1·6	−0·07
Scott	96·46	315·3	− 0·9	+0·09
Sellars	96·59	315·7	+ 0·4	+0·33
Comstock	96·60	314·8	− 0·5	−0·25
See	97·39	306·9	− 3·2	−0·01
Aitken	97·46	309·0	− 0·7	+0·29
Scott	97·47	308·7	− 0·9	+0·09
Lehman	97·52	310·1	+ 0·8	+0·13
Aitken	98·35	303·7	− 1·5	+0·14
,,	99·38	300·1	− 0·5	−0·12
Doolittle	99·42	299·0	− 1·5	−0·18
Aitken	1900·49	295·5	− 0·9	−0·11
Tebbutt	1900·42	298·9	+2·5	−0·07
Lunt	1900·53	298·9	+2·6	+0·14
Doolittle	1900·60	294·1	− 1·9	−0·02
Doberck	1901·76	291·3	− 0·4	−0·05
Innes	1902·36	292·0	+2·4	+0·14
Aitken	1902·44	288·8	−0·5	−0·29
Tebbutt	1902·49	289·9	+0·8	+0·38
Aitken	1903·56	286·1	+0·5	−0·13
Doolittle	1903·62	283·3	− 1·1	−0·17
Scott	1903·66	287·1	+ 1·9	+0·05
Aitken	1904·63	282·8	+ 1·1	−0·18
,,	1905·63	280·1	+ 1·2	+0·08
Tebbutt	1905·62	279·3	+0·2	+0·16
Doberck	1905·66	281·5	+2·5	−0·20
Aitken	1906·46	276·9	+0·5	+0·08
Tebbutt	1907·57	274·2	+0·7	−0·12
Scott	1907·56	273·0	−0·5	+0·09

The table shows us that the differences θ_{O-C}, though small, show some period, but a correction of the elements does not

improve them. Better material for correction may be obtained in the next few years.

Ephemeris.

1908·5	270°·3	2″·20	1912·5	256°·4	2″·08
1909·5	267°·0	2·18	1913·5	252°·6	2·04
1910·5	263°·6	2·15	1914·5	248°·6	2·00
1911·5	260°·1	2·12	1915·5	244°·4	1·96

Orbital Elements of Double Stars. By T. J. J. See, Ph.D.

The following orbits have been communicated by Dr. See. The material on which they are based is for the most part to be found in Burnham's *General Catalogue.*

Σ 2.

R.A. $0^h\ 4^m$ Dec. $+79°\ 10'$

Discovered by W. Struve in 1828. Always a close pair, and measures are therefore difficult with small or moderate apertures. An orbit was found by Glasenapp using measures to 1893. It was again measured by Biesbroeck in 1904.

P =	166·0 years	Ω =	168°·3
T =	1894·50	ι =	72°·3
e =	0·40	λ =	0°·0
a =	0″·65		

β 524 = 20 Persei.

R.A. $2^h\ 47^m$ Dec. $+37°\ 55'$

The principal component of the wide pair Σ 318. It was discovered by Burnham in 1878 and is always difficult. It is assumed that Burnham's angle in 1878 should be altered by 180°. All the observations are represented except the distance 0″·22 by Lewis in 1896. It is not probable that the orbit given is more than a good approximation.

P =	36 years	Ω	indeterminate
T =	1897·0	ι =	0°·0
e =	0·75	λ =	149°·6 = Periastron
a =	0″·16		

Σ 483.

R.A. $3^h\ 57^m$ Dec. $+39^\circ\ 14'$

Discovered by W. Struve in 1830. There are in all 20 measures down to 1904, and in 1906 it appeared single to Burnham in the 40-inch. Early observations are important as soon as separation occurs.

P =	135·5 years	Ω =	23°·1
T =	1907·75	ι =	68·0
e =	0·786	λ =	213·4
a =	1″·77		

β 883.

R.A. $4^h\ 46^m$ Dec. $+10^\circ\ 54'$

Discovered by Burnham in 1879. A long series of measures is available, and Professor Aitken has communicated those of 1907 and 1908.

P =	16·6 years	Ω =	33°·5
T =	1907·33	ι =	35·7
e =	0·47	λ =	197·3
a =	0″·24		

β 581 AB.

R.A. $8^h\ 0^m$ Dec. $+12^\circ\ 35'$

The close pair of a triple system discovered by Burnham in 1878. The distance never exceeds 0″·5, and the object is difficult, but the measures are numerous and good. A few have been rejected. The comes is now passing periastron, and measures, though difficult, will be valuable.

P =	41·2 years	Ω =	133°·7
T =	1909·10	ι =	63·1
e =	0·53	λ =	278·1
a =	0″·61		

AC 5 = 8 Sextantis.

R.A. $9^h\ 48^m$ Dec. $-7^\circ\ 38'$

Discovered by Alvan Clark in 1852. The angles given by Dembowski, by the Cincinnati observers in 1878, and by Hall in 1879 have been reversed. There is a systematic run of negative

residuals in angle in the second quadrant, but the available material is not sufficient to justify a modification of the elements.

P =	68·75 years	Ω	indeterminate
T =	1882·0	ι =	0°·0
e =	0·60	λ =	240°·6 = Periastron
a =	0″·35		

Σ 3123.

R.A. $12^h\ 1^m$ Dec. $+69°\ 15'$

Discovered by W. Struve in 1831. The early measures are few and not very accordant. The early angles have been altered by 180°, as suggested by Lewis. With this proviso an arc of 300° has been described since Struve's measures.

P =	103·3 years	Ω =	56°·9
T =	1860·50	ι =	49°·7
e =	0·49	λ =	79°·1
a =	0″·32		

ξ Bootis = Σ 1888.

R.A. $14^h\ 47^m$ Dec. $+19°\ 31'$

Discovered by Herschel in 1780 and followed regularly. No orbit given as yet represents all the observations. The measures are given in Lewis' *Memoir* and in Burnham's *General Catalogue.*

P =	143·0 years	Ω =	171°·6
T =	1909·36	ι =	45°·8
e =	0·546	λ =	34°·1
a =	5″·02		

Dembowski 15.

R.A. $16^h\ 41^m$ Dec. $+43°\ 40'$

Discovered by Dembowski in 1869. The measures are good, and distributed over an arc of 192°.

P =	96·0 years	Ω =	145°·7
T =	1898·0	ι =	66°·05
e =	0·35	λ =	168°·3
a =	0″·84		

Σ 2438.

R.A. $18^h\ 56^m$ Dec. $+58°\ 5'$

Discovered by Herschel in 1782, and re-examined the next year. Re-measured by Sir John Herschel in 1830, and afterwards followed by several observers. As periastron was passed about 25 years ago, it will be many years before the present elements can be improved. The eccentricity is higher than that of any other known double star except Σ 2525.

P =	233·0 years	Ω	indeterminate
T =	1882·50	ι =	0°·0
e =	0·916	λ =	178°·3 = Periastron
a =	0″·53		

Secchi 2 = Σ 2481 BC.

R.A. $19^h\ 8^m$ Dec. $+38°\ 36'$

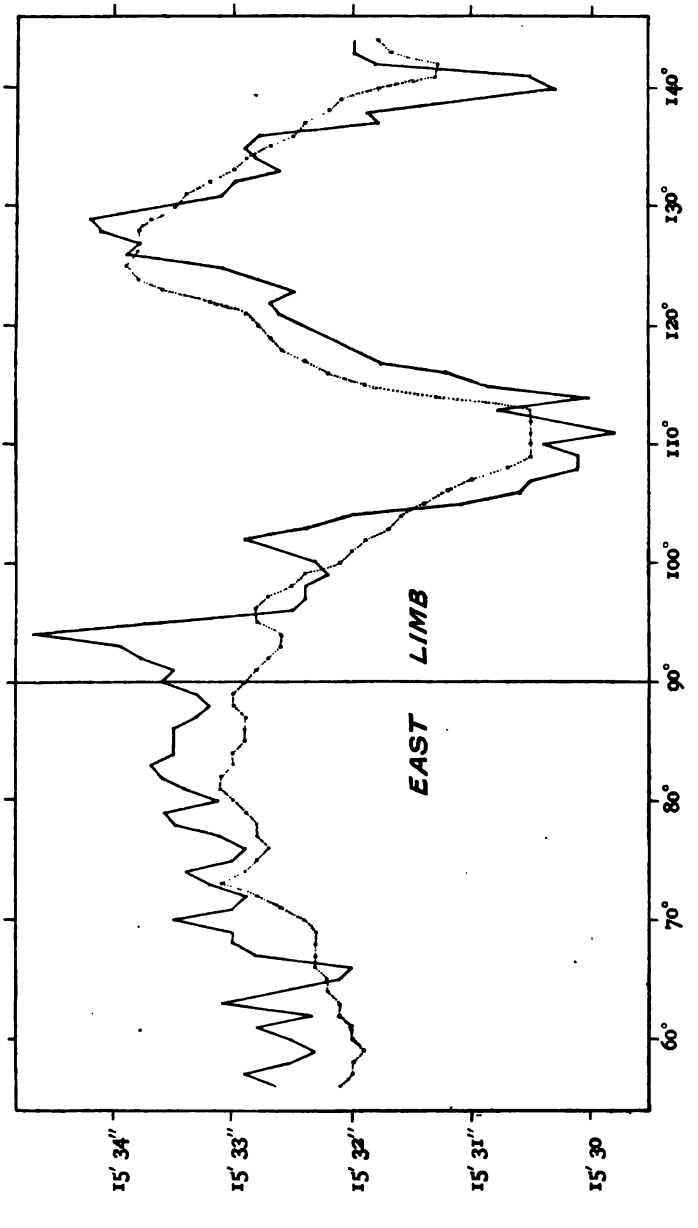
The companion of Σ 2481, found to be a close double by Secchi in 1858. Measured by Otto Struve in 1866, and by Schiaparelli in 1876; there are no measures between 1881 and 1897, since which time it has been followed by Aitken. Aitken's angles before 1901 are altered by 180°.

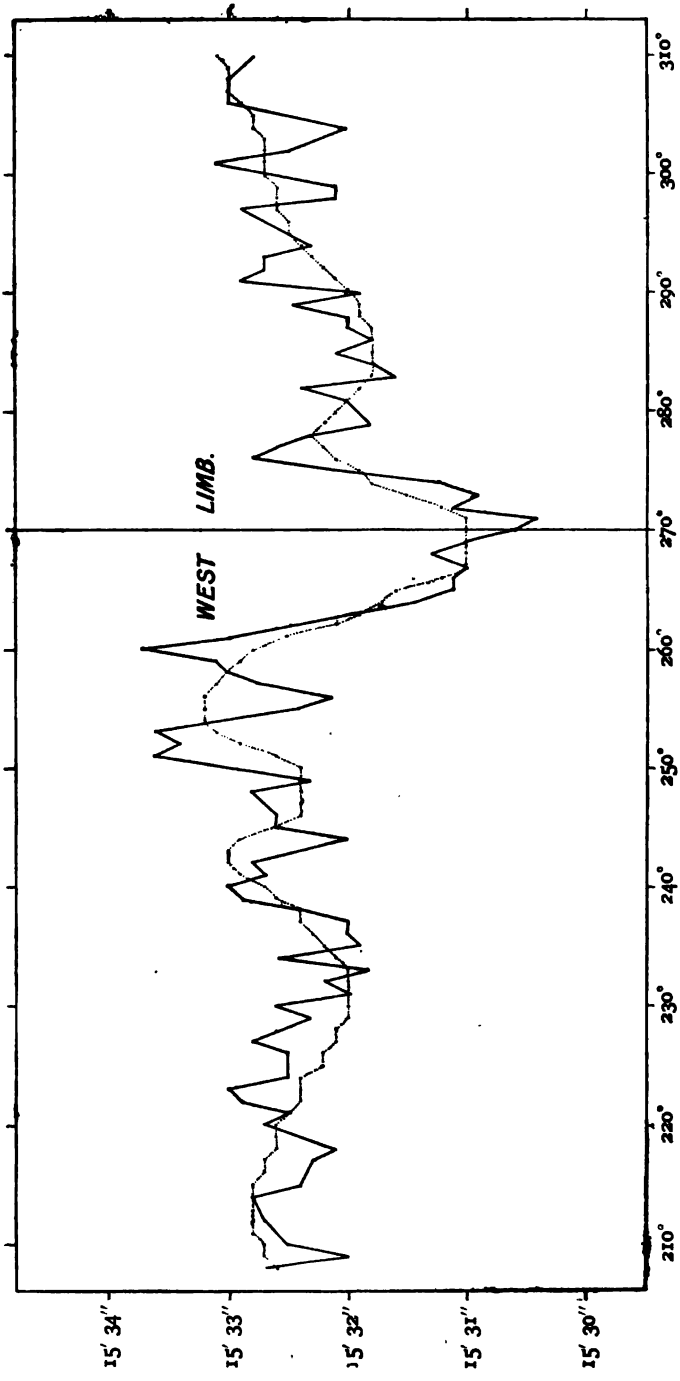
P =	16·0 years	Ω =	109°·7
T =	1902·25	ι =	63°·5
e =	0·68	λ =	218°·3
a =	0″·39		

The Radius of the Moon for libration −4°·5. By Walter Heath.

Dr. L. Struve, in his reduction of the occultations observed during the lunar eclipses of 1884 and 1888 (*Dorpat Observatory Publications*, vol. xx.), made a table of the values of the radius of the Moon's limb at different position-angles; a similar table was published by me last year (*Ast. Nach.*, 4206), including more observations, the geocentric libration in longitude being within the limits −3°·3 and −5°·3.

I have drawn the diagrams below in order to compare the figures in this table with the results obtained by Dr. Hayn from his micrometer measurements at the Leipzig Observatory (*Selenographische Koordinaten*, III Abhandlung, Leipzig, 1907). In the diagrams the figures at the side are for measuring the radius, and those at the bottom denote position-angles; the dotted line shows the values taken from Dr. Hayn's table, and the other line indicates the occultation results. The zero from which the occultation radius is measured is the Moon's centre as determined


15′ 34″
15′ 33″
15′ 32″
15′ 31″
15′ 30
60°
70°
80°
90°
100°
110°
120°
130°
140°
EAST LIMB


WEST LIMB.
15′ 34″
15′ 33″
15′ 32″
15′ 31″
15′ 30″
210°
220°
230°
240°
250°
260°
270°
280°
290°
300°
310°

by Dr. Struve from the occultations of 1884; and in order to compare the curves, I have supposed this point to be the zero for Dr. Hayn's measurements, and that the "mean level" which he adopts as a standard is at 15′ 32″·5 in the diagram.

The occultation curve is compounded of the curves representing the groups of results for 1884 and 1888 and some smaller groups. If the declination of the Moon's centre in respect of any group is corrected, the curve for that group will be turned about the east or west point of the limb; the east and west points for the eclipse of 1884 are at position-angles 114° and 294°, and those for the eclipse of 1888 are at 73° and 253°.

Uplands, Cobham, Surrey:
1908 *June* 7.

The Lunar Bright Rays. By H. G. Tomkins.

It is with some hesitation that I venture to add to the numerous theories which have at one time or another been put forward regarding the lunar bright rays, and my excuse must be the close analogy which exists between them and certain terrestrial phenomena, and the ease with which it can be applied to the Moon.

The present explanation is based, firstly, on the assumption that the configuration of the ray systems may be due to a cause separate from their albedo; secondly, that similar configurations appear to exist on the Earth; and thirdly, that a terrestrial analogy to the white material of the rays is also available.

Taking the albedo first, I am led to think that this can be explained on the supposition that the white material consists of the soluble salts of sodium and other minerals contained in the lunar crust. In the case of the Earth, these salts are brought to the surface by evaporation in a great many parts of the globe. The salts originally exist below the surface, either in the shape of beds of solid salt or salt-impregnated strata; and when they come into contact with the subsoil water, they rise to the surface in solution, and, on the drying up of the water, are left as a white efflorescence. In arid parts such as are found in North India and Persia, the efflorescences are very abundant, and a map of the tracts in the countries named exhibits a distinct tendency towards a radial formation, one of the strips extending about 800 miles towards Bengal from the Salt Range. In Germany also, though, owing to a temperate climate, efflorescences do not occur on a large scale, the brine springs, indicating the presence of the salts, have a distinctly radial configuration. In these cases the white material and the radial formations appear to occur together. This, however, need not always happen, and, as a matter of fact, in many countries the efflorescences do not seem to show any radial tendency, and, on the other hand, radial configurations sometimes appear without efflorescences. This is easily explained when the

origin of the radial configurations on the Earth is considered. They are referred to by several geologists of note, and in particular by Judd in his work on volcanoes, in which he quotes the case of the Lipari Islands, and states that a similar arrangement can be traced in many volcanic regions, especially those in which a great central volcano has existed. They are due to intrusion of liquid matter from below, which, by pushing up a centre of volcanic activity, causes the formation of radial fissures, and upheavals of strata round it. They are not always visible, however, on the surface.

Consequently, if such a system of upheavals occurred in the neighbourhood of saline deposits, the saline strata would be pushed upwards along with other strata, and this would bring them within the reach of evaporation at those places, and result in efflorescences on the surface which would follow the direction of the upheavals, and a radial formation would thus occur. Judd also mentions the presence of saline springs in some cases.

On the Moon, all the conditions of such a state of affairs exist. Careful observation very quickly shows that the ray systems differ greatly in configuration from one another—in fact, their prominent central formations and their albedo are about the only real points of resemblance, and this leads to the conclusion that their albedo may differ in origin from their configuration. As regards the latter, the central formation, the radial configuration of the streamers, the complex forms round Copernicus, and the differences of the systems between themselves, all point to an origin similar to that of volcanic centres on the Earth. Experiments made by Nasmyth and Carpenter and many others also testify to the probability of radial formations accompanying central upheavals or pressure.

Sir George Darwin has demonstrated the probability of the Moon having once been a part of the Earth, and it is reasonable, therefore, to assume that it has a similar constitution; in which case the existence of the salts of sodium and other minerals would be expected on the Moon—more especially sodium, which spectroscopic evidence shows to be present on the Sun and other heavenly bodies. Water does not exist in its free state on the lunar surface, but it is admitted that there is no reason why water should not have existed in the soil; and with the lunar climate, there would therefore have been cause for strong evaporation at some time, even if it does not still continue.

Consequently, any soluble salts in the lunar crust would come to the surface in the shape of efflorescences, and, owing to the absence of clouds or rain which might temporarily operate to reverse the process of evaporation, as on the Earth, the salts would rise more or less continuously, would remain permanently on the surface, and would follow the configurations of the ground, as already explained. If on the Earth water and rain were to disappear from the surface, a similar state of things would inevitably ensue; and in addition to the countries now affected, the areas occupied by the ocean would also have to be taken into account, with the enormous quantities of salts at present in solution.

The only objection of importance to the above explanation is the observed late appearance of the lunar rays at sunrise, and their early disappearance at sunset, which has been regarded by some as indicating physical change of the material of the rays. Such change would not, of course, occur with the salts. The facts, however, that the rays can easily be seen during a total eclipse of the Moon as well as on the dark part of the young Moon, that there are some exceptions to the phenomena, and that, if the site of a ray is watched, sunrise can be seen to extend to it exactly in the same manner as to other formations, are evidence against any physical change, and indicate that their invisibility is probably merely due to the fact that there is little or nothing except their albedo to notify their presence to the eye, and that they are there all the time, and only *seem* to appear when the illumination is sufficient to produce a contrast between the white material of which they are composed and the surrounding country.

I may also add that experiments made by me on this assumption entirely agree with Professor W. H. Pickering's observations that the appearance and disappearance of the rays take place when the solar altitude is from 5 to 10 degrees.

For purposes of the explanation of the rays, therefore, these phenomena can be put down to variations of illumination, and neglected.

Canterbury:
1908 *June* 3.

Observations of Daniel's Comet (*d* 1907) *at the Radcliffe Observatory, Oxford.*

(*Communicated by the Radcliffe Observer.*)

The following observations were made with the 10-inch Barclay Equatorial, using the Grubb wire-micrometer with power 200.

Observer—Mr. W. H. Robinson.

Date.	G.M.T.	Local Sidereal Time.	Comet *minus* Star, (Corrected for Refraction only). R.A.	Comet *minus* Star, (Corrected for Refraction only). N.P.D.	No. of Comps.	Apparent R.A. of Comet.	Corrections for Parallax in R.A. *p*.	Log. ($p \times \Delta$).	Apparent N.P.D. of Comet.	Corrections for Parallax in N.P.D. *q*.	Log. ($q \times \Delta$).	Ref.
1907.	h m s	h m s	m s	′ ″		h m s	s		° ′ ″	″		
July 30	14 40 54	23 6 9	−0 23·64	...	12	4 15 13·88	−0·48	9·5658	...	...	...	(*a*)
30	14 45 31	23 10 48	...	−1 32·2	15	...	...	...	74 15 45·4	−8·2	0·7964	(*b*)
Aug. 11	15 6 44	0 19 22	+1 46·77	+1 10·6	4	6 8 1·20	−0·48	9·5798	72 37 17·1	−8·1	0·8110	(*c*)
13	15 49 58	1 10 37	−2 34·35	...	2	6 26 19·65	−0·46	9·5721	...	...	...	(*d*)
13	15 55 2	1 15 42	...	+0 3·2	2	...	...	...	72 42 12·2	−7·6	0·7931	(*e*)
13	16 3 55	1 24 36	−4 2·97	−1 17·3	1	6 26 24·67	−0·45	9·5661	72 42 12·7	−7·5	0·7889	(*f*)
18	15 38 57	1 19 16	+0 34·79	−0 25·1	8	7 9 6·57	−0·43	9·5785	73 15 14·9	−7·4	0·8134	(*g*)
20	15 57 13	1 45 29	+1 15·29	...	5	7 25 24·06	−0·41	9·5762	...	...	...	(*h*)
20	15 58 35	1 46 51	...	−3 8·8	3	...	...	...	73 35 32·5	−7·1	0·8093	(*i*)
26	15 48 44	2 0 37	−5 50·15	−3 3·5	3	8 10 54·23	−0·37	9·5750	74 53 6·1	−6·6	0·8255	(*j*)
Sept. 8	16 16 24	3 19 38	+5 56·09	−3 33·4	1	9 37 25·18	−0·29	9·5672	78 40 40·6	−5·3	0·8335	(*k*)
8	16 41 24	3 44 42	+3 45·77	−7 4·4	4	9 37 32·08	−0·29	9·5683	78 40 59·2	−5·2	0·8258	(*l*)
11	16 41 14	3 56 21	−7 38·65	+7 44·9	2	9 55 19·48	−0·27	9·5671	79 40 32·0	−5·0	0·8293	(*m*)
11	16 47 46	4 2 54	−3 16·37	+1 35·7	1	9 55 20·33	−0·27	9·5671	79 40 39·8	−5·0	0·8276	(*n*)

Observer's Remarks.

(*a*) (*b*) The nucleus or condensation of comet (magnitude 8, and diameter 2″ or 3″) shows up on the preceding side of Coma, the radius of which is about 1′. For a long time after these observations were taken the comet continued to be easily seen in strong twilight.

(*c*) Comet conspicuous to the naked eye, magnitude 2. Tail precedes, 2° or 3° in length. In the 10-inch, the condensation, though large and diffused, is not brighter than magnitude 6·5.

(*d*, *e*, *f*) To the naked eye the comet is nearly as bright as γ Geminorum, magnitude 2. The tail can be traced to a distance approximately equal to that of Pollux from Castor. In the 10-inch the condensation appears large, diffused, and somewhat triangular in form, with a brightness much below that of the second comparison-star, magnitude 7·8 (*A.G.*), but superior to that of the first, magnitude 9·0.

(*g*) Condensation diffused and large, not brighter than the comparison-star, magnitude 8. To the naked eye, tail appeared 7° or 8° long, and nearly reaching to γ Geminorum.

(*h*, *i*) Images very diffused this morning. A nucleus occasionally shows up, slightly south of the observed centre of condensation, which spreads over many seconds of arc. The tail of the comet extends to about midway between the nucleus and γ Geminorum. Altitude small; twilight coming on.

(*j*) Observed through haze in intervals of cloud. The comet's tail is distinctly visible to the naked eye, notwithstanding low altitude, haze, moonlight, and twilight.

(*k*) Altitude of comet only 6°.

(*k*, *l*) Condensation large and diffused.

(*m*, *n*) Comet very low. The diffused condensation is slightly brighter than the second comparison-star, 7·2 magnitude, but much fainter than the first, 6·0 magnitude. The comet was just visible in the 10-inch at $17^h\ 10^m$ G.M.T.

On September 19, the comet was seen for a short interval at a low altitude; magnitude in the 10-inch, 7·5. Observations were prevented by the rapidly increasing twilight, and by the remoteness of the only available comparison-star, which preceded the comet $8^m\ 30^s$ in R.A.

Assumed Places of the Comparison-Stars.

Ref.	Mean R.A. 1907·0.	Reduction to Appar. R.A.	Mean N.P.D. 1907·0.	Reduction to Appar. N.P.D.	Authority.
	h m s	s	° ′ ″	″	
(*a*) (*b*)	4 15 37·21	+0·31	74 17 16·8	+0·8	Küstner, *Bonn Veröff.*, No. 4, p. 97.
(*c*)	6 6 14·32	+0·11	72 36 4·7	+1·8	Berlin, A (*A.G.*) 1973
(*d*, *e*)	6 28 53·94	+0·06	72 42 7·1	+1·8	Berlin, A (*A.G.*) 2231
(*f*)	6 30 27·59	+0·05	72 43 28·2	+1·8	Berlin, A (*A.G.*) 2252
(*g*)	7 8 31·76	+0·02	73 15 38·2	+1·8	Berlin, A (*A.G.*) 2702
(*h*) (*i*)	7 24 8·76	+0·01	73 38 39·6	+1·7	Berlin, A (*A.G.*) 2862
(*j*)	8 16 44·43	−0·05	74 56 8·1	+1·6	Berlin, A (*A.G.*) 3303
(*k*)	9 31 29·11	−0·02	78 44 12·8	+1·2	Leipzig, I (*A.G.*) 3809
(*l*)	9 33 46·34	−0·03	78 48 2·5	+1·1	Leipzig, I (*A.G.*) 3820
(*m*)	10 2 58·18	−0·05	79 32 46·1	+1·0	Leipzig, I (*A.G.*) 3952 (P. M.'s −0ˢ·0082 and +0″·038.
(*n*)	9 58 36·74	−0·04	79 39 3·1	+1·0	Leipzig, I (*A.G.*) 3933

In the computation of the parallaxes the adopted value of the Sun's mean horizontal parallax is 8″·80, and the geocentric distances Δ are taken from the *Astronomische Nachrichten*, Nos. 4191, 4194, and 4196.

Radcliffe Observatory, Oxford:
1908 *May* 30.

The Orbit of Jupiter's Eighth Satellite. By P. H. Cowell and A. D. Crommelin.

In this paper the time is measured in units of 16 days from 1908 January 12·0 Paris mean time. The astronomical unit of length is used.

Paris mean time has been used because the Connaissance des Temps gives both heliocentric and geocentric longitudes and latitudes of Jupiter referred to the same equinox, the true equinox of date.

With 16 days as the unit of time,

$$\log \text{mass of Sun} = 8{\cdot}87\ 943$$
$$\text{„ „ Jupiter} = 5{\cdot}85\ 934$$

It will be seen in the later parts of this paper that the Sun's disturbing force varies during the early part of 1908 between 10 and 6 per cent. of the attractive force of Jupiter, and is therefore by no means negligible. Moreover, if the attempt were made to express the co-ordinates of the satellite with the help of Delaunay's algebraical lunar theory, it would be found that Delaunay's quantities e, e' and m take such large arithmetical values that Delaunay's series are insufficient for the purpose. Hence it is

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best to calculate the orbit by quadratures, discarding the idea of an ellipse.

Let the jovicentric co-ordinates at time $n+t$ referred to three rectangular directions be

$$x_n + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5 + a_6t^6 +$$
$$y_n + b_1t + b_2t^2 + b_3t^3 + b_4t^4 + b_5t^5 + b_6t^6 +$$
$$z_n + c_1t + c_2t^2 + c_3t^3 + c_4t^4 + c_5t^5 + c_6t^6 +$$

then we have $x_{n+1} = 2x_n - x_{n-1} + 2a_2 + 2a_4 + 2a_6$.

Again, given the co-ordinates at any time, the accelerations can be calculated in accordance with Newton's law of gravitation. If X denote the x acceleration at time $n+t$,

$$X = 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3 + 30a_6t^4.$$

Hence $$X_n = 2a_2$$

and $$\tfrac{1}{12}(X_{n+1} - 2X_n + X_{n-1}) = 2a_4 + 5a_6 + \ \ldots$$

The left-hand side of this last equation may be described in words as one-twelfth of the second difference of X_n, and it will be denoted for brevity by P_n.

Hence we have $x_{n+1} = x_n + (x_n - x_{n-1}) + X_n + P_n$

with one error of $3a_6$, which we shall assume to be negligible.

Now let us assume that x_{n-2}, x_{n-1}, x_n have been calculated, and X_{n-2}, X_{n-1}, X_n, and therefore also P_{n-2}, P_{n-1}. We cannot at present calculate P_n, for this requires a knowledge of X_{n+1}, and therefore of $x_{n+1}, y_{n+1}, z_{n+1}$; but we may guess at the value of P_n with the help of the values of P_{n-2}, P_{n-1} (or for the first few values of n, where P_{n-2} has not been calculated, by trial and error); we then obtain a provisional value of x_{n+1} (and similarly y_{n+1}, z_{n+1}); hence we calculate X_{n+1}, and so obtain a revised value of P_n; the correction ΔP_n is then added to x_{n+1}, and in practice it is found that $X_{n+1}, Y_{n+1}, Z_{n+1}$ are unaffected.

We give a numerical application of the above method, taken from our calculations of a provisional orbit. In the illustration it is assumed that the orbit has already been calculated as far as $t=5$ (April 1); it is required to extend it to $t=6$ (April 17). For the sake of clearness, quantities already calculated are written without brackets, quantities guessed at are written in round brackets, and fresh quantities calculated accurately are enclosed in square brackets.

$x_5 = +\cdot0511\ 598$	$y_5 = -\cdot1691\ 203$	$z_5 = -\cdot0173\ 081$
$x_5 - x_4 = -\cdot0160\ 908$	$y_5 - y_4 = -\cdot0004\ 222$	$z_5 - z_4 = +\cdot0095\ 069$
$X_5 = -\cdot0005\ 159$	$Y_5 = +\cdot0020\ 967$	$z_5 = +\cdot0002\ 300$
$(P_5) = +\cdot0000\ 027$	$(Q_5) = +\cdot0000\ 015$	$(R_5) = -\cdot0000\ 020$

Hence by addition,

$$(x_6) = +\cdot 0345\ 558,\ (y_6) = -\cdot 1674\ 443,\ (z_6) = -\cdot 0075\ 732.$$

Hence we calculate X_6, Y_6, Z_6, and hence

$$[P_5] = +\cdot 0000\ 029,\ [Q_5] = +\cdot 000\ 014,\ [R_5] = -\cdot 0000\ 020.$$

Hence applying the small differences between the true and provisional values of P_5, Q_5, R_5,

$$[x_6] = +\cdot 0345\ 560,\ [y_6] = -\cdot 1674\ 444,\ [z_6] = -\cdot 0075\ 732.$$

The foregoing sufficiently explains how to calculate the motion from assumed initial positions; we now deal with the problem how to find an approximate orbit from the observations.

Retaining the previous notation, and interpolating a little among the observations, we shall assume as known the geocentric angular positions at three equidistant times $n-\tau$, n, and $n+\tau$.

Reasoning exactly as before we have

$$x_{n+\tau} - 2x_n + x_{n-\tau} = X_n\tau^2 \text{ with an error of } 2a_4\tau^4.$$

If we take the x direction at right angles to the geocentric directions of the satellite at times $n-\tau$ and $n+\tau$, then our ignorance of the geocentric distances at these times produces no uncertainty in the numerical values of $x_{n+\tau}$ and $x_{n-\tau}$; we therefore have one unknown quantity only occurring, viz. the geocentric distance at time n; and the above equation, by trial and error, will determine this quantity.

The corresponding equations for y and z then determine the geocentric distances on the first and third occasions.

We may then revise our solution so as to correct for the residuals $2a_4\tau^4$, etc.

In practice it is not necessary to take the x axis at right angles to the first and third geocentric distances; we shall then have to deal with three simultaneous equations for three unknown geocentric distances. The idea of resolution perpendicular to the outer geocentric distances is perhaps useful for forming a distinct mental conception of the process.

We shall not go into numerical details in connection with the method just sketched. It may be anticipated, however, that the solution is a dual one, or that two orbits which may be distinguished as direct and retrograde can be found to satisfy three observations. It unexpectedly turned out that the jovicentric distance on February 28 was less on the supposition of a direct orbit than for a retrograde orbit; but a very large eccentricity resulted from the former hypothesis. For this reason combined with the reasons stated in *M.N.*, lxviii. p. 457, the retrograde solution was chosen as the one to be worked up. It will be seen that the corresponding orbit closely follows the observations. Now the dual solution implies that there are two distinct ways of making the coefficients of the squares of the time agree with observation. Of these two solutions the retrograde solution makes the coefficients of the third and fourth

powers of the time also agree with observation. Of course it is not inconceivable that the direct solution might also give cube and fourth terms agreeing with observation, but it is *a priori* distinctly improbable. We have not, therefore, examined the direct solution (or the question of a possible planetary solution) with any minuteness. However closely a direct solution was found to fit the observations, it would be impossible at the present moment to reject the possibility that the retrograde solution was the right one.

The following table contains full details of our provisional orbit. The axes are drawn as follows:—x towards the first point of Aries for 1908·0, y towards longitude 90°, z towards the pole of the ecliptic

$x\ y\ z$ are the jovicentric co-ordinates of the satellite,
$x'\ y'\ z'$,, ,, ,, Sun
$X_1\ Y_1\ Z_1$ are the accelerations of Jupiter on the satellite
$X_2\ Y_2\ Z_2$,, ,, the Sun ,, ,,
$X_3\ Y_3\ Z_3$,, ,, the Sun on Jupiter reversed
X Y Z are the sums of the three preceding quantities.
P Q R are the twelfth parts of the second differences of X Y Z.

The values of P, Q, R for April 17 are enclosed in brackets to indicate that their values have been inferred, and not calculated: the values of x, y, z for May 3 are enclosed in brackets to indicate that they depend upon the preceding values of P, Q, R.

Table exhibiting Details of Calculation of provisional Orbit of J. VIII.

Jan. 28.	Feb. 13.	Feb. 29.	Mar. 16.	Apr. 1.	Apr. 17.	May 3.
+·1113290	+·0974232	+·0827044	+·0672506	+·0511598	+·0345560	(+·0175977)
−·1570880	−·1624748	−·1664110	−·1686981	−·1691203	−·1674444	(−·1634199)
−·0531070	−·0447827	−·0359985	−·0268150	−·0173081	−·0075732	(+·0022697)
+3·2999	+3·3949	+3·4882	+3·5797	+3·6694	+3·7572	+3·8432
−4·1496	−4·0791	−4·0065	−3·9318	−3·8551	−3·7764	−3·6960
−0·0587	−0·0611	−0·0635	−0·0658	−0·0681	+0·0704	−0·0727
−·0010107	−·0009553	−·0008821	−·0007863	−·0006613	−·0004987	
+·0014262	+·0015931	+·0017749	+·0019724	+·0021861	+·0024163	
+·0004821	+·0004391	+·0003839	+·0003135	+·0002237	+·0001093	
+·0018109	+·0018614	+·0019080	+·0019507	+·0019889	+·0020229	
−·0022675	−·0022109	−·0021515	−·020899	−·0020262	−·0019611	
−·0000032	−·0000092	−·0000154	−·0000217	−·0000279	−·0000341	
−·0016771	−·0017204	−·0017625	−·0018036	−·0018435	−·0018823	
+·0021090	+·0020670	+·0020243	+·0019809	−·0019368	+·0018920	
+·0000298	+·0000310	+·0000321	+·0000332	+·0000342	+·0000353	
−·0008769	−·0008143	−·0007366	−·0006392	−·0005159	−·0003581	
+·0012677	+·0014492	+·0016477	+·0018634	+·0020967	+·0023472	
+·0005087	+·0004609	+·0004006	+·0003250	+·0002300	+·0001105	
...	+·0000013	+·0000016	+·0000022	+·0000029	(+·0000036)	
...	+·0000014	+·0000014	+·0000015	+·0000014	(+·0000014)	
...	−·0000010	−·0000013	−·0000016	−·0000020	(−·0000025)	

From the co-ordinates x, y, z of the preceding table, the tabular differences of geocentric right ascension and declination between the satellite and Jupiter were obtained. We exhibit also the interpolation for every fourth day.

Table for Paris Mean Noon, giving excess of Satellite's tabular R.A. and Decl. over that of Jupiter.

Date. 1908.	R.A. s	Decl. ″	Date.	R.A. s	Decl. ″
Jan. 28	−7·29	−2588·7	Mar. 20	+112·71	−1419·6
Feb. 1	+1·19	2511·9	24	122·83	1332·4
5	9·72	2430·4	28	132·99	1247·7
9	18·34	2345·0	Apr. 1	143·15	1165·7
13	27·07	2256·4	5	153·27	1086·7
17	35·96	2165·3	9	163·32	1010·7
21	45·01	2072·2	13	173·25	937·8
25	54·22	1977·8	17	183·02	868·0
29	63·60	1883·0	21	192·60	801·4
Mar. 4	73·15	1788·3	25	201·95	737·8
8	82·85	1694·0	29	211·03	677·3
12	92·69	1602·3	May 3	+219·81	−619·7
16	+102·65	−1509·3			

The next table gives the dates of the observations reduced to Paris time with the light times subtracted; the place of observation, and the observed geocentric distances in R.A. and declination of the satellite from Jupiter; also the excess of the corresponding calculated co-ordinates over the observed.

Comparison of Provisional Orbit with Observation.

Date. 1908.	Hour. h	Place.	Observed R.A. J. VIII−$\mathcal{Y}$. s	Calculated minus Observed. s	Observed Decl. J. VIII−$\mathcal{Y}$. ″	Calculated minus Observed. ″
Jan. 27	12·2	Greenwich	− 8·29	−0·04	−2597·3	−0·6
Feb. 1	11·4	,,	+ 2·20	0·00	2501·1	−1·3
3	10·0	,,	6·37	−0·04	2464·2	+1·0
22	10·5	,,	48·38	−0·07	2038·1	−0·3
23	8·1	,,	50·31	+0·07	2015·7	−1·4
24	12·0	,,	53·15	−0·09	1989·2	−0·4
27	10·5	,,	59·94	−0·02	1919·7	−0·4
28	11·0	,,	62·34	−0·02	1895·2	−0·7
Mar. 3	9·3	Heidelberg	71·72	−0·03	1804·6	+1·8
8	19·9	Lick	84·72	+0·16	1674·4	−0·4

Comparison of Provisional Orbit with Observation—continued.

Date.	Hour.	Place.	Observed R.A. J. VIII—η.	Calculated minus Observed.	Observed Decl. J. VIII—η.	Calculated minus Observed.
1908.	h		s	s	"	"
Mar. 23	7·1	Heidelberg	121·12	−0·07	1347·1	−0·5
27	8·4	Greenwich	131·35	−0·01	1259·9	−1·4
31	8·8	,,	141·55	−0·01	1178·1	−0·4
Apr. 1	16·3	Lick	145·00	−0·13	1153·7	+1·6
3	9·3	Greenwich	149·33	−0·14	1120·2	+2·0
24	9·2	,,	201·03	−0·52	1745·7	−1·7
29	16·3	Lick	+213·08	−0·55	− 669·9	+2·6

It will be noticed that the accordance is exceedingly close, considering that this is a first approximation to the orbit, and that nothing whatever has been done in the way of modifying the orbit to produce a better accordance.

It remains to modify the orbit, and then to carry it forward to the end of the year, to assist in picking up the satellite at the next opposition of Jupiter.

For general interest, from the first three lines of the first table of this paper the following jovicentric longitudes, latitudes, and distances are computed, also L the longitude in the orbit reckoned from the ascending node on the ecliptic.

1908.	Long.	Lat.	L.	Distance.
Jan. 28	305°·32	−15°·42	328°·42	·1997
Feb. 13	300·95	−13·31	333·18	·1946
29	296·43	−10·96	338·17	·1893
Mar. 16	291·74	− 8·40	343·45	·1836
Apr. 1	287·83	− 5·59	349·09	·1775
17	281·66	− 2·54	355·09	·1711
May 3	276·15	+ 0·79	1·53	·1644

The instantaneous orbit is inclined 31°·14 to the ecliptic, and 29°·84 to Jupiter's orbit, and the instantaneous eccentricity has a value about $\frac{1}{3}$; the mean distance is about 0·1702, and the period about two years and two months.

The longitude of ascending node is 277°·46.

Observations of Jupiter's Sixth, Seventh, and Eighth Satellites from photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich, in 1907–8.

(*Communicated by the Astronomer Royal.*)

The sixth, seventh, and eighth satellites of Jupiter have been under observation during the recent opposition of Jupiter with the 30-inch reflector, photographs having been obtained whenever possible. In all 38 photographs of Satellite VI, 21 photographs of Satellite VII, and 13 photographs of Satellite VIII have been secured between 1907 November 13 and 1908 April 24.

The positions of the satellites have been measured on the photographs taken with the reflector with reference to faint comparison stars (of eleventh or twelfth magnitude), symmetrically distributed about the satellites. The positions of these faint comparison stars were then measured relatively to the reference stars (of eighth to ninth magnitude) on photographs (with two exposures of 20^m or 30^m) taken with the Astrographic 13-inch refractor. The field, sensibly free from distortion, being much larger with this telescope than with the reflector, from 24 to 39 reference stars were available on each plate. As Jupiter moved slowly, it was possible to make one reference plate serve for several photographs, which were each referred to it. The constants were determined in the usual manner, all the stars on the plate given in the Astronomische Gesellschaft catalogues being used for this purpose. Right ascensions and declinations of the satellites were then determined, and compared with the corrected tabular places of Jupiter after applying the corrections

$$\text{in R.A.} + 0^s{\cdot}110, \quad \text{and} \quad \text{Dec.} + 0''{\cdot}72$$

as determined below.

The G.M.T. is the arithmetical mean of the times of beginning and end of exposure.

SATELLITE VI.

Date and G.M.T.				Exposure.	Apparent R.A.			Apparent Dec.			Sat. VI—Jupiter. R.A.		Sat. VI—Jupiter. Dec.	
1907.	d	h	m	min.	h	m	s	°	′	″	m	s	′	″
Nov.	13	13	47·8	60	8	59	54·16	+17	27	6·2	−3	10·90	+ 3	40·9
,,	15	14	57·9	70	9	0	11·88	17	24	54·3	3	18·89	+ 2	52·2
Dec.	5	14	15·7	40	9	0	38·63	17	15	14·9	4	14·48	− 5	24·2
,,	5	16	1·7	140	9	0	38·31	17	15	15·7	4	14·52	5	25·6
,,	10	15	7·3	66	9	0	2·87	17	16	17·9	4	21·25	7	34·7
,,	13	13	14·8	13	8	59	34·29	17	17	34·1	4	23·89	8	49·6
,,	14	14	48·9	120	8	59	22·64	17	18	7·9	4	24·44	9	17·8

SATELLITE VI—*continued.*

Date and G.M.T.				Exposure.	Apparent R.A.			Apparent Dec.			Sat. VI—Jupiter. R.A.		Sat. VI—Jupiter. Dec.	
1908.	d	h	m	min.	h	m	s	°	′	″	m	s	′	″
Jan.	3	13	11·6	120	8	53	41·16	17	39	26·7	4	7·05	17	29·6
,,	9	12	41·9	16	8	51	20·86	17	49	11·6	3	51·03	19	36·9
,,	11	13	10·6	90	8	50	30·68	17	52	44·3	3	44·50	20	17·8
,,	11	14	12·8	15	8	59	29·61	17	52	48·9	3	44·35	20	18·6
,,	27	12	41·4	65	8	43	24·62	18	23	58·9	2	35·64	24	20·6
Feb.	1	11	52·1	112	8	41	9·34	18	34	15·4	2	9·24	25	2·8
,,	1	13	13·6	15	8	41	7·84	18	34	22·5	2	8·92	25	3·1
,,	3	10	27·3	80	8	40	17·18	18	38	17·2	1	58·46	25	13·3
,,	3	11	28·4	25	8	40	16·09	18	38	21·9	1	58·18	25	14·2
,,	6	11	35·4	30	8	38	56·58	18	44	32·9	1	41·12	25	26·0
,,	12	8	16·8	10	8	36	28·09	18	56	20·2	1	6·66	25	29·5
,,	12	9	25·1	10	8	36	26·91	18	56	28·1	1	6·39	25	27·2
,,	22	10	56·6	100	8	32	40·70	19	15	22·3	0	6·44	24	26·3
,,	23	8	33·8	42	8	32	22·58	19	16	57·2	−0	1·20	24	16·8
,,	24	12	24·9	80	8	31	59·88	19	18	58·7	+0	5·55	24	2·5
,,	27	10	59·4	70	8	31	5·65	19	23	53·5	0	22·59	23	24·9
,,	28	10	2·9	14	8	30	48·92	19	25	27·2	0	28·04	23	10·7
,,	28	11	28·6	80	8	30	47·89	19	25	33·0	0	28·36	23	9·6
,,	29	9	39·8	50	8	30	32·32	19	27	0·4	0	33·57	22	56·3
Mar.	9	8	40·4	15	8	28	28·12	19	39	48·8	1	21·33	20	8·2
,,	9	10	13·8	30	8	28	27·40	19	39	54·0	1	21·64	20	6·6
,,	9	10	55·6	9	8	28	27·03	19	39	56·0	1	22·74	20	6·1
,,	19	8	41·9	13	8	27	10·46	19	50	43·5	2	6·70	16	3·2
,,	21	9	48·7	120	8	27	2·82	19	52	29·2	2	14·75	15	6·9
,,	23	8	59·0	120	8	26	58·20	19	54	1·4	2	22·12	14	11·3
,,	27	8	54·3	43	8	26	56·53	19	56	38·1	2	35·44	12	16·0
,,	28	9	11·1	180	8	26	57·85	19	57	12·5	2	38·54	11	45·0
,,	31	9	16·4	165	8	27	5·78	19	58	36·7	2	46·98	10	14·5
Apr.	3	9	51·7	90	8	27	19·81	19	59	38·9	2	54·31	− 8	40·7
,,	20	9	21·0	18	8	30	24·95	19	58	4·9	3	12·34	+ 0	11·6
,,	24	9	43·5	100	8	31	32·95	+19	55	47·5	+3	10·54	+ 2	11·3

SATELLITE VII.

Date and G.M.T.				Exposure.	Apparent R.A.			Apparent Dec.			Sat. VII—Jupiter. R.A.		Sat. VII—Jupiter. Dec.	
1907.	d	h	m	min.	h	m	s	°	′	″	m	s	′	″
Dec.	5	16	1·7	140	9	1	56·44	+17	54	46·7	−2	56·39	+34	5·3
,,	10	15	7·3	66	9	1	14·38	17	56	27·6	3	9·74	32	35·0
,,	14	14	48·9	120	9	0	28·40	17	58	24·7	3	18·68	30	58·9
1908.														
Jan.	3	13	11·6	120	8	54	18·91	18	14	29·6	3	29·30	17	33·3
,,	11	13	10·6	90	8	51	3·83	18	22	51·1	3	11·35	+ 9	49·0
,,	27	12	41·4	65	8	44	9·55	18	40	54·4	1	50·71	− 7	25·1
Feb.	1	11	52·1	112	8	42	3·27	18	46	46·4	1	15·31	12	31·8
,,	3	10	27·3	80	8	41	14·97	18	49	10·9	1	0·67	14	19·6
,,	3	11	28·4	25	8	41	13·65	18	49	12·2	−1	0·61	14	23·9
,,	22	10	56·6	100	8	34	13·10	19	13	36·7	+1	25·96	26	11·9
,,	23	8	33·8	42	8	33	55·86	19	14	49·7	1	32·08	26	24·3
,,	24	12	24·9	80	8	33	34·27	19	16	22·5	1	39·94	26	38·7
,,	27	10	59·4	70	8	32	41·67	19	20	18·0	1	58·61	27	0·5
,,	28	11	28·6	80	8	32	24·20	19	21	37·4	2	4·67	27	5·1
,,	29	9	39·8	50	8	32	8·79	19	22	50·9	2	10·04	27	5·7
Mar.	21	9	48·7	120	8	28	12·66	19	46	39·3	3	24·59	20	56·8
,,	23	8	59·0	120	8	28	3·35	19	48	19·8	3	27·27	19	52·9
,,	27	8	54·3	43	8	27	51·66	19	51	12·5	3	30·57	17	41·6
,,	28	9	11·1	180	8	27	50·17	19	51	49·8	3	30·86	17	7·7
,,	31	9	16·4	165	8	27	50·10	19	53	30·2	3	31·30	15	20·9
Apr.	24	9	43·5	100	8	31	23·31	+19	53	8·3	+ 3	0·90	− 0	29·4

SATELLITE VIII.

Date and G.M.T.				Exposure.	Apparent R.A.			Apparent Dec.			Sat. VIII—Jupiter. R.A.		Sat. VIII—Jupiter. Dec.	
1908.	d	h	m	min.	h	m	s	°	′	″	m	s	′	″
Jan.	27	12	41·4	65	8	45	51·86	+18	5	1·4	− 0	8·40	−43	18·1
Feb.	1	11	52·1	112	8	43	20·67	18	17	36·3	+ 0	2·08	41	41·8
,,	3	10	27·3	80	8	42	21·90	18	22	25·7	0	6·26	41	4·9
,,	3	11	28·4	25	8	42	20·52	18	22	34·7	0	6·26	41	1·3
,,	22	10	56·6	100	8	33	35·41	19	5	49·8	0	48·27	33	58·9
,,	23	8	33·8	42	8	33	13·98	19	7	37·6	0	50 20	33	36·4
,,	24	12	24·9	80	8	32	47·37	19	9	51·3	0	53·04	33	9·9
,,	27	10	59·4	70	8	31	42·89	19	15	18·1	0	59·83	32	0·4
,,	28	11	28·6	80	8	31	21·76	19	17	6·7	1	2·23	31	35·9
Mar.	27	8	54·3	43	8	26	32·33	19	47	53·5	2	11·24	21	0·6
,,	31	9	16·4	165	8	26	40·24	19	49	12·3	2	21·44	19	38·8
Apr.	3	9	51·7	90	8	26	54·72	19	49	38·8	2	29·22	18	40·9
,,	24	9	43·5	100	8	31	43·33	+19	41	8·9	+3	20·92	−12	28·8

Errors of Tabular Place of Jupiter.

The positions of the satellites given above depend on the positions of certain stars taken from catalogues of the Astronomische Gesellschaft, and will be affected by the mean error in the places of these stars. To eliminate this it is necessary to determine the position of Jupiter referred to the same stars.

For this purpose a number of photographs of Jupiter were taken with the 26-inch refractor, using the occulting shutter. By this means good measurable images of Jupiter were obtained, together with sufficiently exposed images of the reference stars. The field of the 26-inch on a 16 cm. plate is one square degree, and six comparison stars in this area were measured together with Jupiter. The adopted places of these six stars were deduced from measures made on the Astrographic reference plates, the constants of the reference plates being, as mentioned above, derived from all the available stars in the Astronomische Gesellschaft catalogues. The places of the six stars, and therefore the deduced place of Jupiter, will be affected by the mean systematic error of all the catalogue stars on the reference plates, and will thus be comparable with the deduced places of the satellites.

Four photographs were selected for measurement. From two to four images of Jupiter and of each of the stars were measured on each plate in the Astrographic micrometer, and the following are the results obtained :—

Errors of R.A. Tabular Place of Jupiter.

	Tab.—Obs.	
	R.A.	Dec.
	s	"
Jan. 27	−·10	−0·6
Feb. 10	−·14	−0·8
,, 18	−·12	−0·6
May 6	−·08	−0·9
Mean	−·110	−0·72

Corrections for these mean errors have been applied to the tabular places of Jupiter, as already stated.

Royal Observatory, Greenwich:
1908 *June* 10.

Observations of the Satellite of Neptune, from photographs taken at the Royal Observatory, Greenwich, between 1907 *December* 10 *and* 1908 *March* 19.

(*Communicated by the Astronomer Royal.*)

The following measures of position-angle and distance of Neptune's satellite were made from photographs taken with the 26-inch refractor of the Thompson equatorial. The occulting shutter was used as in previous years. The photographs were taken by Messrs. Davidson, Edney, or Melotte, and were measured in a position-micrometer in direct and reversed positions by Messrs. Davidson and Melotte. The tabular positions with which comparison is made were computed from data given in the *Connaissance des Temps*, based on Dr. Hermann Struve's elements, the eccentricity of the orbit being neglected. A discussion of these residuals gives the following differences from Dr. Hermann Struve's elements in the sense Tabular—Observed:

$$du = -0^{\circ}\cdot69, \quad dN = -0^{\circ}\cdot54, \quad dI = +0^{\circ}\cdot11, \quad da = +0''\cdot244$$

giving for the epoch 1908·2

$$a = 16''\cdot027, \quad N = 188^{\circ}\cdot38, \quad I = 116^{\circ}\cdot84$$

Neptune and Satellite.

Position-angle and Distance from photographs taken with the 26-inch Refractor.

Date and G.M.T.					Position-angle.			Distance.		
					Obs.	Tab.	T—O.	Obs.	Tab.	T—O.
1907.	d	h	m	s	°	°	°	"	"	"
Dec.	10	13	47	48	241·68	243·09	+1·41	13·82	14·00	+0·18
	14	12	49	34	341·21	339·80	−1·41	12·04	11·93	−0·11
	14	13	15	59	338·86	338·36	−0·50	11·88	12·02	+0·14
1908.										
Jan.	3	11	35	18(*a*)	217·90	215·22	(−2·68)	11·16	11·86	(+0·70)
	4	11	13	40(*b*)	132·17	134·64	+2·47	13·35	13·97	+0·62
	4	11	39	10	133·26	133·58	+0·32	13·75	14·09	+0·34
	9	11	58	11	202·98	203·38	+0·40	10·87	11·37	+0·50
	11	11	27	45	82·99	83·75	+0·76	16·36	16·28	−0·08
	11	11	53	29	82·24	82·99	+0·75	16·30	16·22	−0·08
	27	9	49	5	179·03	178·08	−0·95	10·70	11·20	+0·50
	27	10	13	39	175·19	176·50	+1·31	11·34	11·23	−0·11
Feb.	3	9	15	21(*c*)	108·18	108·29	+0·11	16·25	16·42	+0·17
	6	10	2	5(*d*)	285·08	284·32	−0·76	16·23	16·62	+0·39
	6	10	35	58(*c*)	282·40	283·37	+0·97	16·29	16·67	+0·38
	10	10	0	38(*e*)	53·51	54·88	+1·37	13·08	13·35	+0·27

Position-angle and Distance—continued.

Date and G.M.T.				Position-angle.			Distance.		
				Obs.	Tab.	T—O.	Obs.	Tab.	T—O.
1908.	d	h	m	s	°	°	°	"	"	"
Feb.	10	10	23	22	52·81	53·86	+1·05	13·33	13·25	−0·08
	10	10	43	6	51·44	53·00	+1·56	13·16	13·17	+0·01
	18	9	18	23	275·88	275·48	−0·40	16·50	16·72	+0·22
	18	9	41	43	274·99	274·83	−0·16	16·66	16·72	+0·06
	18	10	5	24	272·87	274·15	+1·28	16·31	16·71	+0·40
Mar.	5	8	1	6	23·12	23·92	+0·80	11·20	11·24	+0·04
	5	8	33	32(*f*)	19·93	21·95	+2·02	11·11	11·18	+0·07
	14	9	9	29(*b*)	180·41	182·81	+2·40	10·55	10·91	+0·36
	14	9	49	10	177·86	180·31	+2·45	10·61	10·93	+0·32
	19	10	39	41	243·84	245·23	+1·39	13·97	14·21	+0·24
	19	11	4	17	243·49	244·28	+0·79	14·06	14·11	+0·05

(*a*) Very poor photograph.
(*b*) Very diffused.
(*c*) Very faint.
(*d*) Poor photograph, satellite faint.
(*e*) Satellite diffused.
(*f*) Satellite faint.

The satellite of Neptune has now been regularly observed at Greenwich during seven successive oppositions. The differences from Dr. Hermann Struve's elements found from the observations are as follows:—

Epoch.	No. of plates.	*du*.	*d*N.	*d*I.	*da*.	2 *e*. sin Q.	2 *e*. cos Q.
		°	°	°	"		
1902·1	51	−0·95	−0·42	−0·12	+0·087	+·0005	−·0004
1903·1	63	−0·67	−0·40	−0·30	+0·092	+·0005	+·0016
1904·1	51	−0·83	−0·72	−0·20	+0·033	−·0017	+·0002
1905·1	57	−0·82	−0·95	−0·37	+0·069	+·0038	+·0053
1906·1	59	−0·90	−1·17	+0·12	+0·085	−·0043	−·0027
1907·2	29	−1·02	−0·93	−0·20	−0·008	+·0024	+·0009
1908·2	25	−0·64	−0·44	+0·11	+0·244	−·0110	+·0032

The very small eccentricity has been treated as a quantity to be determined in these solutions; accordingly the figures for *du*, *d*N, *d*I, and *da* differ slightly from those published previously in the *Monthly Notices*, in which the eccentricity is taken as 0.

Royal Observatory, Greenwich:
1908 *June* 11.

On the Mathematical Theory of Two Star-drifts, and on the Systematic Motions of Zodiacal Stars.

By A. S. Eddington, B.A., M.Sc.

In a former paper,* the distribution of the proper motions of the stars was discussed from the point of view of the hypothesis of two drifts of stars. The method of analysis consisted in finding by trial and error the constants of two drifts which would lead to a distribution of proper motions (as regards direction) agreeing as nearly as possible with the observed distribution. There are evident disadvantages in a method of this kind; the difficulty of simultaneously adjusting five disposable constants, so as to obtain the best agreement with observation, is considerable; there is room for bias in deciding what is the best agreement with observation; and the method is inapplicable to the discussion of a series of proper motions of stars sparsely distributed over a large region of the sky, such as the Bradley proper motions.

It is the object of the first part of this paper (*Mathematical Theory*) to obtain from the observational data direct equations to determine the constants of the drifts. In the second part the theory is applied to the analysis of the proper motions of the Catalogue of Zodiacal Stars (*Astronomical Papers of the American Ephemeris*, vol. viii., part iii.); this is an example of a series of proper motions which, while sufficiently numerous to afford a good determination of the velocities of the drifts, are much too sparsely distributed to be analysed by the method of trial and error.

Mathematical Theory.

I have defined a "drift" to be a system of stars in which the motions of the individual stars relative to the mean of the system are haphazard; the whole system has in general a drift-motion relative to the Sun. For the sake of precision, the distribution of the haphazard or "peculiar motions" in the ideal drift is taken to be according to Maxwell's law.

In "Systematic Motions," p. 36, the law of distribution of the proper motions in the different directions was discussed for a system of stars forming a drift thus defined; the analysis may be briefly recapitulated here.

A small region of the sky is considered, and motions in the line of sight are neglected, so that the problem is two-dimensional.

Let the number of stars having component peculiar (linear) velocities between the limits (u, v) and $(u+du, v+dv)$ be

$$\frac{n_1 h^2}{\pi} e^{-h^2(u^2+v^2)} du dv \quad . \quad . \quad . \quad . \quad (1)$$

in accordance with Maxwell's law.

* "The Systematic Motions of the Stars," *Monthly Notices*, vol. lxvii. p. 34.

n_1 is the total number of stars of the drift.
h a constant inversely proportional to the mean peculiar speed of the stars.

Let v_1 be the velocity of the drift.
θ_1 the inclination of the direction of this drift-velocity to the axis of x.
r, θ the amount and direction of the resultant linear velocity of a star (*i.e.* drift velocity and peculiar velocity compounded).
$\rho\, d\theta$ the number of stars having proper motions in directions inclined to Ox between θ and $\theta + d\theta$.

Then $$u^2 + v^2 = r^2 + v_1^2 - 2v_1 r \cos(\theta - \theta_1)$$

$$du\,dv = r\,dr\,d\theta$$

and hence $$\rho d\theta = \frac{n_1 h^2}{\pi} d\theta \int_0^\infty e^{-h^2\{r^2 + v_1^2 - 2v_1 r\cos(\theta-\theta_1)\}} r\,dr \quad . \quad (2)$$

By means of the substitution

$$x = hr - hv_1 \cos(\theta - \theta_1)$$

this expression can easily be reduced to

$$\rho = \frac{n_1 h^2}{\pi} e^{-h^2 v_1^2} \left\{ \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^{\infty} e^{-x^2} dx \right\} \quad . \quad . \quad (3)$$

where $$\tau = hv_1 \cos(\theta - \theta_1)$$

or setting $$f(\tau) = \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^{\infty} e^{-x^2} dx$$

$$\rho = \frac{n_1}{\pi} e^{-h^2 v_1^2} f\big(hv_1 \cos(\theta - \theta_1)\big) \quad . \quad . \quad . \quad (4)$$

The function f was tabulated in "Systematic Motions," p. 36; a factor $\frac{2}{\sqrt{\pi}}$ was, however, inserted in defining it there, which it has been more convenient to drop now.

Consider now a series of proper motions of stars belonging to two drifts; distinguishing the corresponding constants by suffixes 1 and 2, we shall have

$$\rho = \frac{n_1}{\pi} e^{-h^2 v_1^2} f(hv_1 \cos\overline{\theta - \theta_1}) + \frac{n_2}{\pi} e^{-h^2 v_2^2} f(hv_2 \cos\overline{\theta - \theta_2}) \quad . \quad (5)$$

If the observed proper motions are considered to correspond to two drifts, a series of equations of condition will be obtained by giving various values to θ in (5), the corresponding values of ρ being derived from observation. From these the six unknowns, n_1, n_2, v_1, v_2, θ_1, θ_2 are to be derived; these, however, may be at

once reduced to five unknowns, for n_1+n_2 is equal to the total number of stars.

It is of course impossible, with non-linear equations of condition such as these, rigorously to determine the values of the constants which shall make the sum of the squares of the residuals a minimum. In order to obtain what may serve, as it were, for normal equations, I have expanded each side of equation (5) in a Fourier series, and have equated the first few coefficients on each side until sufficient equations were thus formed. It is clear that, at least if the observed distribution really does correspond to two drifts, a good agreement between the observed and theoretical distributions will have been obtained when the Fourier series representing them agree as far as the coefficients of $\cos 3\theta$ and $\sin 3\theta$, and it is not likely that the agreement could be much further improved. Discordances represented by differences in the coefficients of higher harmonics are likely to be of an accidental and unimportant character.

In practice, the process of equating Fourier coefficients is equivalent to equating the sums $\Sigma\cos\theta$, $\Sigma\sin\theta$, $\Sigma\cos 2\theta$, $\Sigma\sin 2\theta$, etc., for the observed proper motions, to the corresponding integrals for the theoretical distributions. These integrals are evaluated in the note at the end of this paper. We find there that for a single drift (n_1, v_1, θ_1),

$$\left.\begin{aligned}
\int_0^{2\pi}\rho\cos(\theta-\theta_1)d\theta &= n_1\frac{\sqrt{\pi}}{2}hv_1e^{-\frac{1}{4}h^2v_1^2}\left\{J_0(\iota\tfrac{1}{2}h^2v_1^2)-\iota J_1(\iota\tfrac{1}{2}h^2v_1^2)\right\}\\
&= n_1\,|\,C(hv_1)\,|\qquad \text{say}\\
\int_0^{2\pi}\rho\cos 2(\theta-\theta_1)d\theta &= n_1\left(1-\frac{1-e^{-h^2v_1^2}}{h^2v_1^2}\right)= n_1\,|\,D(hv_1)\,|\qquad \text{say}\\
\int_0^{2\pi}\rho\cos 3(\theta-\theta_1)d\theta &= n_1\frac{\sqrt{\pi}}{2}hv_1e^{-\frac{1}{4}h^2v_1^2}\left\{J_0(\iota\tfrac{1}{2}h^2v_1^2)-\iota J_1(\iota\tfrac{1}{2}h^2v_1^2)\left(1-\frac{4}{h^2v_1^2}\right)\right\}\\
&= n_1\,|\,E(hv_1)\,|\qquad \text{say}
\end{aligned}\right\}\quad(6)$$

and evidently from the symmetry of the drift

$$\int_0^{2\pi}\rho\sin(\theta-\theta_1)d\theta=\int_0^{2\pi}\rho\sin 2(\theta-\theta_1)d\theta=\int_0^{2\pi}\rho\sin 3(\theta-\theta_1)d\theta=0\quad .\quad .\quad (7)$$

A table of the functions here introduced is given later (see Table I.).

It is here convenient to introduce complex quantities into the analysis, which will be denoted always by capital letters.

Let
$$\left.\begin{aligned}
V_1&=v_1e^{\iota\theta_1}\\
C_1&=C(hV_1)=|\,C(hv_1)\,|\,e^{\iota\theta_1}\\
D_1&=D(hV_1)=|\,D(hv_1)\,|\,e^{2\iota\theta_1}\\
E_1&=E(hV_1)=|\,E(hv_1)\,|\,e^{3\iota\theta_1}
\end{aligned}\right\}\quad .\quad .\quad .\quad (8)$$

The moduli have already been defined by equations (6).

Still considering one drift only

$$\int_0^{2\pi}\rho e^{\iota\theta}d\theta = e^{\iota\theta_1}\int_0^{2\pi}\rho e^{\iota(\theta-\theta_1)}d\theta$$

$$= e^{\iota\theta_1}\int_0^{2\pi}\rho\cos(\theta-\theta_1)d\theta + \iota e^{\iota\theta_1}\int_0^{2\pi}\rho\sin(\theta-\theta_1)d\theta$$

$$= e^{\iota\theta_1}n_1|\,C(hv_1)\,|.$$

Thus
$$\int_0^{2\pi}\rho e^{\iota\theta}d\theta = n_1C_1$$

Similarly
$$\int_0^{2\pi}\rho e^{2\iota\theta}d\theta = n_1D_1$$
$$\int_0^{2\pi}\rho e^{3\iota\theta}d\theta = n_1E_1 \qquad \ldots\ldots (9)$$

Return now to the consideration of two drifts. Knowing for every star the apparent direction θ (measured from an arbitrary initial line) of its observed motion, we can calculate $\Sigma\cos\theta$, $\Sigma\sin\theta$, $\Sigma\cos 2\theta$, etc. for all the stars.

Let
$$L = \frac{1}{n}\Sigma e^{\iota\theta} = \frac{1}{n}\left(\Sigma\cos\theta + \iota\Sigma\sin\theta\right)$$

$$M = \frac{1}{n}\Sigma e^{2\iota\theta} = \frac{1}{n}\left(\Sigma\cos 2\theta + \iota\Sigma\sin 2\theta\right)$$

$$N = \frac{1}{n}\Sigma e^{3\iota\theta} = \frac{1}{n}\left(\Sigma\cos 3\theta + \iota\Sigma\sin 3\theta\right)$$

where n is the total number of stars discussed.

And let $\frac{n_1}{n} = \frac{1}{2}(1+a)$

so that $\frac{n_2}{n} = \frac{1}{2}(1-a)$ since $n = n_1 + n_2$.

The equation of the observed and theoretical values of $\Sigma e^{\iota\theta}$, $\Sigma e^{2\iota\theta}$, and $\Sigma e^{3\iota\theta}$, in the case of two drifts, therefore leads to

$$\left.\begin{aligned} 2L &= (1+a)C_1 + (1-a)C_2 \\ 2M &= (1+a)D_1 + (1-a)D_2 \\ 2N &= (1+a)E_1 + (1-a)E_2 \end{aligned}\right\} \qquad \ldots\ldots (10)$$

To solve these equations write

$$\left.\begin{aligned} C &= \zeta P \\ D &= \zeta P^2 \\ E &= \frac{1}{\gamma}\zeta P^3 \end{aligned}\right\} \qquad \ldots\ldots (11)$$

Since the arguments of C_1, D_1, and E_1 are θ_1, $2\theta_1$, and $3\theta_1$, ζ and γ are real, and the argument of P_1 is θ_1. Further, it is found that

for the range of values of hV (up to about $|hV| = 1{\cdot}8$) occurring in the actual drifts, γ may be assumed constant and equal to $1{\cdot}163$. In Table I. are given the values of ζ and the moduli of C, D, E, δE and P for various values of hV. The values of $\delta E = E - \frac{1}{1{\cdot}163}\zeta P^3$ are given in order to show the accuracy of the approximation made.

TABLE I.

hV.	P.	ζ.	C.	D.	E.	δE.
0·0	·0000	1·585	·0000	·0000	·0000	·000
0·1	·0566	1·573	·0884	·0050	·0002	·000
0·2	·1124	1·561	·1755	·0197	·0017	·000
0·3	·1680	1·548	·2600	·0437	·0057	−·001
0·4	·2227	1·531	·3409	·0759	·0134	·001
0·5	·2762	1·510	·4171	·1152	·0253	·002
0·6	·3284	1·486	·4879	·1602	·0420	·003
0·7	·3789	1·459	·5528	·2094		·005
0·8	·4275	1·431	·6115	·2614	·0901	·006
0·9	·4739	1·401	·6640	·3147		·007
1·0	·5179	1·371	·7103	·3679	·1558	·008
1·1	·5595	1·342	·7507	·4200		·008
1·2	·5984	1·313	·7856	·4701	·2337	·008
1·3	·6345	1·285	·8156	·5175		·007
1·4	·6678	1·259	·8410	·5617	·3172	·005
1·5	·6984	1·235	·8626	·6024	·3590	−·003
1·6	·7262	1·213	·8807	·6396	·4000	·000
1·7	·7514	1·192	·8959	·6732		+·004
1·8	·7742	1·174	·9087	·7035	·4774	·009
1·9	·7946	1·157	·9194	·7305		·014
2·0	·8128	1·142	·9284	·7546	·5468	+·019

With this substitution, (10) becomes

$$2L = (1+\alpha)\,\zeta_1 P_1 + (1-\alpha)\,\zeta_2 P_2\,;\ \text{etc.}$$

Now let
$$\left.\begin{aligned}(1+\alpha)\zeta_1 &= (1+\beta)k\\ (1-\alpha)\zeta_2 &= (1-\beta)k\end{aligned}\right\} \quad . \quad . \quad . \quad (12)$$

so that
$$k = \tfrac{1}{2}(1+\alpha)\zeta_1 + \tfrac{1}{2}(1-\alpha)\zeta_2 \quad . \quad . \quad . \quad (13)$$

We obtain finally

$$2L/k = (1+\beta)P_1 + (1-\beta)P_2 \quad . \quad . \quad . \quad (14)$$
$$2M/k = (1+\beta)P_1^2 + (1-\beta)P_2^2 \quad . \quad . \quad (15)$$
$$2N\gamma/k = (1+\beta)P_1^3 + (1-\beta)P_2^3 \quad . \quad . \quad (16)$$

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It will be seen from (13) that k is a weighted mean between ζ_1 and ζ_2, the weights being proportional to the number of stars in the two drifts. Now looking at Table I. we see that ζ_1 and ζ_2 vary within fairly narrow limits; actually the values of $h\nu_1$ and $h\nu_2$ in the different parts of the sky are such that, except in a few unusual cases, k will lie between 1·35 and 1·45. For a first approximation, we may either calculate a provisional value of k from the roughly known speeds of the drifts, or we may assume for it the value 1·40, which will certainly be not far from the truth. A second approximation may afterwards be made if desired, using the value of k deduced from the results of the first approximation; in practice, however, this is usually quite unnecessary.

Thus k may be assumed to be known, and the solution of (14), (15), and (16) is then easy. We find

$$\frac{4}{k^2}(N\gamma k - LM) = (1-\beta^2)(P_1-P_2)^2(P_1+P_2) \quad . \quad (17)$$

$$\frac{4}{k^2}(Mk - L^2) \quad = (1-\beta^2)(P_1-P_2)^2 \quad . \quad . \quad . \quad (18)$$

therefore $$\frac{N\gamma k - LM}{Mk - L^2} = P_1 + P_2 = 2K \quad \text{say} \quad . \quad . \quad (19)$$

whence solving (14) and (19) for P_1 and P_2

$$\left.\begin{aligned} P_1 &= K + \left(\frac{L}{k} - K\right)\Big/\beta \\ P_2 &= K - \left(\frac{L}{k} - K\right)\Big/\beta \end{aligned}\right\} \quad . \quad . \quad . \quad (20)$$

and substituting these values in (15) we easily find

$$\beta = \frac{L - Kk}{\sqrt{(L-Kk)^2 + (Mk - L^2)}} \quad . \quad . \quad . \quad (21)$$

We also find from (14) and (18)

$$\left.\begin{aligned} P_1 &= \frac{1}{k}\left(L + \sqrt{\frac{1-\beta}{1+\beta}}\sqrt{Mk - L^2}\right) \\ P_2 &= \frac{1}{k}\left(L - \sqrt{\frac{1+\beta}{1-\beta}}\sqrt{Mk - L^2}\right) \end{aligned}\right\} \quad . \quad . \quad (22)$$

Equations (21) and (22) constitute the solution of the problem.

The argument of P_1 is θ_1, and $h\nu_1$ can be found from the modulus by means of Table I. ζ_1 and ζ_2 can be taken from the same table, and then α is found from the formula

$$\frac{1-\alpha}{1+\alpha} = \frac{1-\beta}{1+\beta}\cdot\frac{\zeta_1}{\zeta_2}.$$

In the practical application of these formulæ a difficulty arises from the fact that, whereas β is by definition a real quantity, the right-hand side of (21), which is derived from observation, is in general a complex quantity.

The reason is this: equations (14), (15), and (16) practically constitute six equations, for each of them has a real and an imaginary part, which must be satisfied separately. But there are only five unknowns. In the above solution the equations have been reconciled by introducing a sixth fictitious unknown, viz. the imaginary part of α. If, however, α and β are to be purely real, the six equations cannot be exactly satisfied. Now it is clear that, just as we were justified in rejecting equations derived from equating the higher harmonics, we should attach more weight to equations (14) and (15) than to (16). It seems reasonable to satisfy the former exactly, and to throw the discordance entirely on (16). In other words, the observed and theoretical distributions will be made to agree exactly as regards coefficients of $\sin\theta$, $\cos\theta$, $\sin 2\theta$, $\cos 2\theta$, in the corresponding Fourier series, the coefficients of $\sin 3\theta$ and $\cos 3\theta$ will be made to agree as well as possible, and all higher harmonics will, as before, be entirely disregarded. If this rule seems rather arbitrary, it must be remembered that the whole question is one of weighting the equations of condition, that the ideal system of weighting is impracticable, and that we are seeking a practicable system which shall not unduly waste the material afforded by observation.

As (16) is not to be exactly satisfied, our solution must be chosen to make the sum of the squares of the two residuals (of the real and imaginary parts of the equation) a minimum. This is equivalent to making a minimum the modulus of the residual of the complex equation. Or we may satisfy the equation exactly by replacing N by $N+\delta N$, where $|\delta N|$ is to be a minimum.

Following this change through the solution previously given, let the alteration in N change $L-Kk$ to $(L-Kk)_0$; we see from (19) that if $|\delta N|$ is a minimum,

$$|(L-Kk)-(L-Kk)_0| \text{ will be a minimum.}$$

But (21) may be written

$$\frac{1}{\beta^2}=1+\left\{\frac{\sqrt{Mk-L^2}}{L-Kk}\right\}^2;$$

therefore if β is real, we must choose $(L-Kk)_0$ so that its argument is the same as, or differs by 180° from, that of $\sqrt{Mk-L^2}$.

(The alternative $\arg(L-Kk)_0=\arg\sqrt{Mk-L^2}\pm 90°$ leads to a value of β greater than unity, which must evidently be excluded.)

The conditions are satisfied if $(L-Kk)_0$ is the projection of the vector $L-Kk$ (in an Argand diagram) on the direction of the vector $\pm\sqrt{Mk-L^2}$; for this makes $|L-Kk-(L-Kk)_0|$ a minimum, subject to β being real.

Therefore

$$|(L - Kk)_0| = |L - Kk| \cos\{\arg(L - Kk) - \arg\sqrt{Mk - L^2}\} \quad (23)$$

and

$$\frac{1}{\beta^2} = 1 + \frac{|Mk - L^2|}{|(L - Kk)_0|^2} \quad . \quad . \quad . \quad (24)$$

and P_1 and P_2 are given by (22) without modification.

The sign of β is positive or negative according as $(L - Kk)_0$ is of the same or opposite sign to $\sqrt{Mk - L^2}$. Either square root of $Mk - L^2$ may be taken, but the same root must be adhered to throughout. It is, however, desirable to choose the root whose argument agrees roughly with the direction of motion of Drift I relative to Drift II, otherwise the suffixes of the constants will not agree with the usual designations of the drifts.

Example.

The following example shows the practical application of the method. It refers to Region C (R.A. 2^h–6^h, N.P.D. 20°–52°) of the Groombridge proper motions.

From the observed proper motions I find

Total number of stars $n = 512$.

$$\Sigma\cos\theta = +73{\cdot}0 \qquad \Sigma\sin\theta = +219{\cdot}0$$
$$\Sigma\cos 2\theta = -42{\cdot}3 \qquad \Sigma\sin 2\theta = +124{\cdot}1$$
$$\Sigma\cos 3\theta = -98{\cdot}45 \qquad \Sigma\sin 3\theta = +23{\cdot}1$$

hence, dividing by n,

$$L = +{\cdot}143 + {\cdot}428\,\iota$$
$$M = -{\cdot}083 + {\cdot}242\,\iota$$
$$N = -{\cdot}192 + {\cdot}045\,\iota$$
$$\text{and} \quad \gamma N = -{\cdot}223 + {\cdot}052\,\iota \qquad (\gamma = 1{\cdot}163).$$

Assume provisionally $k = 1{\cdot}40$, we find

$$N\gamma k - LM = -{\cdot}1967 + {\cdot}0737\iota = {\cdot}2100 \text{ expi}^* \ 159°{\cdot}5$$
$$Mk - L^2 = +{\cdot}0466 + {\cdot}2164\iota = {\cdot}2214 \text{ expi} \quad 77°{\cdot}9$$

therefore by division (equation (19))

$$2K = {\cdot}948 \text{ expi } 81°{\cdot}6$$
$$K = +{\cdot}069 + {\cdot}469\,\iota$$

therefore

$$L - Kk = +{\cdot}046 - {\cdot}229\,\iota$$
$$= {\cdot}234 \text{ expi } 281°{\cdot}4;$$

also

$$\sqrt{Mk - L^2} = {\cdot}470 \text{ expi } 39°{\cdot}0.$$

If we continue the solution with $L - Kk$ unmodified, equation (21) gives

$$\beta = -{\cdot}300 - {\cdot}442\,\iota.$$

* Expi $x = e^{\iota x}$.

But modifying it in accordance with (23)

$$(L - Kk)_0 = \cdot 234 \cos (281^\circ \cdot 4 - 39^\circ \cdot 0) \operatorname{expi} 39^\circ \cdot 0$$
$$= - \cdot 1083 \operatorname{expi} 39^\circ \cdot 0$$

and $$\frac{\sqrt{Mk - L^2}}{(L - Kk)_0} = - 4 \cdot 53,$$

whence $$\beta = - \frac{1}{\sqrt{1 + (4 \cdot 53)^2}} = - \cdot 215.$$

This value of β must now be substituted in (22),

$$\sqrt{\frac{1 - \beta}{1 + \beta}} = 1 \cdot 244.$$

$$\sqrt{\frac{1 - \beta}{1 + \beta}} \sqrt{Mk - L^2} = \cdot 610 \operatorname{expi} 39^\circ \cdot 0$$
$$= \cdot 474 + \cdot 384\, i$$

and $$L = \cdot 143 + \cdot 428\, i.$$

Sum $$= kP_1 = \cdot 617 + \cdot 812\, i,$$
$$= 1 \cdot 020 \operatorname{expi} 52^\circ \cdot 8.$$

Therefore $$P_1 = \cdot 729 \operatorname{expi} 52^\circ \cdot 8.$$

Similarly we find $$P_2 = \cdot 174 \operatorname{expi} 132^\circ \cdot 7.$$

The directions of the two drifts are accordingly

$$\theta_1 = 52^\circ \cdot 8 \qquad \theta_2 = 132^\circ \cdot 7.$$

To find their velocities we make use of Table I., which gives the corresponding values of $|P|$ and hv,

$$hv_1 = 1 \cdot 61 \qquad hv_2 = 0 \cdot 31 ;$$

also from Table I.,

$$\zeta_1 = 1 \cdot 211 \qquad \zeta_2 = 1 \cdot 547$$

but $$\frac{1 - a}{1 + a} = \frac{1 - \beta}{1 + \beta} \frac{\zeta_1}{\zeta_2}$$
$$= 1 \cdot 548 \times \frac{1 \cdot 211}{1 \cdot 547}$$
$$= 1 \cdot 212,$$

whence $$a = - \cdot 096$$

$$\tfrac{1}{2}(1 + a) = \cdot 452 \quad \tfrac{1}{2}(1 - a) = \cdot 548.$$

Thus 45·2 per cent. of the stars belong to Drift I and 54·8 per cent. to Drift II.

Finally, we can check the provisional value of k,

$$k = \tfrac{1}{2}(1 + a)\zeta_1 + \tfrac{1}{2}(1 - a)\zeta_2 = 1 \cdot 395,$$

agreeing with the adopted value 1·40, so that no further approximation is needed.

The results obtained by this method agree closely with those previously obtained by the method of trial and error ("Systematic Motions," p. 50). I give below a comparison of the constants determined in the two ways for four of the Groombridge Regions; it affords some sort of indication of the reliability of both methods.

		Analytical Method.		Trial and Error.	
		Drift I.	Drift II.	Drift I.	Drift II.
Region A	$h\nu$	1·38	0·31	1·65	0·30
	θ	0°	157°	0°	155°
	n*	52·5	47·5	48·5	51·5
Region B	$h\nu$	1·47	0·48	1·55	0·45
	θ	12°	131°	10°	130°
	n	49·5	50·5	49·5	50·5
Region C	$h\nu$	1·61	0·31	1·65	0·30
	θ	53°	133°	55°	125°
	n	45·2	54·8	41·5	58·5
Region F	$h\nu$	0·86	0·77	1·20	0·45
	θ	226°	75°	225°	80°
	n	63·0	37·0	46·5	53·5

* Per cent.

The weakest point in the solution is exemplified in the case of Region F; the determination of α (or β) is not very satisfactory. The difference in the two solutions in the case of Region F depends entirely on the division of the stars between the two drifts; if we had adopted the same value of α in the two cases, the remaining four constants found from (22) would have agreed almost exactly with those found by trial and error. The weakness of the determination of β can be shown analytically.

Differentiating (21),

$$\frac{d\beta}{dN} = -\tfrac{1}{2}k^2\gamma\{(Mk - L^2) + (L - Kk)^2\}^{-\frac{3}{2}}$$

$$= -\frac{4\gamma}{k}(P_1 - P_2)^{-3}.$$

Now $|P_1 - P_2|$ is generally about 0·7, but may be less, hence $\left|\frac{d\beta}{dN}\right|$ is generally about 10, but may be more.

Thus in a region containing 500 stars, a change of 2·5 in $\Sigma \cos 3\theta$ or $\Sigma \sin 3\theta$ would produce a change in N of ·005, and the corresponding change in β or α might be $\pm$ ·05.

The question arises whether the weakness of the determination of β is a defect of the method of analysis or is necessarily involved in the nature of the observational data. I think there is little doubt that the latter alternative is correct; I have examined various other methods of determining β, but all are rather insensitive. In the case of Region F, both the solutions given above are found to agree with the observed distributions almost equally well.

Thus it seems likely that even in an ideal solution we should have one equation very much weaker than the other four (the weakness may be more pronounced for some regions of the sky than for others). As the five constants depend on one another and on this fifth equation, they may all share in the uncertainty.

One or two considerations help to avoid this difficulty to some extent. We may be content to assume $a=0$, *i.e.* that the stars are evenly divided between the two drifts; all evidence seems to indicate that this is approximately true, and it is conceivable that there may be some physical reason for it. Or, instead of adopting the value of a found for the particular region, we may adopt a mean found from all the regions discussed; this will have a much smaller probable error. But the most fortunate circumstance is that *we may determine the relative motion of the two drifts almost independently of* a.

Equation (18) gives

$$P_1 - P_2 = \frac{2}{k}(1-\beta^2)^{-\frac{1}{2}}\sqrt{Mk - L^2}.$$

If β lies anywhere between $-0{\cdot}3$ and $+0{\cdot}3$, $(1-\beta^2)^{-\frac{1}{2}}$ may be put equal to ·98 with an error certainly less than 3 per cent.; but this range of β includes all values likely to occur. Only in the case of a very great disparity in the distribution between the two drifts could a value outside these limits occur. Thus $P_1 - P_2$ is nearly independent of β or a. P is a sufficiently nearly linear function of hV for $h(V_1 - V_2)$, to be also nearly independent of a. Thus although adopting $a=0$ may lead to some error in the determinations of hV_1 and hV_2, the error will nearly be eliminated from the determination $h(V_1 - V_2)$. This relative motion of the two drifts is the quantity which most interests us, especially as a systematic error in the proper motions does not affect its determination so adversely as it affects the determinations of hV_1 and hV_2.

Systematic Motions of Zodiacal Stars.

I have applied the theory given above to the discussion of the proper motions of the zodiacal stars. The proper motions were taken from the Catalogue of Zodiacal Stars, *Astronomical Papers of the American Ephemeris*, vol. viii., part iii. Excluding the stars of the Pleiades, this contains 1533 proper motions. I divided the zodiac into sixteen regions, each extending $22\frac{1}{2}^\circ$ in longitude by about 16° in latitude; these are denoted successively by I*a*, II*a*, . . . VIII*a*, I*b*, ... VIII*b*, the centre of region I*a* being at the first point of Aries. As regions I*a* and I*b* are diametrically opposite to one another, the observed motions are in parallel planes, and the two regions may be treated together; similarly, the other regions can be treated together in pairs. Thus the number of regions is virtually reduced to eight, each containing from 150 to 250 stars.

The distribution of the proper motions as regards direction in the eight regions is shown in Table II. Opposite $\theta=0^\circ$ in the

first column is given the number of stars whose observed motions are in directions between $\theta = 355^\circ$ and $\theta = 5^\circ$, and so on. The numbers have not been smoothed. For stars in I*a*, II*a*, etc., $\theta = 0^\circ$ is in the direction of increasing R.A., and $\theta = 90^\circ$ in the direction of increasing Dec. For stars in I*b*, II*b*, etc., the reckoning of θ agrees with that in the opposite parallel planes I*a*, II*a*, etc.

TABLE II.

Distribution of the Proper Motions in Direction.

	Regions.							
θ	I.	II.	III.	IV.	V.	VI.	VII.	VIII.
0°	16	14	9	5	7	2	1	2
10	11	8	3	8	0	0	5	4
20	3	3	3	2	1	1	2	2
30	2	6	0	2	3	4	2	1
40	1	2	2	0	1	2	0	2
50	4	3	0	2	3	2	3	0
60	2	1	0	1	0	1	1	0
70	3	2	0	0	0	0	2	2
80	2	1	0	1	1	0	0	2
90	3	3	1	1	1	0	1	1
100	1	1	1	0	1	1	0	0
110	1	1	0	2	1	1	2	0
120	0	2	1	1	0	4	2	3
130	0	1	1	2	3	2	2	4
140	1	0	1	2	0	3	5	4
150	2	1	0	1	4	1	3	5
160	3	2	1	2	5	6	4	3
170	1	2	3	2	3	2	9	11
180	0	1	2	2	6	10	8	13
190	6	4	1	1	6	7	18	14
200	6	8	4	3	8	9	12	12
210	5	5	6	3	5	9	12	14
220	6	4	3	5	11	16	14	8
230	10	5	6	8	11	20	10	6
240	2	7	2	4	13	14	16	4
250	2	6	3	10	9	17	13	4
260	1	3	6	7	9	9	9	5
270	6	5	2	7	21	12	3	8
280	3	6	9	12	17	10	6	4
290	4	7	13	20	10	8	10	4
300	6	7	11	18	16	8	0	10
310	12	7	8	13	13	6	7	3
320	17	11	15	27	13	6	2	2
330	14	4	12	18	8	4	2	5
340	9	20	18	23	6	1	0	3
350	10	17	9	25	5	0	8	4
Total	175	180	156	240	221	198	194	169

It will be seen, by looking down the columns of the Table that the two streams are plainly marked in Regions I, II, III, and VIII. In the other four regions their directions (projected on the sky) are inclined at an acute angle, and the existence of the two streams is rather concealed. The success of the analysis in these four regions is on that account especially interesting. It may be noticed that this belt of the sky is not so favourable as the Groombridge region for showing conspicuously the separation into two streams; the centre of the latter region lies between the two apices, so that in it the streams are in nearly opposite directions.

Although, in the main, pairs of regions such as I*a* and I*b* were treated together, I thought it safer to calculate $\sqrt{Mk-L^2}$ and L for I*a* and I*b* separately, and to take the mean afterwards. This was in order to avoid the possible effects of systematic error, by ensuring that the difference of motion of the two drifts found from the observations was a difference of motion of intermingled systems of stars, and not the difference in the apparent motion of stars in I*a* from that of stars in I*b*. Actually, however, the precaution might have been omitted. I found that in every region very nearly the same result was obtained whether the two halves were treated separately or together.

For the solutions I used entirely the equations—

$$P_1=\frac{1}{k}\left(L+\sqrt{\frac{1-\beta}{1+\beta}}\sqrt{Mk-L^2}\right)$$

$$P_2=\frac{1}{k}\left(L-\sqrt{\frac{1+\beta}{1-\beta}}\sqrt{Mk-L^2}\right)$$

I did not calculate N, or attempt to find β from the observations. The final results given below depend, therefore, on the assumption (based on previous experience) that the stars are evenly distributed between the two drifts; but calculations are given which show to what extent the results obtained would need to be modified if the assumption is incorrect.

As a preliminary I made two solutions, (*a*) assuming $\sqrt{\frac{1-\beta}{1+\beta}}=1{\cdot}0$ and (*b*) assuming $\sqrt{\frac{1-\beta}{1+\beta}}=1{\cdot}1$; in both cases k was assumed to be 1·40. These correspond to assuming that the stars belonging to Drift I are about (*a*) 55 per cent., (*b*) 50 per cent. of the whole.

The combined results from all the regions were—

Velocity of Drift I	(*a*)	1·61	towards Latitude		−36°,	Longitude	105°
	(*b*)	1·74	,,	,,	−34°,	,,	106°
Velocity of Drift II	(*a*)	0·64	,,	,,	−42°,	,,	302°
	(*b*)	0·60	,,	,,	−48°,	,,	304°
Velocity of Drift I relative to Drift II	(*a*)	1·84	,,	,,	−16°·3,	,,	109°·4
	(*b*)	1·90	,,	,,	−16°·5,	,,	109°·6

The unit of velocity is $1/h$.

The mutual relative velocity of the two drifts is thus determined nearly independently of the assumed division of the stars between them, a result which has already been arrived at theoretically.

For a final solution, using (b) as a first approximation, I calculated for each region values of k and β, assuming that the stars were equally divided between the drifts.

The values of k for the Regions I, II, ... VIII were respectively—

1·35, 1·35, 1·36, 1·40, 1·44, 1·44, 1·43, 1·38,

and of $\sqrt{\frac{1-\beta}{1+\beta}}$

1·12, 1·12, 1·11, 1·09, 1·07, 1·07, 1·08, 1·10.

Performing the analysis with these values, the constants of the drifts in the eight regions were found as follows:—

Region.	Longitude of Centre.	Drift I.		Drift II.	
		hv_1.	θ_1.	hv_2.	θ_2.
I	0° , 180°	1·57	345°·4	0·46	244°·8
II	22½ , 202½	1·46	350 ·0	0·58	249 ·4
III	45 , 225	1·86	330 ·0	0·63	252 ·2
IV	67½ , 247½	1·68	323 ·5	0·51	271 ·3
V	90 , 270	1·08	299 ·7	0·70	231 ·2
VI	112½ , 292½	1·04	245 ·4	0·54	233 ·3
VII	135 , 315	1·35	208 ·8	0·40	275 ·8
VIII	157½ , 337½	1·49	189 ·2	0·54	292 ·3

If we resolve these drift-velocities along and perpendicular to the plane of the ecliptic, the components perpendicular to the ecliptic derived from each region should agree.

The determinations of these components are—

for Drift I, −·97, −·77, −1·35, −1·19, −·94, −1·00, −·96, −·76, Mean −·99

for Drift II, −·30, −·43, −·52, −·51, −·54, −·48, −·37, −·39, Mean −·44

The accordance of these seems decidedly good when it is remembered that they are derived from regions containing on an average less than 200 stars.

The component drift-velocities in the ecliptic are more or less foreshortened according to the longitude of the region; I determined the mean values by a least-squares solution.

The final results are—

Velocity of Drift I,	1·78	towards	Latitude	−33°·8,	Longitude	105°·5
Velocity of Drift II,	0·59	,,	,,	−47°·9,	,,	303°·7
Velocity of Drift I, relative to Drift II,	1·94	,,	,,	−16°·4,	,,	109°·3

The slight difference between this result and that of solution (*b*) represents the effect of allowing for the variation of β and k from region to region, instead of adopting a mean. The comparison between solutions (*a*) and (*b*) serves to indicate how the results would be changed if the stars were not evenly divided between the drifts. The constants most affected by such a change are the speed of Drift I and the direction of Drift II; the other constants are nearly independent of the assumption, and are therefore more reliably determined.*

From an examination of residuals, I estimate that the probable accidental error of the determination of kV_1 and kV_2 is about $\pm$ ·06, and the probable errors of the apices of Drifts I and II are respectively about 2° and 6° of a great circle.

Converting latitude and longitude into R.A. and Dec., the positions of the antapices may be compared with previous determinations as follows—

	Drift I.		Drift II.	
Kapteyn . . .	R.A. 85°	Dec. −11°	R.A. 260°	Dec. −48°
Dyson	,, 94°	,, − 7°	,, 240°	,, −74°
Groombridge stars	,, 90°	,, −19°	,, 292°	,, −58°
Zodiacal stars . .	,, 103°	,, −11°	,, 330°	,, −64°

The great R.A. of the antapices of both drifts found in the present discussion is rather hard to account for.

The velocities of the two drifts $hv_1 = 1{\cdot}78$, $hv_2 = 0{\cdot}59$ are in excellent agreement with those found from the Groombridge stars $hv_1 = 1{\cdot}7$, $hv_2 = 0{\cdot}5$.

For the velocity of one drift relative to the other, the Zodiacal stars give the value 1·94. From the Groombridge stars (by a least-squares solution from the results of the separate regions) I have found the value 1·90. The determinations of the point towards which this relative velocity is directed (called by Professor Kapteyn the *true* vertex) are

Kapteyn	R.A. 91°	Dec. + 13°
From Groombridge stars .	,, 95°	,, + 3°
,, Zodiacal ,, .	,, 109°	,, + 6°

Professor Schwarzschild's determination of the line of symmetry of motion may be added. This, although based on a rather different theory, is directly comparable with the above. He found,

From Groombridge stars . . R.A. 93° Dec. + 6°

* It should be understood that it is not impossible to determine a from the observations, but simply that when, as in the present case, a few stars are discussed, the value of a is liable to a greater uncertainty than some of the other results. Undoubtedly from the whole 1533 stars a fairly good mean value of a could be determined; but the task of computing it for sixteen regions separately, and taking the mean, would be laborious.

Further, from Professor Dyson's investigation, an R.A. of about 92° for this point may be inferred.

Thus the Right Ascension given by the Zodiacal proper motions is discordant as compared with the other determinations. The number of stars here considered is fewer, and the proper motions are perhaps not so well determined, but I do not think the discordance can be altogether attributed to this. Nor can it be traced to a local anomaly, for it seems to be indicated by the proper motions all round the ecliptic. I have verified by calculation that the Regions I*a* to VIII*a* agree with Regions I*b* to VIII*b* in leading to this high value of the Right Ascension.

Note on the evaluation of certain integrals required in the analysis.

(1) To calculate $n_1 C_1 = \int_0^{2\pi} \rho \cos(\theta - \theta_1) d\theta$, for a single drift.

We may choose the initial line, along the direction of the drift, so that $\theta_1 = 0$.

Then $$\rho = \frac{n_1}{\pi} e^{-h^2 v_1^2} \left\{ \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^{\infty} e^{-x^2} dx \right\}$$

where $\tau = h v_1 \cos\theta$.

Now $\left\{ \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^{0} e^{-x^2} dx \right\} \cos\theta$ is an odd function of $\cos\theta$ and vanishes when integrated from 0 to 2π.

Hence $$\int_0^{2\pi} \rho \cos\theta d\theta = \frac{n_1}{\pi} e^{-h^2 v_1^2} \frac{\sqrt{\pi}}{2} \int_0^{2\pi} \tau e^{\tau^2} \cos\theta d\theta$$

$$= \frac{n_1 h v_1}{2\sqrt{\pi}} e^{-h^2 v_1^2} \int_0^{2\pi} \cos^2\theta e^{h^2 v_1^2 \cos^2\theta} d\theta.$$

Now $$\int_0^{2\pi} e^{2\kappa \cos^2\theta} d\theta = e^{\kappa} \int_0^{2\pi} e^{\kappa \cos 2\theta} d\theta$$

$$= 2\pi e^{\kappa} J_0(\iota\kappa).$$

Whence differentiating with respect to κ,

$$2\int_0^{2\pi} \cos^2\theta e^{2\kappa \cos^2\theta} d\theta = 2\pi e^{\kappa} \left(J_0(\iota\kappa) - \iota J_1(\iota\kappa) \right).$$

Now write $\kappa = \frac{1}{2} h^2 v_1^2$ and substitute above

$$C_1 = \frac{\sqrt{\pi}}{2} h v_1 e^{-\frac{1}{2} h^2 v_1^2} \left\{ J_0(\iota \tfrac{1}{2} h^2 v_1^2) - \iota J_1(\iota \tfrac{1}{2} h^2 v_1^2) \right\}.$$

Tables of the Bessel Functions of an imaginary argument are given in *Brit. Assoc. Reports*, 1893 and 1896, and have been used in calculating Table I.

(2) To calculate $n_1 D_1 = \int_0^{2\pi} \rho \cos 2\theta d\theta.$

We have $$\rho = \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2-2v_1 r\cos\theta)} r dr$$

and identically

$$n_1 = \int_0^{2\pi} \rho d\theta = \frac{n_1 h^2}{\pi} \int_0^{2\pi} d\theta \int_0^\infty e^{-h^2(r^2+v_1^2-2v_1 r\cos\theta)} r dr$$

$$= \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr \int_0^{2\pi} e^{z\cos\theta} d\theta . \quad . \quad . \quad . \quad (1)$$

where $z = 2h^2 v_1 r$.

Hence $$n_1 \equiv \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr . 2\pi J_0(\iota z) . \quad (2)$$

But from (1)

$$\int_0^{2\pi} \rho \cos^2\theta d\theta = \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr \frac{d^2}{dz^2} \int_0^{2\pi} e^{z\cos\theta} d\theta$$

$$= \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr \frac{d^2}{dz^2} \left(2\pi J_0(\iota z)\right)$$

$$= \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr \left(2\pi J_0(\iota z) - \frac{1}{z} \frac{d}{dz} \left(2\pi J_0(\iota z)\right)\right)$$

by Bessel's differential equation

$$= n_1 - \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} \frac{1}{4h^4 v_1^2} \frac{d}{dr} \left(2\pi J_0(\iota z)\right) dr$$

Integrate by parts:

$$= n_1 - \frac{n_1 h^2}{\pi} \frac{1}{4h^4 v_1^2} \left[e^{-h^2(r^2+v_1^2)} 2\pi J_0(\iota 2h^2 v_1 r) \right]_0^\infty$$

$$+ \frac{n_1 h^2}{\pi} \frac{1}{4h^4 v_1^2} \int_0^\infty dr(-2h^2 r) e^{-h^2(r^2+v_1^2)} 2\pi J_0(\iota z)$$

$$= n_1 + \frac{n_1}{2h^2 v_1^2} e^{-h^2 v_1^2} - \frac{n_1}{2h^2 v_1^2}$$

(the integrated part vanishes for $r = \infty$ and reduces to $\frac{n}{2h^2 v_1^2} e^{-h^2 v_1^2}$

at the lower limit; the part remaining to be integrated is simplified by means of the identity (2)),

$$\text{hence} \quad \int_0^{2\pi} \rho \cos 2\theta d\theta = 2\int_0^{2\pi} \rho \cos^2\theta d\theta - \int_0^{2\pi} \rho d\theta$$

$$= n_1\left(1 - \frac{1 - e^{-h^2 v_1^2}}{h^2 v_1^2}\right).$$

(3) The integral $\int_0^{2\pi} \rho \cos 3\theta d\theta$, and corresponding integrals for any odd multiples of θ, can be found by a simple extension of the method employed for $\int_0^{2\pi} \rho \cos\theta d\theta$. The integrals for even multiples of θ are more troublesome to evaluate, but the method employed for $\int_0^{2\pi} \rho \cos 2\theta d\theta$ always succeeds.

Tables of the two hypergeometrical functions, $F\left(1/6, 5/6, 2, \sin^2\frac{\iota}{2}\right)$ *and* $F\left(-1/6, 7/6, 2, \sin^2\frac{\iota}{2}\right)$, *between the limits iota equals* 90 *and* 180 *degrees.* By C. J. Merfield.

In the method of Mr. R. T. A. Innes for the determination of the secular perturbations,* there are two hypergeometrical functions to be deduced. In an appendix † to this valuable paper, tables of the logarithms of these functions are given with the argument iota for each degree ‡ for the first quadrant.

Tables of these functions facilitate the application of this method in no small degree, and it seemed desirable to extend them, as in many future investigations it will be found that the angle iota will much exceed a quadrant. Taking an example, Eros §—Earth, it will be noted that the modular angle theta, the argument to the tables of elliptical integrals, reaches the value 60°, corresponding to iota 138° 18′, and there will be many other cases in which it exceeds this value.

In the preparation of the tables here given the formulæ ‖ (*l.c.*,

* "Computation of Secular Perturbations," by R. T. A. Innes, *Monthly Notices*, vol. lxvii. 427.

† Tables for the application of Mr. Innes's Method, by Frank Robbins. *l.c.*, 444.

‡ The values of these functions for iota equals 0° have been omitted in the tabulation by Mr. Robbins.

§ "Secular Perturbations of Eros," by C. J. Merfield, *Astr. Nachr.*, 4178–79, Band 175.

‖ The values of these functions may be deduced from series. I have given the coefficients of twenty terms, *Astr. Nachr.*, 4215, Band 176, p. 246.

442) of Mr. Innes have been adopted. Seven-figure logarithms have been used, and the *Additions- und Subtractions-Logarithmen* by Zech where necessary, but the powers of the trigonometrical ratios have been formed from the values of these functions tabulated in Vega's ten-figure table, and the results used to seven figures.

Taking

$$\iota_1 = 180 - \iota$$

then $$\cos \frac{\iota_1}{2} = \sin \frac{\iota}{2}$$

and the following relation holds,

$$F\left(-1/6 \ldots \sin^2 \frac{\iota}{2}\right) F\left(1/6 \ldots . \sin^2 \frac{\iota_1}{2}\right) +$$

$$F\left(1/6 \ldots . \sin^2 \frac{\iota}{2}\right) F\left(-1/6 \ldots \sin^2 \frac{\iota_1}{2}\right) = 216/35\pi.$$

The logarithm of the right-hand member equals 0·29323583 . . . . Writing in this formula the quantities taken from the tables, then it will generally be found that the residual does not exceed a unit in the seventh decimal place of the logarithm in any portion of the tables. This formula might have been used in calculating either function after one of them had been deduced between the limits 90 and 180 degrees, combined with the values tabulated by Mr. Robbins, *loco citato*.

Sydney:
1908 *February* 10.

TABLES.

ι	Log F (1/6 . . . .)	Δ1	Δ2	Log F (−1/6 . . . .)	Δ1	Δ2
90°	0·0183800	+3991	+41	9·9738258	−5778	−62
91	0·0187832	4032	40	9·9732418	5840	62
92	0·0191904	4072	38	9·9726516	5902	62
93	0·0196014	4110	39	9·9720552	5964	61
94	0·0200163	4149	39	9·9714527	6025	61
95	0·0204351	4188	38	9·9708441	6086	60
96	0·0208577	4226	39	9·9702295	6146	62
97	0·0212842	4265	38	9·9696087	6208	60
98	0·0217145	4303	39	9·9689819	6268	60
99	0·0221487	4342	37	9·9683491	6328	59
100	0·0225866	4379	+37	9·9677104	6387	−60
		+4416			−6447	

t	Log F (1/6 . . . .)	Δ_1	Δ_2	Log F (−1/6 . . . .)	Δ_1	Δ_2
101	0·0230282	+4416	+36	9·9670657	−6447	−58
102	0·0234734	4452	36	9·9664152	6505	58
103	0·0239222	4488	35	9·9657589	6563	57
104	0·0243745	4523	36	9·9650969	6620	58
105	0·0248304	4559	34	9·9644291	6678	56
106	0·0252897	4593	34	9·9637557	6734	56
107	0·0257524	4627	35	9·9630767	6790	55
108	0·0262186	4662	34	9·9623922	6845	56
109	0·0266882	4696	33	9·9617021	6901	54
110	0·0271611	4729	33	9·9610066	6955	53
111	0·0276373	4762	31	9·9603058	7008	53
112	0·0281166	4793	32	9·9595997	7061	52
113	0·0285991	4825	31	9·9588884	7113	52
114	0·0290847	4856	30	9·9581719	7165	50
115	0·0295733	4886	28	9·9574504	7215	48
116	0·0300647	4914	29	9·9567241	7263	49
117	0·0305590	4943	29	9·9559929	7312	48
118	0·0310562	4972	27	9·9552569	7360	45
119	0·0315561	4999	26	9·9545164	7405	46
120	0·0320586	5025	26	9·9537713	7451	44
121	0·0325637	5051	24	9·9530218	7495	44
122	0·0330712	5075	24	9·9522679	7539	41
123	0·0335811	5099	22	9·9515099	7580	41
124	0·0340932	5121	23	9·9507478	7621	39
125	0·0346076	5144	21	9·9499818	7660	37
126	0·0351241	5165	20	9·9492121	7697	36
127	0·0356426	5185	19	9·9484388	7733	34
128	0·0361630	5204	18	9·9476621	7767	33
129	0·0366852	5222	16	9·9468821	7800	30
130	0·0372090	5238	15	9·9460991	7830	29
131	0·0377343	5253	13	9·9453132	7859	26
132	0·0382609	5266	13	9·9445247	7885	25
133	0·0387888	5279	11	9·9437337	7910	24
134	0·0393178	5290	10	9·9429403	7934	20
135	0·0398478	5300	7	9·9421449	7954	17
136	0·0403785	5307	7	9·9413478	7971	15
137	0·0409099	5314	5	9·9405492	7986	13
138	0·0414418	5319	+4	9·9397493	7999	9
139	0·0419741	5323	0	9·9389485	8008	7
140	0·0425064	5323	−1	9·9381470	8015	−3
		+5322			−8018	

t	Log $F(1/6\ldots.)$	Δ_1	Δ_2	Log $F(-1/6\ldots.)$	Δ_1	Δ_2
141	0·0430386	+5322	−4	9·9373452	−8018	0
142	0·0435704	5318	5	9·9365434	8018	+4
143	0·0441017	5313	8	9·9357420	8014	6
144	0·0446322	5305	10	9·9349412	8008	11
145	0·0451617	5295	12	9·9341415	7997	14
146	0·0456900	5283	16	9·9333432	7983	20
147	0·0462167	5267	19	9·9325469	7963	25
148	0·0467415	5248	21	9·9317531	7938	30
149	0·0472642	5227	25	9·9309623	7908	34
150	0·0477844	5202	28	9·9301749	7874	40
151	0·0483018	5174	31	9·9293915	7834	45
152	0·0488161	5143	35	9·9286126	7789	52
153	0·0493269	5108	40	9·9278389	7737	55
154	0·0498337	5068	43	9·9270707	7682	65
155	0·0503362	5025	48	9·9263090	7617	73
156	0·0508339	4977	52	9·9255546	7544	79
157	0·0513264	4925	58	9·9248081	7465	89
158	0·0518131	4867	64	9·9240705	7376	96
159	0·0522934	4803	68	9·9233425	7280	107
160	0·0527669	4735	76	9·9226252	7173	116
161	0·0532328	4659	83	9·9219195	7057	128
162	0·0536904	4576	89	9·9212266	6929	137
163	0·0541391	4487	98	9·9205474	6792	151
164	0·0545780	4389	106	9·9198833	6641	165
165	0·0550063	4283	115	9·9192357	6476	180
166	0·0554231	4168	126	9·9186061	6296	194
167	0·0558273	4042	136	9·9179959	6102	211
168	0·0562179	3906	148	9·9174068	5891	231
169	0·0565937	3758	163	9·9168408	5660	253
170	0·0569532	3595	177	9·9163001	5407	274
171	0·0572950	3418	194	9·9157868	5133	300
172	0·0576174	3224	213	9·9153035	4833	331
173	0·0579185	3011	238	9·9148533	4502	362
174	0·0581958	2773	263	9·9144393	4140	403
175	0·0584468	2510	294	9·9140656	3737	448
176	0·0586684	2216	334	9·9137367	3289	507
177	0·0588566	1882	385	9·9134585	2782	581
178	0·0590063	1497	462	9·9132384	2201	687
179	0·0591098	1035	−607	9·9130870	1514	+890
180	0·0591526	+428		9·9130246	−624	

Further Note on the Position of the Sun's Axis of Rotation, as deduced from Greenwich Sun-spot Measures 1874–1885. *Papers of the I.U.S.R. Computing Bureau, No. II.* By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. This note gives for the period 1874–1885 computations similar to those given for the period 1886–1901 in the December No. (p. 98). The discussion is limited to spot-groups which live for ten days. The tables are numbered to correspond with those of the former note.

TABLE I.

Latitude drift between +65° *and* −65° *in each month* (*of the years* 1874–1885).

Latitude.	−25°	−20°	−15°	−10°	0°	+10°	+15°	+20°	+25°
Jan.	+57	+170	−64	−19	−33	+25	00	−21	—	+5
Feb.	+113	−41	−48	−61	+62	−29	+50	−16	+77	+70
Mar.	−27	+145	+15	+45	+16	+30	+80	+97	—	—
Apr.	−55	+100	+79	+23	+34	+55	+113	+61	+118	—
May	−85	—	+12	+39	+45	+5	+62	−22	+10	+5
June	—	−171	+43	+31	−50	+70	−61	+125	+73	+125
July	−23	—	−33	−97	−18	+5	−52	−116	−2	+115
Aug.	—	+15	+35	+28	+17	−34	+27	+23	+57	−5
Sept.	+35	−90	−11	−136	+8	+31	−55	+11	+38	0
Oct.	—	−13	+18	+57	−38	−53	−4	+46	+53	—
Nov.	—	−37	+60	+12	−11	−17	−52	−7	−83	—
Dec.	−75	—	+62	+240	−25	+33	−80	−54	−1	+215
Mean	—	—	+14	+14	+1	+10	+2	+11	—	—

The unit in the tables is 0°·01, and the figures are, in Table I., differences between the mean residual at +65° and −65° in

TABLE II.

Latitude drift between +35° *and* −35° *in each month* (*of the years* 1874–1885).

Latitude.	−25°	−20°	−15°	−10°	0°	+10°	+15°	+20°	+25°
Jan.	+48	+95	+25	+12	−37	+25	−29	−53	—	+15
Feb.	+50	−6	−51	−30	+20	+23	+3	−5	+40	+152
Mar.	+32	+71	−6	+32	+23	+39	+70	−2	—	—
Apr.	−35	+80	+49	+12	−1	+7	+33	+10	+98	—
May	−87	+10	+24	+31	+36	−31	+31	+68	+50	+88
June	+12	−70	+10	+34	+61	+60	−23	+75	+43	+135
July	+51	−88	−1	−16	+25	−10	−10	−38	+12	+87
Aug.	—	+47	+15	−26	+10	−63	+6	+90	+8	+15
Sept.	+2	−6	−24	−87	+26	−4	+3	+8	+41	+12
Oct.	—	+5	+9	+25	+4	−25	−15	+5	+10	—
Nov.	—	−43	+74	+17	−1	−67	−15	+2	−58	—
Dec.	−15	—	+43	+137	+30	+59	−88	−194	+22	+67
Mean	—	—	+14	+12	+16	+1	−3	−3	—	—

longitude from the centre of the disc; and in Table II., similar differences for longitudes $\pm 35^\circ$ from the centre.

Excluding,* at any rate in the first instance, the outside groups (for latitudes above 20°), we can now analyse the remaining 6 columns in each table harmonically, to find an expression of the form

$$A \sin \theta + B \cos \theta + C,$$

where $\theta = 0$ for the middle of January.

Multiplying the coefficients A and B by $\frac{1}{2}$ cosec 65° for Table I., and $\frac{1}{2}$ cosec 35° for Table II., we get the error of the Sun's axis, which we may now express (as Carrington does) in minutes of arc. The separate determinations are given in Table III.

TABLE III.

Quantities representing the Errors of the Sun's adopted axis.

Latitudes.	−20°	−15°	−10°	0°	+10°	+15°	+20°	Mean.
A from Table I.	−0′·5	−0′·1	+7′·5	+8′·0	+19′·6	+8′·0		+7′·1
A ,, ,, II.	−3·3	+4·3	+3·9	+12·6	+15·5	+12·2		+7·5
B ,, ,, I.	−2·7	+14·3	+1·2	−1·0	+3·9	−2·4		+2·2
B ,, ,, II.	+2·8	+14·5	−8·8	+9·8	−5·9	−32·2		−3·3

Inspection of Table III. suggests the following conclusions:—

(*a*) The accordance between determinations from meridians $\pm 65^\circ$ and $\pm 35^\circ$ is not so marked as before; but the discordances are not systematic. We must remember that the former series was based on 16 years' observations, and the present on less than 11 years'.

(*b*) The mean error of the adopted axis in A is confirmed within very narrow limits, the mean value of A being +7′·3 as against +7′·4 for the period 1886–1901. But the mean value of B for the same period was −7′·8, whereas the measures here discussed give nearly zero. Can we ascribe this difference to accidental error? It is possible that B is more liable to error than A, owing to the fact that its coefficient is large in December and June, and it gets the full effect of any difference between summer and winter. If, for instance, there are fewer good photographs taken at Greenwich during the winter, there will be more Indian used in the series; and any differences between the two would have maximum effect. Such points must be separately examined. Meanwhile, we may remark that the arithmetical sum of the differences between pairs of values of B from Tables I. and II. in the present paper is 66′·1 as against 23′·7 for A; in

* In the former paper two sentences have here been inadvertently transposed. They are repeated here in the correct order.

the former paper these numbers were 20″·7 and 14″·3; so that there is some reason for regarding A as better determined than B, and also for regarding the former results as entitled to greater weight than the present. The best provisional determination of the errors of Carrington's axis, assuming that it remained fixed in space during the period 1874–1901, would therefore be

$$A = +7'' \qquad B = -5''$$

giving double weight to the results of the former paper.

(*c*) As regards the discordances between the different zones, we may compare the results of the former paper and the present as follows:—

	A			B		
	−20° to −10°	to +10°	to +20°	−20° to −10°	to +10°	to +20°
1874–1885	+0·1	+8·0	+13·8	+7·2	+0·3	−9·2
1886–1901	+3·2	+5·8	+13·3	−1·3	−13·5	−8·7
Weighted Mean	+2·2	+6·5	+13·5	+1·5	−8·9	−8·9

double weight being given to the second series in taking the mean.

(*d*) We now come to the constant C, the mean value of which is positive (as already remarked in a note on p. 103 of the last paper), instead of negative as before. Rearranging the material in consecutive years, as in the former paper, we get

TABLE IV.

Drift in Latitude between +65° and −65° from central meridian.

Year.	−25°	−20°	−15°	−10°	0°	+10°	+15°	+20°	+25°
1874	—	—	$+54_3$	$+63_3$	-45_4	$+70_6$	-61_4	-245_2	—	—
1875	—	—	—	$+75_5$	$+87_4$	-34_5	0_1	$+67_2$	—	—
1876	—	—	—	-112_4	$+55_2$	—	$+57_2$	—	—	—
1877	—	—	—	$+20_1$	—	-6_5	$+90_1$	—	—	—
1878	—	—	—	—	—	$+46_2$	$+5_1$	—	—	—
1879	—	$+45_1$	—	—	—	—	—	—	—	-80_1
1880	$+31_3$	-171_3	$+19_9$	$+40_1$	—	—	$+100_3$	-4_6	$+49_{12}$	$+13_3$
1881	$+40_2$	$+38_3$	-5_7	$+142_2$	$+203_1$	-35_1	-41_{12}	$+22_9$	$+18_{10}$	$+86_5$
1882	-72_2	-14_6	$+82_{10}$	$+64_{12}$	-38_5	$+33_5$	$+11_{12}$	$+21_9$	$+21_7$	—
1883	$+13_2$	-23_8	$+5_{16}$	$+50_{14}$	-52_{17}	0_7	-5_{16}	$+37_3$	—	—
1884	-27_1	$+45_1$	$+51_8$	-1_{15}	$+24_{22}$	-12_{22}	$+46_{19}$	-22_3	—	—
1885	—	$+27_3$	-9_{15}	-19_{18}	$+25_{15}$	$+10_{12}$	$+5_{14}$	-27_2	—	—

In Table V. the results of the former paper are repeated, so as to show the annual results for the whole series of years.

TABLE V.

Year.	Mean Drift.	No. Spots.	Year.	Mean Drift.	No. Spots.
1874	−7	22	1888	−90	10
1875	+40	17	1889	−34	10
1876	−28	8	1890	−3	6
1877	+11	7	1891	−2	57
1878	+32	3	1892	−30	107
1879	−18	2	1893	−19	135
1880	+17	40	1894	−24	118
1881	+18	52	1895	−40	94
1882	+27	68	1896	−39	49
1883	−4	83	1897	−39	44
1884	+16	91	1898	−47	32
1885	+2	79	1899	+21	14
1886	−4	37	1900	−7	9
1887	0	18	1901	+50	3

The unit of the table is 0°·01, and the drift tabulated takes place in 130° of longitude. The figures have not yet been corrected for error of the Sun's axis, so that the results for years with few spots are subject to sensible corrections, depending on the particular months in which the spots were visible.

But the main feature of the table seems to be an oscillation from a drift of about +20, or +33′ per rotation, in 1880 to an opposite drift of −25, or −40′ per rotation in about 1893.

Now one possible explanation of these figures is that the Sun's axis is not stationary, but is describing a cone in space, in a period which may be one of several.

(α) If the period were about 26 years, the effect would be reversed in 13 years, but it is easily seen that the angular radius of the cone would have to be too large to fit the facts.

(β) If the period were nearly a submultiple of a year, say six months, Tables I. and II. would show it. Analysis does not yield a suitable result.

(γ) It seems most likely that the period is nearly a year. If it were exactly a year, and we had adopted a wrong axis so as to get an apparent drift in latitude, the error and its effects would travel round with the Earth, so that we should get a spurious constant drift. If the period differs from a year, the spurious drift would slowly change, going through a complete cycle in the L.C.M. of the period and the year. For instance, if the cycle is about 26 years, the period of the wobble of the Sun's axis must differ from one year by about half a month.

(δ) The drift may, however, be a physical drift on the surface of the Sun; and we can distinguish between these rival hypotheses by examining spots which return after disappearing. If the drift is physical, it will persist on the side of the Sun remote from the Earth; if due to the rotation of the axis, it will be reversed on the other side. Since the amount per rotation is large near a maximum (say 40′ per rotation), even a cursory examination should settle the point. Carrington's observations show no tendency to physical drift, as may easily be seen from the results 130 to 213, or the table pp. 213–219, or the summary on p. 220. Nor does an examination of the spot returns tabulated by Father Corte (*Mon. Not.*, vol. lx.) give much support to the hypothesis of a physical drift.

On the whole, hypothesis (γ) seems best worthy of further study.

But before deducing definitive elements for the axis, we await the results deduced from the measures of C. H. F. Peters, and recently published by Professor Frost. These are nearly completed, and can probably be given in the next number of the *Monthly Notices*.

Magnitude of η Argus, 1908. By R. T. A. Innes.

Comparison star used, Gilliss 1332 of 7·6 mag., yellowish or reddish, 4 on Chandler's colour-scale. This star is 8′ N.pr. η Argus.

1908.	Mag.	Colour.
Apr. 2	7·8	5
7	7·8	
13	7·7	6
June 2	7·7	6
Mean	7·75	5·7

A low-power field, with both stars in view, on the 9-inch refractor.

Johannesburg:
1908 *June* 3.

Photometric Measurements of Neptune, January to April 1908. By J. M. Baldwin, M.A., 1851 Exhibition Scholar (Melbourne).

(*Communicated by Sir David Gill, K.C.B., F.R.S.*)

1. The observations which are recorded in this paper were undertaken chiefly to ascertain whether it were possible to recognise a short-period variation in the brightness of Neptune such as that which Maxwell Hall believed he had found in 1883 and again in 1884.

The first announcement of a variation was made in the *Monthly Notices* (xliv. 257, 1884). The observations made by Mr. Hall were *estimations*, generally in fourths of a magnitude, of the difference between Neptune and a star in the same field, assumed as of magnitude 8·5 (B.D. + 15°, 453, mag. 8·5). On the night of 1883 November 29 Neptune seemed to him fainter than on the two preceding evenings, being "as nearly as possible" of the same brightness as the comparison star, but later in the evening Neptune appeared somewhat the brighter. From then till December 14 the estimated magnitude varied between 7·5 and 8·5, but during the nights of December 15, 16, and 18 no variation could be observed. From his observations he deduced a period of 7·92 hours. On December 27 and again on 1884 January 8 Neptune was compared with another star (B.D. 15°, 446, mag. 7·5), but "no variation was noticed on either night, yet Neptune was undoubtedly brighter on January 8 than on December 27" (*Observatory*, vii. 73, 1884).

He observed Neptune again at the next opposition, and in a short note in the *Observatory* (viii. 26, 1885) says: "Careful photometric measures gave November $29^d\ 9^h\ 44^m$ G.M.T. as an epoch of maximum brilliancy, but the variation was only 0·4 magnitude between max. and min. The period of $7^h{\cdot}92$ observed last year has been fully confirmed." I have unfortunately been unable to find any information as to the observations beyond that contained in this brief paragraph.

2. A considerable number of *measurements* of Neptune with the meridian photometer have been made at Harvard College Observatory,* almost all by E. C. Pickering, λ Ursæ Minoris

* *Observatory*, vii. 134, 1884, and viii. 111, 1885; *Harvard Annals*, xxiv. p. 265 and xlvi., part ii., p. 203. In this paper the early comparisons, 4 in number, of Neptune with Uranus, made by Zöllner (*Photometrische Untersuchungen*, p. 150) and the 8 Harvard comparisons with α Arietis in 1878 (*Harvard Annals*, xl. p. 224) have not been made use of.

being used as standard. The number of observations and results obtained are given in Table I.

TABLE I.

Year.	Observations.	Nights.	Mean Mag.	Mean Error.
1882–83	11	11	7·71	±·11
1883–84	5	5	7·77	±·09
1884–85	9	9	7·63	±·11
1894	5	5	7·71	±·13
1895–96	5	5	7·50	±·10
1896	5	5	7·63	±·02
1897–98	16	10	7·63	±·10

3. The observations of Müller * at Potsdam are more numerous. These are *measurements* with the Zöllner photometer of the difference in magnitude between Neptune and a suitable comparison star or stars, the same throughout any one opposition. The mean results, etc. for the separate years will be found later on in this paper (Table IV). The mean of all the 138 observations is 7·66 on the scale used in Band viii.; this is exactly the same as the mean of Pickering's 56 observations between 1882 and 1898.

4. In the observations given below, a Zöllner photometer attached to the Steinheil refractor (13·5 cm. aperture and 216 cm. focal length, called photometer D in the *Potsdam Publications*) was used throughout. Two comparison stars were chosen near Neptune, namely B.D. +24°, 1457 and +22°, 1531, the magnitudes given in the Potsdam General Catalogue being respectively 7·97 and 7·18. As usual, four readings were taken for each object, one in each quadrant of the intensity circle (which was always turned in the same sense), the star being respectively to the left of, above, to the right of, below, the artificial star. The complete observation consisted in observing first one comparison star, then Neptune, then the other comparison star. For the next observation the stars were taken in the reverse order, and so on throughout the whole series. The complete results are given in Table II.

* *A.N.*, 2600, Bd. cix. 121, 1884, or *Observatory*, vii. 264, 1884; *Pots. Publ.*, Bd. viii. It is to be noticed that the magnitudes as given in the *A.N.* are not referred to the same scale as those in the collected results in the *Pots. Publ.*

[TABLE

TABLE II.

1908.	d	G.M.T. h	m	Z.D. °	M_1.	M_2.	Mean.	Reduction.	M_0.	Phase. °	Residual.
Jan.	25	6	8	51·7	7·85	7·98	7·92	+·01	7·93	0·68	−·06
		7	6	43·5	8·00	7·93	7·97		7·98		−·01
		9	46	30·4	8·02	7·85	7·94		7·95		−·04
Feb.	3	10	12	32·5	8·04	7·89	7·96		7·97	0·98	−·02
		11	3	37·1	7·80	7·93	7·86	+·01	7·87		−·12
Feb.	9	11	18	41·9	7·96	7·96	7·96	·00	7·96	1·15	−·03
	10	7	24	34·0	7·93	7·91	7·92		7·92	1·17	−·07
		8	5	31·3	7·88	8·01	7·95		7·95		−·04
		9	43	32·5	7·97	7·99	7·98		7·98		−·01
	14	7	22	32·9	7·93	8·06	7·99	·00	7·99	1·27	·00
Feb.	19	8	42	30·5		8·04	8·04	−·01	8·03	1·38	+·04
Mar.	2	9	52	41·9	8·10	8·11	8·11	−·02	8·09	1·63	+·10
		10	44	49·3	8·17	8·13	8·15		8·13		+·14
		11	10	53·1	8·06	8·02	8·04	−·02	8·02		+·03
	16	9	7	43·4	8·23	8·12	8·17	−·03	8·14	1·82	+·15
Mar.	16	9	19	45·2	7·91	8·00	7·95	−·03	7·92		−·07
	22	9	21	48·7	8·01	8·07	8·04	−·04	8·00	1·87	+·01
		9	55	53·7	8·08	8·09	8·08		8·04		+·05
	24	9	47	53·7	8·14	7·99	8·06		8·02	1·89	+·03
		10	16	58·1	8·07	8·07	8·07		8·03		+·04
Mar.	24	10	47	62·8	7·99	8·04	8·01		7·97		−·02
	25	9	2	47·7	8·01	8·02	8·01		7·97		−·02
		10	25	60·1	8·05	7·98	8·01		7·97		−·02
		10	48	63·6	8·09	8·06	8·07	−·04	8·03	1·89	+·04
	26	9	0	48·0	8·02	8·02	8·02	−·05	7·97	1·90	−·02
Mar.	26	10	21	60·1	7·94	8·03	7·99		7·94		−·05
		10	48	64·2	8·00	8·10	8·05		8·00		+·01
	27	8	31	44·5	7·95	8·03	7·99		7·94		−·05
		9	19	51·3	8·09	8·07	8·08	−·05	8·03	1·90	+·04
Apr.	26	8	28	61·3	8·22	8·06	8·14	−·08	8·06	1·76	+·07
Apr.	28	7	39	54·9	7·98	8·00	7·99		7·91	1·74	−·08
		8	19	61·0	8·15	8·07	8·11	−·08	8·03		+·04
								Mean	7·990		±·059

Jan. 25. Slightly foggy.
Feb. 14. Hazy, but no clouds visible in bright moonlight.
Feb. 19. Clouded over, weight ½.
Mar. 2. Not very clear, hazy near horizon.
Mar. 16. Sky bad, and so observations not reliable, weight ½.
Apr. 26. Cloud slowly rising, but observations probably good.

In this table the first two columns give the date and G.M.T. of observation, and the corresponding zenith distance of Neptune, the next columns the observed magnitudes (corrected for atmospheric absorption) referred to the brighter and fainter comparison stars respectively, and the mean. Each observation gave the difference in magnitude between the comparison stars; from the whole series the mean was 0·83 mag., and so the values for the comparison stars are taken as 7·16 and 7·99 respectively. In the sixth and seventh columns are given the correction to mean opposition and the magnitude in mean opposition. The eighth column gives the phase angle; this remains always so small that it is not to be expected that any variation of magnitude with phase angle can be recognised with certainty. The last column gives the difference, observed mag. – mean magnitude.

A glance at the column of residuals shows that the agreement between the observations is extremely satisfactory, the mean error of one observation, giving a weight ½ to certain observations as indicated, being only ± ·059 mag.

5. Amongst the foregoing material there are three series of observations which are especially suited to detect a short period variation in brightness such as that announced by Hall, the epoch and amplitude of which, owing to movement of the markings supposed to give rise to the variation, may be gradually changing.

(1) Müller's 72 observations on 37 evenings between 1884 Aug. 24 and 1885 March 10.

(2) Pickering's 16 observations on 10 evenings between 1897 Dec. 11 and 1898 March 3.

(3) The above 32 observations on 15 evenings.

If these observations be examined it will be at once evident that they show no trace of a short-period variation, the residuals being just such as occur in photometric work. However, as a direct test, the series have been taken separately, the phase computed for a period 7·92 hours and an arbitrary epoch, and the light curve plotted. In the case of Müller's longer series, this has been done also for parts of the series. The result is in every case the same; the observations furnish no evidence whatever of a variation of period 7·92 hours. In particular, Müller's observations from 1885 Jan. 8 to Feb. 24 and those given in this paper are especially conclusive on this point, as the observations are well distributed over the curve. With regard to his observations from 1884 Aug. 24 to 1885 March 10, Müller says (*Pots. Publ.*, viii. p. 352), "during the whole period of nearly 7 months not the slightest trace of a periodic variation in light can be recognised. The magnitude in mean opposition varies between 7·39 and 7·81, and such a difference can be very well explained by uncertainty in the measurements if the fact that these extreme values were obtained when the planet was very low, and under unfavourable conditions, be taken into account"; and a little further on, "the observations of the following years also, although not nearly so numerous, are in direct opposition to the hypothesis of a periodic variation in the light from Neptune."

This latter remark may be extended to the whole of the photometric observations of Neptune.

6. Looking now at the evidence which Hall gave as showing a periodic variation of brightness, it consists of—

(1) A series of 25 estimations of difference of magnitude.

(2) Some photometric observations, of which particulars are not available.

If the first set of observations be plotted to obtain the light curve, it will be found that the assumption of variation practically rests on 3 observations, for, apart from these, no residual occurs which could not be explained by the method of observing, the greatest being 0·3 mag. from a mean 7·8. Of these three, two are the early observations of Nov. 29 (mag. 8·5) and Dec. 1 (mag. 8·3), and it was not until after the first observation that a diagonal prism was used to alter the relative positions of Neptune and the comparison star; and the third, on Dec. 7 (mag. 8·3), requires in any case a correction of more than −0·4 to bring it on to a curve through the other observations.

From the second set a variation of 0·4 mag., occurring about 1885 Nov. 29, was deduced. In the absence of details it is impossible to say what instrument was used, how many observations were made, or over what period they extend. Against the hypothesis of variation, however, there is very conclusive evidence in Müller's observations before and after this epoch, for there are continuous observations from Oct. 23 to Nov. 19, and again from Jan. 8 to Feb. 24, which effectively include any assumption of variation during these periods, and observations on Dec. 1, 2, and 16, which give no evidence in favour of it.

It would appear, then, in the highest degree probable that the "variation" announced is due to errors of observation, especially when account is taken of the small amplitude deduced.

7. The mean value of the brightness of Neptune in mean opposition, as determined by the present series of observations, is 7·99, the mean error of one observation being ±·059, the system to which the magnitude is referred being that of the Potsdam *Durchmusterung*.

The comparison stars used by Müller, the magnitude on the system of his Potsdam planetary observations, and the magnitude from the Potsdam General Catalogue are given in Table III.

TABLE III.

Comparison Star.	B.D.	Mag.	Mag. P.G.C.	Reduction.
52	+13°, 411	7·25	7·52	+·27
43	+15°, 430	6·29	6·68	+·39
34	− 4°, 4376	5·41	5·54	+·13
49	+16°, 458	6·95	7·34	+·39
56*	+16°, 432	7·96	8·23	+·27

* Owing to a clerical error, this comparison star has been printed as 55 instead of 56 throughout the observations of Neptune, and the discussion of the result of these in vol. viii. of the *Potsdam Publications*. Beyond writing 56 for 55 no change requires to be made.

Stars 34 and 56 are not contained in the General Catalogue, but from *Pots. Publ.*, Band viii. p. 233,

$34 - \beta$ Herculis is 2·53 mag.
and $56 - \sigma$ Arietis is 2·51 mag.

Also from the General Catalogue,

β Herculis is 3·01 mag.
and σ Arietis is 5·72 mag.,

and hence the magnitudes given in Table III are obtained.

Next, combining these results with the table given on p. 354 of Müller's paper we get Table IV.

TABLE IV.

Year.	Number of Observation.	Mag.	Prob. error of one Observation.	Comparison Star.	Reduction.	Reduced Magnitude.
1878	11	7·70	±0·051	52	+·27	7·97
1881-2	17	7·75	·071	52	+·27	8·02
1883	6	7·75	·095	43	+·39	8·14
1883	7	7·70	·085	34	+·13	7·83
1884-5	72	7·62	·056	49 and 56	+·33	7·95
1885-6	15	7·69	·074	49 and 56	+·33	8·02
1886-7	7	7·62	·045	49 and 56	+·33	7·95

The mean magnitude on the system of the Potsdam *Durchmusterung* is thus 7·97.

For the comparison with Pickering's observations, the difference Potsdam – Pickering is given on p. 34, of the introduction to the Potsdam General Catalogue, as follows:—

Magnitude.	W	GW
7·50–7·99	+0·36	+0·29
8·00–8·49	+0·37	+0·30

If the colour of Neptune be assumed as between W and GW the reduction +0·34 cannot be far from the truth, and this gives 8·00 as the reduced value of Pickering's mean result.

It is seen, then, that the results of the present series are in very close agreement with the previous observations of Müller and Pickering.

8. The results arrived at from the present series of observations in conjunction with previous observations of Neptune may be summarised as follows:—

(1) The variations of the estimated brightness of Neptune announced by Hall are probably due to errors of observation, as the measurements of other observers about the same time, and also

at other oppositions, give not the slightest trace of any such variation. Consequently the time of rotation of 7·92 hours deduced by him must be regarded as unsupported by observation.

(2) The observed magnitude of Neptune in mean opposition is 7·99 on the system of the Potsdam *Durchmusterung*, showing, in extremely satisfactory agreement with Müller's value 7·97, and Pickering's value 8·00.

In conclusion, I wish to record my thanks to Professor Müller, at whose suggestion these observations were made, for the interest which he has shown throughout this work.

Astrophysical Observatory, Potsdam:
1908 *June* 6.

Observations of helium D_3 absorption in the neighbourhood of Sun-spots in 1907. By Captain R. A. C. Daunt, D.S.O.

The following observations of D_3 absorption in the neighbourhood of sun-spots were made with a 3-inch equatorial refractor and a large Thorp, prism-grating spectroscope, 15,000 lines to the inch, the eyepiece ordinarily employed being a Steinheil monocentric 20 m/m power 52. The identification of the several groups according to the Greenwich numeration will, it is hoped, be of service for comparison with spectro-heliographic and cognate studies of the same groups. In Table I. the numbers of the various groups and the dates on which they were observed are given, as well as the type or phase in the life-history of the groups at the times of observation, according to the classification of Father Cortie in his paper "On the Types of Sun-spot Disturbances," *Astrophysical Journal*, xiii., 4th May 1901.

In all the cases of absorption of D_3 observed, the dark line was never seen over the umbræ or penumbræ of the spots of any group, but between the spots, or on the photosphere in their neighbourhood. It would appear that some of these groups showed D_3 in absorption on certain days only, and not on others. This may be real; but as the appearance of D_3 is very intermittent, it is more likely to be due to the hour of observation; and had I been able to keep the groups under observation throughout the day, D_3 would probably have been seen near the spots some time or other of the day during the apparition, provided the groups remained active and agitated.

In Table II. are given the Greenwich numbers of those groups in the neighbourhood of which D_3 was not observed. For the reason stated above, some of these groups may have shown the absorption effect of D_3 at some time, but it was not observed.

As, however, most of these spots were small, or of a regular and quiescent nature, it is perhaps unlikely that D_3 was reversed.

A table follows showing the proportion of the total number of fresh groups observed each month with those that showed D_3.

There would appear to be a discrepancy here, and the proportions as shown by the Greenwich numbers appear to be higher. This is due to a different system of numbering. For instance, the group consisting of the Greenwich Nos. 6075, 6076, 6081, 6085 has for the purposes of the table been counted as one group. This is, I think, under the circumstances, more accurate, as by the Greenwich numbers it would appear that four different groups showed D_3 in absorption, whereas these spots were obviously part of one disturbance, and should only be counted as one disturbance for this purpose.

TABLE I.

Group in which D_3 was observed as a dark line.

Greenwich No. of Group.	Dates of Observation.	Type of the Group.
	1907	
6059	Jan. 2, 3	IV_b.
* 66	,, 3	IV
67	,, 3	III
70	,, 3	I
75		
* 76	,, 16	III_b
81		
85		
94	,, 24	III_a
6099	,, 30, Feb. 4	III_a, IV_b
6100	,, 30, ,,	III_a, IV_b
* 03	Feb. 4, 5	III, IV_b
04	,, 4, 5, 11	I, III_b, III_a
* 08	,, 9, 11, 13, 16, 17	V, III_a, III_a, V, V
* 36	Mar. 3, 4	IV_b, IV_a
37	,, 4	III
39	,, 5, 6	IV_b, IV_b, IV_a
* 40	,, 5, 6, 14, 17	IV_b, IV_a
50	,, 22	I
55	,, 23	IV_c
* 56	,, 30	I
58	Apr. 4	III, IV
59	Mar. 31, Apr. 2, 4, 9	IV, III_a, III_a, IV_b
68	Apr. 15, 16, 17	I, I, II
72	,, 21, 23, 29	III, IV_b, IV_a
75	,, 26, 27	I, I
81	May 4, 5, 6	III, III, IV_a
* 84	,, 4, 5, 6, 14	IV_c, IV_c, IV_c, IV_c
6185	,, 13, 14	III_a, III

TABLE I.—*continued.*

Greenwich No. of Group.	Dates of Observation.	Type of the Group.
	1907	
6187	May 14, 16, 17	I, II_c, III
6188	,, 14, 16	III, II
6230	Aug. 5	I
31	,, 5	I
36	,, 18, 23, 24	III, IV_b, IV_b
* 38	,, 24	IV_a
39	,, 23	IV_b
* 41	,, 30, Sept. 6	III_a, III_a
43	Sept. 6	IV_b
45	,, ,,	I
47	,, ,,	III
55	,, 20, 21	I, I
56	,, 22	IV_a
59	,, 20, 21, 22, 25, 26, 27, 29	III_b . . . . III_a
60	,, 20, 21, 22, 26	IV_a, IV_d, IV_d, IV_d
62	Sept. 20, 21	IV_a, IV_d
65	,, 25, 26, 27, 29	III_a . . . . III_a
68	,, 27, 29	I, IV_b
70	,, 29	I
75	Oct. 9	I
76	,, 9, 13, 14, 16	I, III . . . . III_a
77	,, 9, 13, 14, 16	I, III_a . . . . III_a
78	,, 13, 14	III_a, III_a
79	,, 9, 14, 16	I, IV_c, IV_b
81	,, 14, 16	IV_c, IV_b
82	,, 14, 16, 19, 23, 24	IV_b, IV_c, IV_b, IV_b, IV_b
* 83	,, 14, 16, 19, 23, 24, 26, 27	III_a . . . . IV_c, IV_c
* 84	,, 16, 19, 23, 24, 26, 27	III_a . . . . IV_c, IV_c
85	,, 19, 23, 24, 26, 27	III_a . . . . IV_c, IV_c
* 89	,, 23, 26, 27, Nov. 1	IV_a, IV_b, IV_b, I
91	Nov. 1	II_c
6292	,, ,,	I
*6316	Dec. 2, 3	IV_b, IV_b
18	,, 6, 7, 9	IV_a, IV_a, IV_a
* 19	,, 6, 7, 9, 12, 14	IV_b V
20	,, 9	III
* 21	,, 9, 12	IV_b, II_a
23	,, 14, 18	I, II_c
6324	,, 18	IV

An asterisk denotes that the group so marked was the return of a former group. The appearance of the absorption-line in the cases observed was lumpy, patchy, broken, and thickened, sometimes distorted towards the violet, as in groups 6259, 65, 68, and once, in groups 6283, 84, distorted towards the red. Group 6137, March 4th, 10·30 G.M.T., furnished the most remarkable appearances of D_3 absorption observed since those of the big spot of July 1906. The appearance of dark D_3 as a fine continuous dark line has not been recorded.

In every case of observation of reversal of D_3 the C line of hydrogen was sympathetically affected, being twisted and lumpy, distorted towards the red and the violet, accompanied by brilliant reversals, sometimes over considerable areas in the neighbourhood of spots, as in the groups 6283, 84, 85, Oct. 21, when, too, the sodium lines D_1 and D_2 were reversed.

Magnificent reversals were seen in group 6103, Feb. 5th, when a large prominence arising amongst scattered spots could be traced across the disc and out beyond the limb. In group 6188 the line was thick, with black lumps and bright reversals in the faculæ surrounding the group, and in group 6247 violently disturbed, greatly distorted to the red and violet, and very lumpy, with brilliant reversals.

TABLE II.

List of Spot-groups in which the D_3 line was not observed.

Greenwich No.	Type.	Greenwich No.	Type.
6061	I	*6113	IV, I
62	I	* 15	I
68	I	* 17	IV
* 74	I	18	III
77	IV_a	19	I
79	I	20	I
* 86	I	21	I
* 87	IV_a, I	* 23	IV
88	IV_b	27	IV, I
* 90	IV	28	I
93	I	29	I, III
97	I	31	I
*6098	IV, I	33	I
6101	IV, I	34	IV
02	I	35	I
* 05	I	38	I
07	IV_a	45	I
09	I	46	I
11	I	52	I
*6112	I	*6153	I

TABLE II.—*continued.*

List of Spot-groups in which the D_3 line was not observed.

Greenwich No.	Type.	Greenwich No.	Type.
*6154	I	*6246	I
* 61	IV	48	I
64	IV, I	61	I
65	I	63	IV, I
70	I	64	I
74	I	* 67	IV
*6189	IV	69	I
6220	IV	71	IV
23	I	72	I
25	I	* 73	IV
26	IV	* 80	IV
27	I	*6288	I
29	I	6317	I
32	IV	18	IV
34	IV	22	I
37	I	25	I
6240	I	6326	I

TABLE III.

Showing the proportion of Spot-groups which showed D_3 reversed.

Month.	No. of Days of Observation.	Total No. of Groups observed.	No. of Group in which D_3 Dark.	Percentages.
Jan.	7	21	7	33·3
Feb.	9	18	3	16·7
Mar.	17	21	7	33·3
Apr.	12	13	4	30·8
May	7	6	5	83·3
July	1	4	0	—
Aug.	5	14	6	42·9
Sept.	8	17	9	52·9
Oct.	11	18	7	38·9
Nov.	1	1	1	—
Dec.	9	11	5	43·6
Totals	87	144	54	Percentage on the Totals 37·5

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[*Note added by Father Cortie, S.J.*]

In the above tables I am responsible for the columns headed "Type of the Group," using for the information therein contained, with the kind permission of Captain Daunt, the materials in his observing note-book, the Stonyhurst sun-spot drawings, and the useful monthly "Tables and Notes on Sun-spots" compiled by Mr. Maunder for the *Observatory*. The object of these columns is to ascertain whether the phenomenon of D_3 absorption is confined to, or generally favours, any particular phase in the life-history of sun-spot groups. The more active phases of sun-spot development are represented by Type II. and Type III., and 39·7 per cent. of the groups in which D_3 was observed as a dark line were of these types. In the life-histories of spot-groups it is generally the following part of the group which disappears first, leaving a single spot, with accompanying small companions. This phase, represented by Type IV. with various suffixes, is also largely represented in Table I.; and in his notes on the D_3 reversals, Captain Daunt frequently records that the absorption was observed between the two main spots of a group of Type II. or Type III. form, or in regions following the main spot, that is, in the earlier phases of Type IV. Therefore it follows that in general the phenomenon is indicative of violent local action in the earlier phases of the life-history of groups. However, it is not entirely confined to these phases, as it is recorded in groups 6256 and 6318, examples of round steady spots. But even in these cases, 6256 had a divided nucleus and showed signs of disruptive change, and group 6319 in the neighbourhood of group 6318 was disturbed. The table giving the groups in which D_3 was not observed shows that the spots were in the last stages of their life-history, being in most cases either round steady spots or scattered remnants of groups. But here again there are exceptions, as group 6118 was being developed, and group 6129 passed through an active stage, though on a small scale.

Stonyhurst College Observatory.

Lengths of Axes and Position Angles of 52 Oval Nebulæ.

By Dr. Max Wolf.

Professor Turner has suggested the question whether there is a relation between the apparent form and the situation of oval nebulæ of the type of the Andromeda nebula, viz. Is the ratio of the two axes of the ellipse, visible to us in projection, connected by some rule with the position angle of the major axis?

I have made a first trial with the oval nebulæ of my last catalogue,* putting these together in the following list. The lengths of the two axes *a* and *b* are estimated on the plates by a scale in the eye-piece of my microscope; the position angles are estimated with the aid of a cardboard circle at the same.

The visible dimensions of such diffused objects depend in a high degree upon the exposure, the darkness of the plate, the illumination, and the position on the plate, so that the lengths of the axes are affected with some uncertainty. Notwithstanding, there is no doubt that some simple rule, if it exists, must be perceivable from these numbers.

The distribution of the position angles themselves has a maximum at about 30°, and a second fainter at about 160°.

Kgst. Liste 8. No.	N.G.C. No.	α 1875. h m s	N.P.D. 1875. ° ′ ″	Pos. Angle. °	*a* ″	*b* ″
111	3935	11 45 55·2	56 54 0	110	45	20
130	new	47 13·9	56 32 0	40	20	15
132	,,	47 15·6	60 3 3	105	45	20
138	,,	47 22·5	55 56 12	130	60	30
154	,,	47 49·7	56 26 35	160	45	12
173	,,	48 51·5	56 9 34	160	20	6
177	,,	48 57·2	56 10 38	80	20	8
193	,,	49 19·4	59 21 47	40	60	15
213	,,	49 53·7	57 52 51	10	60	15
216	,,	49 56·2	57 15 55	135	45	15
232	3986	50 17·2	57 17 0	100	100	20
255	new	51 2·1	58 13 6	165	30	12
257	,,	51 3·8	56 58 12	30	45	10
258	3991	51 5·2	56 57 55	15	15	10
263	3994	51 10·3	57 1 40	10	30	15
271	3995	51 17·2	57 0 37	(?) 30	90	30
303	4020	52 30·8	58 53 23	15	80	20
318	new	52 47·1	58 34 15	140	30	15
346	,,	53 42·6	56 10 0	35	30	12
352	,,	11 53 50·7	58 4 44	45	40	15

* *Königstuhl Nebel-Liste* No. 8, Publ. iii.

Kgst. Liste 8. No.	N.G.C. No.	α 1875. h m s	N.P.D. 1875. ° ′ ″	Pos. Angle. °	a ″	b ″
380	new	11 54 39·0	58 27 28	60	30	20
391	,,	54 52·5	58 1 28	35	30	6
444	,,	55 46·5	58 21 11	15	20	6
452	,,	55 58·1	59 26 30	70	75	15
455	,,	56 1·2	59 50 6	60	45	20
483	,,	56 57·4	59 53 1	55	90	20
498	,,	57 25·8	59 35 23	150	60	15
502	,,	57 32·6	58 11 44	10	45	15
510	4062	57 40·2	57 24 25	100	180	60
529	new	58 10·4	58 36 47	30	30	8
541	,,	58 21·9	58 8 41	160	20	12
545	,,	58 24·4	56 48 29	5	45	8
548	,,	58 26·4	57 2 30	30	45	(?) 5
582	,,	59 26·2	59 14 18	60	30	15
584	,,	59 28·1	58 15 27	170	20	8
593	,,	59 34·9	57 57 22	45	22	7
607	,,	11 59 44·9	56 24 27	120	30	12
680	,,	12 0 45·0	58 14 36	30	30	15
682	,,	0 46·4	56 18 28	70	40	15
699	,,	1 12·4	56 28 13	110	30	6
711	?4122	1 39·8	56 21 53	160	20	12
715	new	1 46·3	55 47 52	5	30	15
719	,,	1 58·2	56 42 39	175	30	12
725	,,	2 21·3	55 40 27	40	30	15
726	,,	2 21·8	57 7 59	80	30	15
737	4132	2 39·6	60 3 4	30	45	20
740	new	2 44·4	54 59 56	175	30	12
741	4134	2 47·8	60 7 39	150	90	45
743	new	2 50·4	58 23 47	25	75	20
764	4150	4 12·8	58 54 8	150	90	45
769	new	4 57·2	56 42 35	65	45	30
770	,,	12 5 44·4	56 34 10	150	45	30

Astrophys. Observatory, Königstuhl, Heidelberg,
1900 *July* 2.

[By the courtesy of the secretaries, the above very interesting measures have been included in the present number, but there was not time to prepare any discussion of them. A very rough

preliminary analysis indicates that the measures lend some support to the hypothesis of a systematic orientation of axes parallel to the Galaxy (see *M.N.*, lxvii. p. 333 and p. 498), but a complete discussion must be deferred.—H. H. TURNER.]

Erratum.

In *Monthly Notices*, vol. lxvii. p. 321 (Baxendell's Observations of U Geminorum),

for 1858 Nov. 14 *read* 1858 Nov. 15 (as on p. 324).

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVIII. SUPPLEMENTARY NUMBER. No. 9

On the Parallax and Proper Motion of the Double Star Krueger 60 (*Burnham, Gen. Cat.*, 11761). By E. E. Barnard. (Plate 14.)

The History of Krueger 60.

Professor Krueger, in his catalogue of the Astronomische Gesellschaft, covering the zone $+55^{\circ}$ to $+65^{\circ}$, noted with the meridian instrument many stars which were double. He made a rough estimate of the quadrant and distances of these stars and of their magnitudes.

In 1890 Professor Burnham measured 67 stars from Krueger's list with the 12-inch and 36-inch telescopes of the Lick Observatory (*Pub. L.O.*, vol. ii.). The star No. 60 was among those measured. Krueger's note on this object is:—

"Dupl. 12" pr. com. 9·3."

In measuring the principal star, Professor Burnham found that it was a wide and unequal double.

1890·788	Pos. 178°·8	Dist.	2″·32 (1 n)	9·0 m	– 12 m	A B
	56·3		26·82 (1 n)		9·2	A C

Of these A C is the Krueger pair.

No other measures were made of these stars until 1898, when Dr. Eric Doolittle remeasured them with the 18-inch Brashear telescope of the Flower Observatory (*A.J.*, vol. xxi. p. 47).

Doolittle's measures showed that the pair A B had a strong proper motion (0″·93 in the direction 247°·9). His measures are—

1898·446	140°·66	3″·19	9ᵐ·1 – 10ᵐ·5	A B	5 n.
	58°·71	34″·39	9ᵐ·4	A C	

Subsequent measures fully confirmed these results.

In *A.J.*, vol. xxi. p. 64, I have given measures of these stars which are in accord with the motion derived by Doolittle. (In the

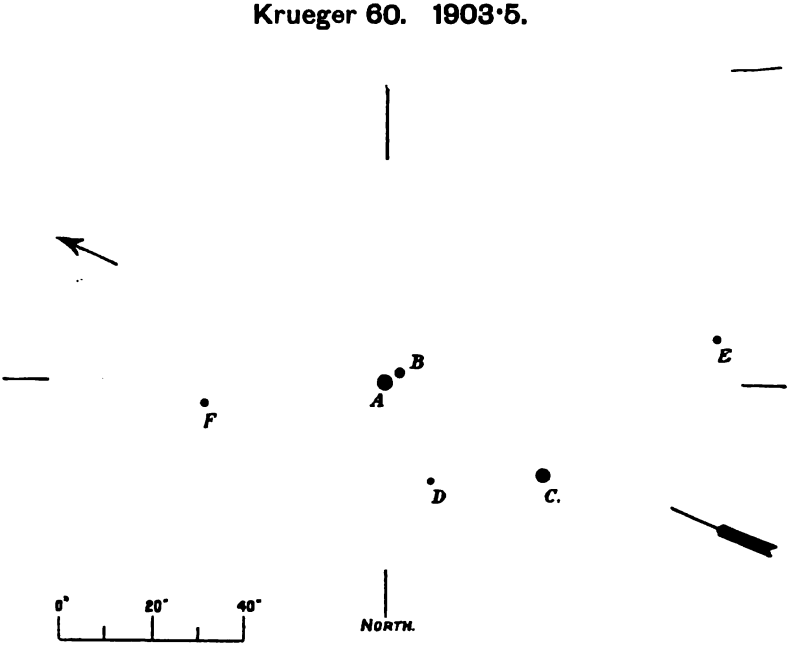


FIG. 1.

measures of A B in that paper there is a misprint of 2″·23 for 3″·23.) In *A.J.*, vol. xxiii.* pp. 169–172, I have given more observations, with a diagram showing the probable orbital motion of A B. A few years' measures soon showed that this was really a physical pair in rather rapid orbital motion. I also called attention to the probable proximity of these stars to our solar system. Acting on this last idea, I began in 1900 a series of measures, with the 40-inch telescope, of A and C for the parallax of A.

* On p. 171 in that paper, the motion in right ascension should be – 0ˢ·107 instead of – 1ˢ·606, and on p. 172 the seconds in the right ascension should be 32ˢ·00 instead of 32ˢ·69. In that paper I derived the proper motion 0″·951 in the direction 246°·3.

In the *Astrophysical Journal*, vol. xx. p. 128, for Sept. 1904, Dr. Schlesinger published his results of the parallax of A derived from photographs made with the 40-inch telescope. Dr. Schlesinger found a parallax of about $\frac{1}{4}''$, verifying the prediction of its nearness to us. I will give, later on, his final values for the parallax.

Additional measures are given by Professor Burnham in *Publications of the Yerkes Observatory*, vol. ii., 1903, p. 67.

This is briefly the history of this remarkable stellar system.

In May of 1903 I measured the position of A with respect to Helsingfors-Gotha A.G.C. 13177. From the measures the place of A is

$$1903{\cdot}0 \quad \alpha = \quad 22^{h}\ 24^{m}\ 32^{s}{\cdot}09$$
$$\delta = +57^{\circ}\ 12'\ 38''{\cdot}5$$

Proper motion $-0^{s}{\cdot}107 - 0''{\cdot}382$.

The star is Helsingfors-Gotha A.G.C. 13170.

Determination of the Parallax of A.

The present paper gives an investigation of the parallax of the star A from the visual micrometer measures made with the 40-inch telescope.

The micrometer screw and the tube of the 40-inch are both of steel. I have found from about ten years' experimental measures in the Pleiades (which are soon to be published) that the focus of the 40-inch glass shortens from summer to winter by an extreme change of $\frac{3}{4}$ inch. In the meantime the steel tube shortens $\frac{1}{2}$ inch. The temperature change in the micrometer screw will correct $\frac{1}{2}$ inch of the total change of the focus of the object-glass. There remains uncorrected, therefore, $\frac{1}{4}$ inch of the extreme focal change. The exact amount of this uncorrected change, from the measures of the Pleiades referred to above, is 0·0032 inch (0·081 mm.) for each degree of temperature. This would cause an extreme error in the measures of Krueger 60 for temperature of not quite $0''{\cdot}01$, and in the main it would not be appreciable. I have, therefore, not thought it necessary to correct the observations for temperature changes. I have not felt sure that the correction would be real. The corrections for aberration, being insensible, have been omitted.

All of the calculations for the parallax of this star were made, under my supervision, by my niece, Miss Mary Ross Calvert, who, through the courtesy of Mr. Andrew Carnegie, is at present my assistant. They have been carefully checked throughout, and seem to be free from errors. The observations, corrected for refraction, were first freed from proper motion. The parallax factors were then computed by Bessel's formulæ, as given in Chauvenet, vol. i. p. 695, fifth edition, where the parallax factor is

$$Rm \cos(\odot - M),$$

in which R and ⊙ are the distance and longitude of the Sun; m and M being derived from the formulæ

$$m \cos M = \sin \alpha \sin P + \cos \alpha \sin \delta \cos P$$
$$m \sin M = (-\cos \alpha \sin P + \sin \alpha \sin \delta \cos P) \cos \omega - \cos \delta \cos P \sin \omega^*$$

where P is the position-angle of the stars, α and δ the right ascension and declination, and ω the obliquity of the ecliptic.

The formula

$$n + ax + by + cp = 0$$

was used for the equations of condition. In all, there were seventy-six equations of condition. From these the normal equations were derived in the usual manner. The normal equations were solved by the ordinary methods of elimination. As a check, they were also solved by Chauvenet's "second method of computing the weights of unknown quantities" (Chauvenet, vol ii., art. 35, p. 516), from which the weights were also derived. From these last, the probable errors were computed by the formulæ.

Probable error of parallax

$$= \frac{P}{\sqrt{wp}}$$

where wp is the weight of the parallax and

$$P = \pm 0{\cdot}6745 \sqrt{\frac{\Sigma v^2}{n-3}},$$

v being the residuals derived by substituting the known values of x, y, and p in the equations of condition. Similarly for the probable errors of x and y, using for them the same value of P.

This method of determining the probable errors was kindly communicated to me by Dr. Schlesinger.

The proper motion used in these calculations was derived by me from a comparison with Professor Burnham's measures of AC in 1890 and my measures in the last of 1905. This value was $0''{\cdot}968$ in the direction $246°{\cdot}49$.

The corrections for motion, to the observed distances, were computed by the formulæ given on page 696, vol. i., of Chauvenet's *Theoretical and Practical Astronomy*.

In Table I. are given the observations and their reduction to 1901·0, the last column being the residuals from the mean of the corrected distances. Table II. contains the equations of condition.

* We have found that this last term is incorrectly given in Bessel's original paper, *A.N.* 366, pp. 83–84, as $-\sin \delta \cos P \sin \omega$.

The same error occurs in the second of the formulæ for determining the parallax from the position-angle on the same page of *A.N.*, 366, the last term of which should also be $-\cos \delta \cos P \cos \omega$.

TABLE I.

Reductions to 1901·0.

			Pos. Ang.	Dist.	Cor'n for motion.	1901·0.	Resid. from mean.
			°	″	″	″	″
1900·937	Dec.	8	59·27	36·88	+0·061	36·941	−0·423
·940		9	59·52	36·67	+0·058	36·728	−0·210
·945		11	59·25	36·69	+0·053	36·743	−0·225
·948		12	59·16	36·66	+0·050	36·710	−0·192
1901·729	Sept.	23	59·65	37·18	−0·700	36·480	+0·038
·731		24	59·84	37·20	−0·702	36·498	+0·020
·748		30	59·86	37·30	−0·719	36·581	−0·063
·751	Oct.	1	59·52	37·15	−0·722	36·428	+0·090
·783		13	59·63	37·43	−0·752	36·678	−0·160
·805		21	59·48	37·36	−0·774	36·586	−0·068
·827		29	59·49	37·34	−0·795	36·545	−0·027
1902·744	Sept.	29	59·64	38·29	−1·676	36·614	−0·096
·764	Oct.	6	59·69	38·10	−1·695	36·405	+0·113
·766		7	59·64	38·32	−1·697	36·623	−0·105
·786		14	59·60	38·50	−1·716	36·784	−0·266
1903·380	May	19	59·55	38·61	−2·287	36·323	+0·195
·396		25	59·57	38·69	−2·302	36·388	+0·130
·418	June	2	58·89	38·84	−2·323	36·517	+0·001
·437		9	59·18	38·76	−2·342	36·418	+0·100
·454		15	59·35	38·71	−2·358	36·352	+0·166
·473		22	59·59	38·74	−2·376	36·364	+0·154
·492		29	59·57	38·74	−2·394	36·346	+0·172
·495		30	59·22	38·71	−2·397	36·313	+0·205
·511	July	6	59·54	38·84	−2·413	36·427	+0·091
·514		7	59·70	38·71	−2·416	36·294	+0·224
·530		13	59·67	38·73	−2·431	36·299	+0·219
·533		14	59·50	38·78	−2·434	36·346	+0·172
·549		20	59·50	38·88	−2·449	36·431	+0·087
·552		21	59·72	38·76	−2·452	36·308	+0·021
·569	July	27	59·35	38·70	−2·468	36·232	+0·286
·590	Aug.	4	59·57	38·72	−2·489	36·231	+0·287
·607		10	59·33	38·86	−2·505	36·355	+0·163
·626		17	59·61	38·88	−2·523	36·357	+0·161
·645		24	59·84	39·02	−2·541	36·479	+0·039
·664		31	59·60	39·07	−2·560	36·510	+0·008
·667	Sept.	1	59·72	39·20	−2·563	36·637	−0·119
·725		22	59·81	39·31	−2·618	36·692	−0·174

TABLE I.—*Reductions to* 1901·0—continued.

			Pos. Ang.	Dist.	Cor'n for motion.	1901·0.	Resid. from mean.
			°	″	″	″	″
1903·741	Sept.	28	60·02	39·20	−2·634	36·566	−0·048
·757	Oct.	5	59·66	39·37	−2·649	36·721	−0·203
·782		13	59·74	39·37	−2·673	36·697	−0·179
·799		19	59·99	39·45	−2·689	36·761	−0·243
·818		26	59·50	39·38	−2·708	36·672	−0·154
·821		27	59·71	39·48	−2·711	36·769	−0·251
·827	Nov.	2	59·53	39·41	−2·726	36·684	−0·166
·897		24	59·45	39·66	−2·784	36·876	−0·358
·974	Dec.	22	59·40	39·61	−2·858	36·752	−0·234
1904·007	Jan	3	59·05	39·76	−2·889	36·871	−0·353
·335	May	2	59·02	39·73	−3·205	36·525	−0·007
·338		3	59·03	39·48	−3·207	36·273	+0·245
·374		16	58·73	39·50	−3·242	36·258	+0·260
·393		23	59·17	39·55	−3·260	36·290	+0·228
·434	June	7	59·16	39·78	−3·300	36·480	+0·038
·502	July	2	58·91	39·62	−3·365	36·255	+0·263
·521		9	59·34	39·62	−3·383	36·237	+0·281
·527		11	59·27	39·73	−3·389	36·341	+0·177
·560		23	59·47	39·80	−3·421	36·379	+0·139
·570		27	59·41	39·78	−3·430	36·350	+0·168
·584	Aug.	1	59·74	39·74	−3·444	36·296	+0·222
·598		6	59·51	39·69	−3·457	36·233	+0·285
·603		8	59·64	39·76	−3·462	36·298	+0·220
·623		15	59·30	39·85	−3·481	36·369	+0·149
·642		22	59·60	40·04	−3·499	36·541	−0·023
·655		27	59·69	40·08	−3·512	36·568	−0·050
·661		29	59·77	39·95	−3·518	36·432	+0·086
·675	Sept.	3	59·29	40·03	−3·531	36·499	+0·019
·680		5	59·57	40·12	−3·536	36·584	−0·066
·732		24	59·79	40·10	−3·586	36·514	+0·004
·751	Oct.	1	59·43	40·22	−3·604	36·616	−0·098
·790		15	59·87	40·25	−3·642	36·608	−0·090
·795		17	59·68	40·19	−3·647	36·543	−0·025
·828		29	59·59	40·25	−3·678	36·572	−0·054
·833		31	59·62	40 40	−3·683	36·717	−0·199
·866	Nov.	12	59·26	40·53	−3·715	36·815	−0·297
1904·905		26	59·33	40·56	−3·752	36·808	−0·290
1905·904	Nov.	26	59 70	41·46	−4·605	36·855	−0·337
·910		28	59·55	41·38	−4·611	36·769	−0·251

TABLE II.

Equations of Condition.

For x.

1900 Dec.	8.	$-0\cdot423 + x - 0\cdot063\,y + 0\cdot939\,p = 0$
	9.	$-0\cdot210 + x - 0\cdot060\,y + 0\cdot944\,p = 0$
	11.	$-0\cdot225 + x - 0\cdot055\,y + 0\cdot951\,p = 0$
	12.	$-0\cdot192 + x - 0\cdot052\,y + 0\cdot966\,p = 0$
1901 Sept.	23.	$+0\cdot038 + x + 0\cdot729\,y - 0\cdot032\,p = 0$
	24.	$+0\cdot020 + x + 0\cdot731\,y - 0\cdot011\,p = 0$
	30.	$-0\cdot063 + x + 0\cdot748\,y + 0\cdot091\,p = 0$
Oct.	1.	$+0\cdot090 + x + 0\cdot751\,y + 0\cdot102\,p = 0$
	13.	$-0\cdot160 + x + 0\cdot783\,y + 0\cdot303\,p = 0$
	21.	$-0\cdot068 + x + 0\cdot805\,y + 0\cdot427\,p = 0$
	29.	$-0\cdot027 + x + 0\cdot827\,y + 0\cdot545\,p = 0$
1902 Sept.	29.	$-0\cdot096 + x + 1\cdot744\,y + 0\cdot068\,p = 0$
Oct.	6.	$+0\cdot113 + x + 1\cdot764\,y + 0\cdot184\,p = 0$
	7.	$-0\cdot105 + x + 1\cdot766\,y + 0\cdot201\,p = 0$
	14.	$-0\cdot266 + x + 1\cdot786\,y + 0\cdot314\,p = 0$
1903 May	19.	$+0\cdot195 + x + 2\cdot380\,y - 0\cdot826\,p = 0$
	25.	$+0\cdot130 + x + 2\cdot396\,y - 0\cdot884\,p = 0$
June	2.	$+0\cdot001 + x + 2\cdot418\,y - 0\cdot936\,p = 0$
	9.	$+0\cdot100 + x + 2\cdot437\,y - 0\cdot986\,p = 0$
	15.	$+0\cdot166 + x + 2\cdot454\,y - 0\cdot997\,p = 0$
	22.	$+0\cdot154 + x + 2\cdot473\,y - 1\cdot006\,p = 0$
	29.	$+0\cdot172 + x + 2\cdot492\,y - 1\cdot002\,p = 0$
	30.	$+0\cdot205 + x + 2\cdot495\,y - 1\cdot010\,p = 0$
July	6.	$+0\cdot091 + x + 2\cdot511\,y - 0\cdot990\,p = 0$
	7.	$+0\cdot224 + x + 2\cdot514\,y - 0\cdot983\,p = 0$
	13.	$+0\cdot219 + x + 2\cdot530\,y - 0\cdot952\,p = 0$
	14.	$+0\cdot172 + x + 2\cdot533\,y - 0\cdot952\,p = 0$
	20.	$+0\cdot087 + x + 2\cdot549\,y - 0\cdot914\,p = 0$
	21.	$+0\cdot210 + x + 2\cdot552\,y - 0\cdot902\,p = 0$
	27.	$+0\cdot286 + x + 2\cdot569\,y - 0\cdot858\,p = 0$
Aug.	4.	$+0\cdot287 + x + 2\cdot590\,y - 0\cdot775\,p = 0$
	10.	$+0\cdot163 + x + 2\cdot607\,y - 0\cdot712\,p = 0$
	17.	$+0\cdot161 + x + 2\cdot626\,y - 0\cdot620\,p = 0$
	24.	$+0\cdot039 + x + 2\cdot645\,y - 0\cdot519\,p = 0$
	31.	$+0\cdot008 + x + 2\cdot664\,y - 0\cdot417\,p = 0$
Sept.	1.	$-0\cdot119 + x + 2\cdot667\,y - 0\cdot400\,p = 0$
	22.	$-0\cdot174 + x + 2\cdot725\,y - 0\cdot055\,p = 0$
	28.	$-0\cdot048 + x + 2\cdot741\,y + 0\cdot050\,p = 0$

TABLE II.—*Equations of Condition—For x*—continued.

1903 Oct.	5.	$-0\cdot203 + x + 2\cdot757\,y + 0\cdot164\,p = 0$
	13.	$-0\cdot179 + x + 2\cdot782\,y + 0\cdot295\,p = 0$
	19.	$-0\cdot243 + x + 2\cdot799\,y + 0\cdot394\,p = 0$
	26.	$-0\cdot154 + x + 2\cdot818\,y + 0\cdot495\,p = 0$
	27.	$-0\cdot251 + x + 2\cdot821\,y + 0\cdot510\,p = 0$
Nov.	2.	$-0\cdot166 + x + 2\cdot837\,y + 0\cdot595\,p = 0$
	24.	$-0\cdot358 + x + 2\cdot897\,y + 0\cdot839\,p = 0$
Dec.	22.	$-0\cdot234 + x + 2\cdot974\,y + 0\cdot977\,p = 0$
1904 Jan.	3.	$-0\cdot353 + x + 3\cdot007\,y + 0\cdot960\,p = 0$
May	2.	$-0\cdot007 + x + 3\cdot335\,y - 0\cdot638\,p = 0$
	3.	$+0\cdot245 + x + 3\cdot338\,y - 0\cdot651\,p = 0$
	16.	$+0\cdot260 + x + 3\cdot374\,y - 0\cdot801\,p = 0$
	23.	$+0\cdot228 + x + 3\cdot393\,y - 0\cdot879\,p = 0$
June	7.	$+0\cdot038 + x + 3\cdot434\,y - 0\cdot980\,p = 0$
July	2.	$+0\cdot263 + x + 3\cdot502\,y - 1\cdot003\,p = 0$
	9.	$+0\cdot281 + x + 3\cdot521\,y - 0\cdot973\,p = 0$
	11.	$+0\cdot177 + x + 3\cdot527\,y - 0\cdot967\,p = 0$
	23.	$+0\cdot139 + x + 3\cdot560\,y - 0\cdot886\,p = 0$
	27.	$+0\cdot168 + x + 3\cdot570\,y - 0\cdot855\,p = 0$
Aug.	1.	$+0\cdot222 + x + 3\cdot584\,y - 0\cdot799\,p = 0$
	6.	$+0\cdot285 + x + 3\cdot598\,y - 0\cdot747\,p = 0$
	8.	$+0\cdot220 + x + 3\cdot603\,y - 0\cdot723\,p = 0$
	15.	$+0\cdot149 + x + 3\cdot623\,y - 0\cdot640\,p = 0$
	22.	$-0\cdot023 + x + 3\cdot642\,y - 0\cdot511\,p = 0$
	27.	$-0\cdot050 + x + 3\cdot655\,y - 0\cdot467\,p = 0$
	29.	$+0\cdot086 + x + 3\cdot661\,y - 0\cdot434\,p = 0$
Sept.	3.	$+0\cdot019 + x + 3\cdot675\,y - 0\cdot364\,p = 0$
	5.	$-0\cdot066 + x + 3\cdot680\,y - 0\cdot326\,p = 0$
	24.	$+0\cdot004 + x + 3\cdot732\,y - 0\cdot007\,p = 0$
Oct.	1.	$-0\cdot098 + x + 3\cdot751\,y + 0\cdot105\,p = 0$
	15.	$-0\cdot090 + x + 3\cdot790\,y + 0\cdot342\,p = 0$
	17.	$-0\cdot025 + x + 3\cdot795\,y + 0\cdot373\,p = 0$
	29.	$-0\cdot054 + x + 3\cdot828\,y + 0\cdot548\,p = 0$
	31.	$-0\cdot199 + x + 3\cdot833\,y + 0\cdot578\,p = 0$
Nov.	12.	$-0\cdot297 + x + 3\cdot866\,y + 0\cdot726\,p = 0$
	26.	$-0\cdot290 + x + 3\cdot905\,y + 0\cdot862\,p = 0$
1905 Nov.	26.	$-0\cdot337 + x + 4\cdot904\,y + 0\cdot863\,p = 0$
	28.	$-0\cdot251 + x + 4\cdot910\,y + 0\cdot875\,p = 0$
		$+0\cdot011 + 76x + 204\cdot053y - 14\cdot804\,p = 0$

Corresponding equations were deduced for y and p, the resulting normal equations being

$$+0{\cdot}0110 + 76{\cdot}000\,x + 204{\cdot}0530\,y - 14{\cdot}8040\,p = 0$$
$$+2{\cdot}2742 + 204{\cdot}053\,x + 642{\cdot}2801\,y - 52{\cdot}9454\,p = 0$$
$$-8{\cdot}5866 - 14{\cdot}804\,x - 52{\cdot}9454\,y + 38{\cdot}0005\,p = 0$$

The solution of these gives the following values:—

$$p = +0''{\cdot}249 \pm 0''{\cdot}0105$$
$$y = +0{\cdot}011 \pm 0{\cdot}0063$$
$$x = +0{\cdot}020 \pm 0{\cdot}0182.$$

The value of the parallax of the star A is, therefore,

$$\pi = +0''{\cdot}249 \pm 0''{\cdot}0105.$$

This gives a distance of over 830,000 times that of the Sun. Light, therefore, requires 13·1 years to come from Krueger 60.

Dr. Schlesinger has kindly supplied me with his final value for the parallax of this star. He determined the parallax by photography (during his connection with the Yerkes Observatory) from both right ascension and declination. These were from 19 plates taken with the 40-inch telescope from 1903 to 1906. Three separate images were impressed on each plate. His results are—

$$\text{From R.A. } \pi = +0''{\cdot}257 \pm 0''{\cdot}007$$
$$\text{,, Decl. } \pi = +0{\cdot}238 \pm 0{\cdot}011.$$

It will be seen that the mean of these,

$$\pi = +0''{\cdot}248,$$

is essentially identical with my value.

The results with the same instrument, by visual and photographic methods respectively, in the one case by position-angle and distance, and in the other by right ascension and declination, should inspire confidence in both methods.

Irregularity in the Proper Motion of A.

At present the stars A and B are moving almost directly away from C. This latter star, therefore, is a good one from which to determine the motion.

If B has a considerable mass ratio to A, a large irregularity in the proper motion of either star must occur because of the large apparent distance between the two. If their masses should be nearly equal, this deviation from uniformity of motion in either star may amount to as much as 1″·5. This last mass ratio is not

improbable, in view of the investigations of Mr. Lewis of the Greenwich Observatory who found * that in some cases the apparently smaller star of a binary is really the more massive of the two. It is important, therefore, frequently to measure the position of A with reference to C and other stars in the field. By this means, in the course of time, its mass will become accurately known. For this reason I have kept the star on my observing list, and have made frequent measures of it and the stars mentioned. Of my own work there are now some seven or eight years' observations. Of course, for various reasons, where it is possible, it is best to use the measures of one individual.

The known history of these stars perhaps covers too short an interval for the observations to show any very decided irregularity in the motion of A. To see if there was any such change, the measures were freed from parallax, and means of four or five taken. These were plotted on a large scale. A thread was then tightly stretched over the measures and carefully adjusted to give the best representation. This straight line does not represent the observations with entire satisfaction. The best average, however, was obtained, and the deviations read off. These differences are given in connection with the mean measures. The ends of the thread would represent the positions 1900·943, distance 36″·49, and 1908·302, distance 43″·53.

As these measures may be of interest to others, they are given below. The first column after the dates is the measured distance corrected for refraction, the next column gives the correction for parallax, while the last is the distance affected only by motion. The means of these and their deviation from the straight line are also given.

Distances (A C) *corrected for parallax.*

			Obs. dist.	Cor'n for pllx.	True dist.	
1900	Dec.	8	36″·88	−0″·23	36″·65	
		9	36·67	−0·23	36·44	
		11	36·69	−0·24	36·45	
		12	36·66	−0·24	36·42	
1900·943					36·49	− 0″·01
1901	Sept.	23	37·18	+0·01	37·19	
		24	37·20	0·00	37·20	
		30	37·30	−0·02	37·28	
	Oct	1	37·15	−0·03	37·12	
1901·740					37·20	− 0·04
1901	Oct.	13	37·43	−0·08	37·35	
		21	37·36	−0·11	37·25	
		29	37·34	−0·14	37·20	
1901·792					37·27	− 0·02

* *Memoirs R.A.S.*, vol. lvi. p. xxi.

Distances (A C) *corrected for parallax*—continued.

		Obs. dist.	Cor'n for pllx.	True dist.	
1902	Sept. 29	38″·29	−0″·02	38″·27	
	Oct. 6	38·10	−0·04	38·06	
	7	38·32	−0·05	38·27	
	14	38·50	−0·08	38·42	
1902·764				38·25	+ 0″·01
1903	May 19	38·61	+0·21	38·82	
	25	38·69	+0·22	38·91	
	June 2	38·84	+0·23	39·07	
	9	38·76	+0·25	39·01	
1903·407				38·95	+ 0·10
1903	June 15	38·71	+0·25	38·96	
	22	38·74	+0·25	38·99	
	29	38·74	+0·25	38·99	
	30	38·71	+0·25	38·96	
1903·480				38·97	+ 0·05
1903	July 6	38·84	+0·25	39·09	
	7	38·71	+0·24	38·95	
	13	38·73	+0·24	38·97	
	14	38·78	+0·24	39·02	
1903·524				39·01	+ 0·04
1903	July 20	38·88	+0·23	39·11	
	21	38·76	+0·22	38·98	
	27	38·70	+0·21	38·91	
	Aug. 4	38·72	+0·19	38·91	
1903·568				38·98	0·00
1903	Aug. 10	38·86	+0·18	39·04	
	17	38·88	+0.15	39·03	
	24	39·02	+0·13	39·15	
	31	39·07	+0·10	39·17	
1903·637				39·10	+ 0·03
1903	Sept. 1	39·20	+0·10	39·30	
	22	39·31	+0·01	39·32	
	28	39·20	−0·01	39·19	
	Oct. 5	39·37	−0·04	39·33	
1903·725				39·29	+ 0·14

Distances (A C) *corrected for parallax*—continued.

			Obs. dist.	Cor'n for pllx.	True dist.	
1903	Oct.	13	39".37	−0".07	39".30	
		19	39·45	−0·10	39·35	
		26	39·38	−0·12	39·26	
		27	39·48	−0·13	39·35	
1903·804					39·31	+ 0".09
1903	Nov.	2	39·41	−0·15	39·26	
		24	39·66	−0·21	39·45	
	Dec.	22	39·61	−0·25	39·36	
1904	Jan.	3	39·76	−0·24	39·52	
1903·927					39·40	+ 0·06
1904	May	2	39·73	+0·16	39·89	
		3	39·48	+0·16	39·64	
		16	39·50	+0·20	39·70	
		23	39·55	+0·22	39·77	
1904·360					39·75	0·00
1904	June	7	39·78	+0·24	40·02	
	July	2	39·62	+0·25	39·87	
		9	39·62	+0·24	39·86	
		11	39·73	+0·24	39·97	
1904·497					39·93	+ 0·04
1904	July	23	39·80	+0·22	40·02	
		27	39·78	+0·21	39·99	
	Aug.	1	39·74	+0·20	39·94	
		6	39·69	+0·19	39·88	
1904·579					39·96	0·00
1904	Aug.	8	39·76	+0·18	39·94	
		15	39·85	+0·16	40·01	
		22	40·04	+0·12	40·16	
		27	40·08	+0·12	40·20	
1904·631					40·08	+ 0·04
1904	Aug.	29	39·95	+0·11	40·06	
	Sept.	3	40·03	+0·09	40 12	
		5	40·12	+0·08	40·20	
		24	40·10	0·00	40·10	
1904·686					40·12	+0·06

Distances (A C) *corrected for parallax*—continued.

		Obs. dist.	Cor'n for pllx.	True dist.	
		"	"	"	
1904 Oct.	1	40·22	−0·03	40·19	
	15	40·25	−0·08	40·17	
	17	40·19	−0·09	40·10	
	29	40·25	−0·14	40·11	
1904·790				40·14	0·00
1904 Oct.	31	40·40	−0·14	40·26	
Nov.	12	40·53	−0·18	40·35	
	26	40·56	−0·21	40·35	
1904·869				40·32	+ 0·09
1905 Nov.	26	41·46	−0·21	41·25	
	28	41·38	−0·22	41·16	
1905·907				41·21	− 0·03
1906 June	28	41·45	+0·25	41·70	
July	7	41·51	+0·24	41·75	
	10	41·54	+0·24	41·78	
	24	41·76	+0·22	41·98	
1906·523				41·80	− 0·03
1906 July	29	41·68	+0·21	41·89	
Aug.	11	41·72	+0·17	41·89	
	14	41·63	+0·16	41·79	
1906·602				41·86	− 0·20
1907·289 Apr.	16	42·52	+0·10	42·62	+ 0·10
1907 July	2	42·73	+0·25	42·98	
	12	42·57	+0·24	42·81	
	28	42·63	+0·21	42·84	
	30	42·64	+0·21	42·85	
1907·544				42·87	+ 0·04
1907 Aug.	6	42·77	+0·19	42·96	
	8	42·62	+0·18	42·80	
	11	42·53	+0·17	42·70	
	13	42·78	+0·17	42·95	
	20	42·68	+0·14	42·82	
1907·612				42·85	− 0·01
1907·749 Oct.	1	42·91	−0·03	42·88	− 0·11
1908 Apr.	19	43·40	+0·12	43·52	
	21	43·25	+0·12	43·37	
1908·302				43·44	− 0·08

Proper Motion of the Double Star Krueger 60.

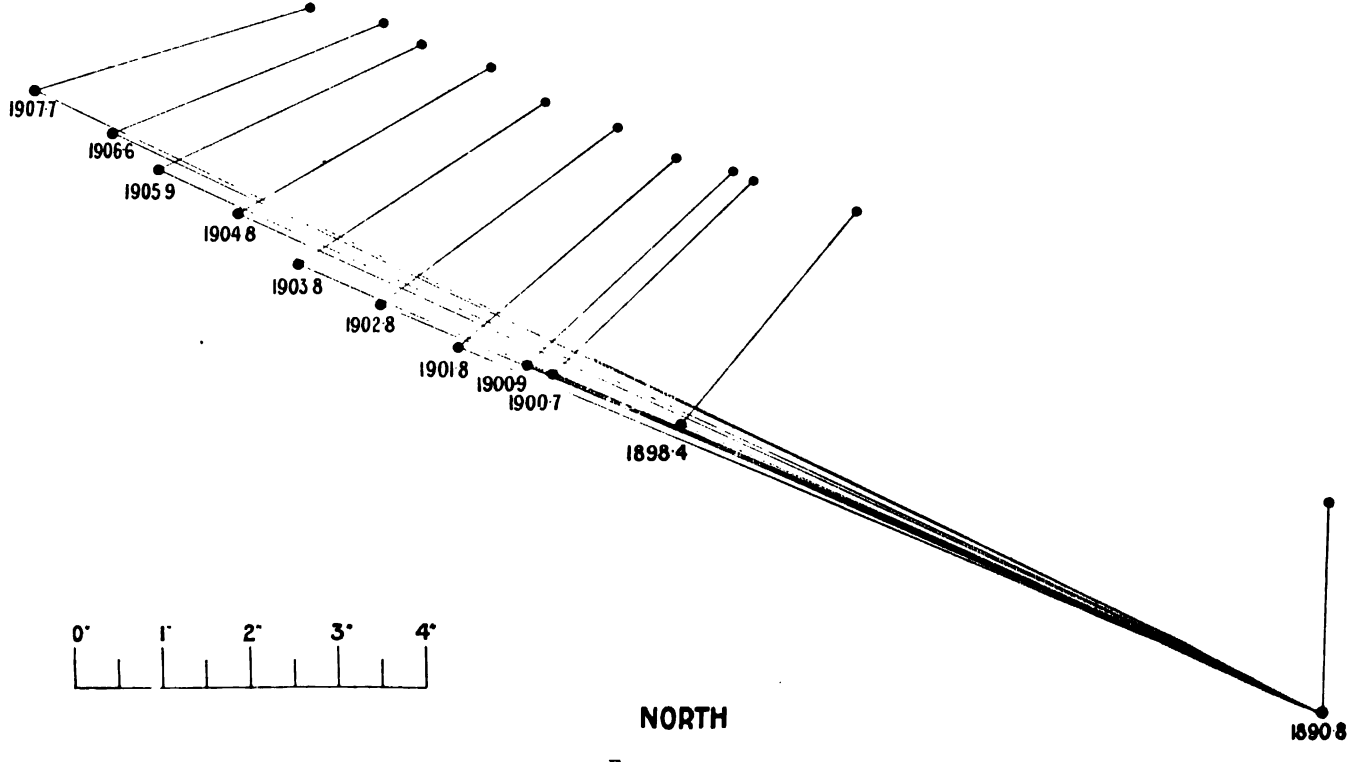


FIG. 2.

This investigation led me to notice that the direction of motion of A was regularly diminishing with the increase of time when derived from a comparison with the measures of 1890·8. The motion was also slightly increasing. To find out the cause of this change I plotted the observations for each year (fig. 2), correcting the distances for parallax, and selecting the positions so that there would be but little parallactic change in the angles. No correction for precession has been applied to the position-angles. The extreme change due to this last cause would be 0°·07. The result shows that between the epochs 1890·8 to 1901·8 and 1901·8 to 1907·7 there has been a total change of some 8° or 9° in the direction of motion. If we take the interval 1901·8–1907·7 the direction of motion is 239°, while the previous values were about 247°. A simple inspection of the diagram will show the change. It is evident, therefore, that the motion of A is not rectilinear. This change is undoubtedly due to the orbital motion of A about the centre of gravity of the system. It is therefore not possible to give a value of its proper motion that will be constant for any considerable interval of time.

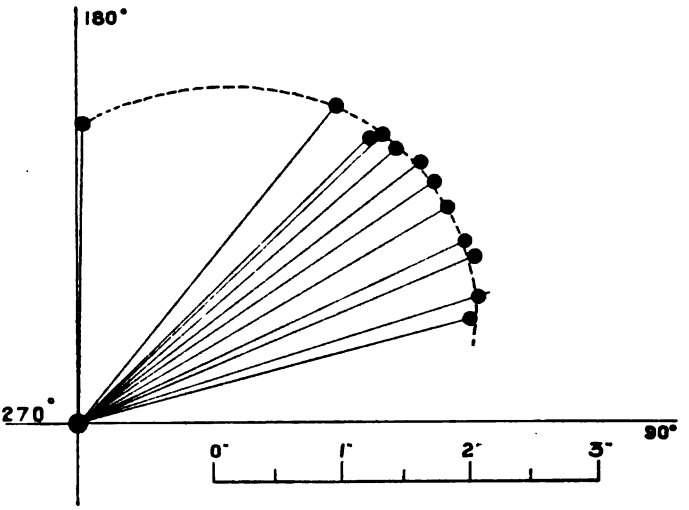
The following measures were used in constructing the diagram. The distances A C are corrected for parallax.

A C.

	°	″	Observer.
1890·788	56·30	26·73	Burnham.
1898·446	58·71	34·64	Doolittle.
1900·737	59·19	36·18	,,
1900·943	59·30	36·49	Barnard.
1901·766	59·62	37·23	,,
1902·764	59·64	38·25	,,
1903·765	59·77	39·30	,,
1904·782	59·55	40·19	,,
1905·907	59·63	41·21	,,
1906·602	59·54	41·86	,,
1907·680	59·57	42·87	,,

A B.

	°	″	Observer.
1890·788	178·80	2·32	Burnham.
1898·446	140·66	3·19	Doolittle.
1900·737	134·02	3·18	,,
1900·943	133·14	3·25	Barnard.
1901·767	130·67	3·28	,,
1902·765	127·05	3·36	,,
1903·761	123·89	3·35	,,
1904·791	120·30	3·33	,,
1905·907	115·34	3·31	,,
1906·601	112·27	3·34	,,
1907·676	107·00	3·27	,,



Orbital Motion of Krueger 60.

FIG. 3.

The Micrometer Measures.

I have collected here for this paper all my published and unpublished measures of these stars, so that my work on the subject up to the present may be presented as completely as possible. All the distances have been corrected for refraction.

The diagram (fig. 1) will show the relative positions of these various stars.

The measures follow.

Measures of Krueger 60 *and Comparison Stars.*

A B.

				mag.	mag.
1900·937 Dec.	8	132°·06	3"·30	9·2	10·5
·940	9	133·19	3·25		
·945	11	133·59	3·24		
·948	12	133·71	3·23		
1900·943		133·14	3·25	9·2	10·5

A B—*continued.*

			°	″	mag.	mag.
1901·729	Sept.	23	130·05	3·37		
·731		24	129·11	3·39	9·5	10·5
·748		30	131·50	3·37		
·751	Oct.	1	131·65	3·25		
·783		13	129·69	3·31		
·805		21	130·59	3·26	9·2	10·5
·827		29	130·59	3·22		
1901·768			130·45	3·31	9·3	10·5
1902·744	Sept.	29	127·25	3·33		
·764	Oct.	6	128·17	3·43		
·766		7	126·73	3·35		
·785		14	126·06	3·35		
1902·765			127·05	3·37		
1903·380	May	19	122·36	3·32		
·418	June	2	122·34	3·42		
·437		9	123·85	3·40		
·454		15	123·49	3·37		
·473		22	123·47	3·33		
·492		29	124·90	3·36		
·495		30	125·96	3·26		
·511	July	6	121·31	3·51		
·514		7	123·18	3·30		
645	Aug.	24	124·52	3·45		
·664		31	122·91	3·35		
·667	Sept.	1	121·98	3·37		
·725		22	124·54	3·27		
·741		28	123·54	3·45		
·818	Oct.	26	123·60	3·33		
1903·562			123·46	3·37		

A B—*continued.*

			°	″	mag.	mag.
1904·335	May	2	119·44	3·35		
·338		3	119·46	3·35		
·373		16	119·92	3·40		
·393		23	118·51	3·43		
·434	June	7	118·82	3·37		
·502	July	2	121·45	3·43		
·521		9	121·28	3·35		
·527		11	120·68	3·31		
·560		23	119·94	3·39		
·570		27	117·99	3·35		
·584	Aug.	1	120·47	3·43		
·598		6	119·24	3·48		
·642		22	120·96	3·42		
·655		27	119·47	3·37		
·675	Sept.	3	119·50	3·41		
·751	Oct.	1	118·98	3·31		
·795		17	121·07	3·28		
·828		29	120·84	3·39		
·905	Nov.	26	117·46	3·33		
1904·578			119·76	3·38		
1905·904	Nov.	26	116·04	3·30		
·910		28	114·64	3·31		
1905·907			115·34	3·31		
1906·490	June	28	113·63	3·28		
·515	July	7	112·44	3·28		
·523		10	109·72	3·34		
·561		24	112·57	3·33		
·575		29	112·71	3·31		
·610	Aug.	11	112·31	3·33		
·619		14	111·78	3·38		
1906·556			112·17	3·32		

A B—*continued.*

					mag.	mag.
1907·289	Apr.	16	108°·06	3″·14		
·500	July	2	108·09	3·26		
·528		12	106·31	3·29		
·571		28	106·59	3·28		
·577		30	107·83	3·23		
·596	Aug.	6	106·06	3·42		
·601		8	107·92	3·28		
·610		11	106·93	3·38		
·615		13	107·47	3·20		
·634		20	107·13	3·17		
·706	Sept.	15	105·34	3·37		
·749	Oct.	1	108·06	3·36		
1907·581			107·15	3·28		
1908·300	Apr.	19	105·65	2·97		
·338	May	3	105·19	3·30		
·376		17	103·99	3·21		
·382		19	104·63	3·23		
·390		22	104·18	3·26		
·401		26	102·70	3·13		
1908·373			104·39	3·18		

A C

In this table all the previous measures of A C—those used in the determination of the parallax—have been omitted, because they are given in Table I.

					mag.	mag.
1906·490	June	28	59°·01	41″·45		
·515	July	7	59·23	41·49		
·523		10	59·23	41·54		
·561		24	59·71	41·76		
·575		29	59·55	41·68		
·610	Aug.	11	59·60	41·72		9·5
·619		14	59·48	41·63		
1906·556			59·40	41·61		9·5

A C—*continued.*

					mag.	mag.
1907·289	Apr.	16	59·18	42·52		
·500	July	2	59·34	42·73		
·528		12	59·55	42·57		
·571		28	59·40	42·63		
·577		30	59·19	42·64		
·596	Aug.	6	59·44	42·77		
·601		8	59·59	42·62		
·610		11	59·14	42·53		
·615		13	59·50	42·78		
·634		20	59·55	42·68		
·749	Oct.	1	59·71	42·91		
1907·570			59·42	42·67		
1908·300	Apr.	19	59·32	43·41		
·305		21	58·99	43·25		
·338	May	3	58·76	43·51		
·357		10	59·15	43·72		
·376		17	58·98	43·32		
·382		19	59·24	43·52		
·390		22	58·98	43·57		
·401		26	59·19	43·54		
1908·356			59·08	43·48		

A D.

1900·945	Dec.	11	21·25	21·26		15
·948		12	20·86	21·34		15·5
1900·947			21·05	21·30		15·3
1901·783	Oct.	13	23·56	21·54		15
·786		14	21·58	21·85		
·805		21	23·04	21·72		15·3
·827		29	23·07	21·78		
1901·800			22·81	21·72		15·1
1903·511	July	6	24·68	22·75		
·514		7	24·09	22·83		15
1903·512			24·39	22·79		15
1908·415	May	31	29·25	26·69		
·467	June	19	29·51	26·99		
·472		21	29·48	26·93		
1908·451			29·41	26·87		

A E.

				mag.	mag.
1900·937	Dec. 8	98°·85	68″·03		12·5
·940	9	99·06	67·79		13·5
·945	11	99·00	67·76		13·5
1900·941		98·97	67·86		13·2
1901·729	Sept. 23	98·93	68·20		12
·731	24	99·14	68·49		12·5
·748	30	99·06	68·64		
·751	Oct. 1	98·92	68·22		
·783	13	99·01	68·32		
·805	21	98·91	68·39		11·3
·827	29	98·83	68·64		
1901·768		98·97	68·41		11·9
1903·437	June 9	97·89	69·17		13
·454	15	97·99	69·37		
·473	22	97·87	69·20		12
·495	30	97·96	69·42		
·511	July 6	98·07	69·47		
·514	7	97·78	69·51		
·530	13	98·26	69·38		
·533	14	98·18	69·44		
·549	20	98·09	69·55		
·569	27	98·01	69·47		
·607	Aug. 10	97·64	69·44		
·626	17	97·84	69·64		
·645	24	97·97	69·90		
·664	31	97·86	69·72		
·667	Sept. 1	97·93	69·71		
·725	22	98·02	70·12		
·741	28	97·90	69·89		12
·757	Oct. 5	97·86	69·93		
·782	13	97·59	70·11		
·799	19	97·80	70·26		13
·818	26	97·78	70·22		
·821	27	97·70	70·08		12·5
·837	Nov. 2	97·64	70·11		12
·897	24	97·56	70·26		12
·974	Dec. 22	97·41	70·22		12
1904·007	Jan. 3	97·49	70·47		12
1903·670		97·85	69·81		12·3

A E—*continued.*

				mag.	mag.
1904·335	May 2	97·14	70·10		12
·338	3	97·44	69·91		12
·527	July 11	97·42	70·38		12
·560	23	97·53	70·19		12
·570	27	97·43	70·13		13
·584	Aug. 1	97·64	70·21		13
·598	6	97·32	70·47		
·603	8	97·89	70·76		
·623	15	97·66	70·76		
·642	22	97·54	70·49		12
·655	27	97·50	70·61		13
·661	29	97·56	70·93		
·675	Sept. 3	97·44	70·61		12
·680	5	97·66	70·81		12·5
·751	Oct. 1	97·53	70·80		12·5
·790	15	97·54	71·00		12
·795	17	97·40	70·77		12
·828	29	97·48	70·97		12·5
·833	31	97·27	70·99		12·8
·866	Nov. 12	97·23	71·06		
·905	26	97·29	70·94		12
1904·653		97·47	70·61		12·3
1905·910	Nov. 28	96·87	71·97		
1906·610	Aug. 11	97·08	72·04		12·5
·619	14	96·61	72·20		12·5
1906·615		96·85	72·12		12·5
1907·596	Aug. 6	96·04	72·51		13
·601	8	95·96	72·66		
·610	11	95·70	72·80		
·615	13	96·29	72·80		
·634	20	95·93	72·59		
·667	Sept. 1	95·82	72·88		12·8
·706	15	96·24	72·89		
·749	Oct. 1	96·14	72·80		
1907·647		96·01	72·74		12·9

A E—*continued.*

				mag.	mag.	
1908·415	May	31	95°·53	73″·05		
·417	June	1	95·39	73·00		
·428		5	95·47	73·10		
·467		19	95·41	73·14		
·472		21	95·50	73·08		
1908·440			95·46	73·07		

A F.

1901·729	Sept.	23	275°·36	39″·61	14	
·731		24	274·78	39·53	14	
·751	Oct.	1	275·20	39·61		
·784		13	275·33	39·52	15	
·805		21	275·34	39·53	15	
·827		29	275·53	39·57		
1901·771			275·26	39·56	14·5	

1903·473	June	22	277°·24	38″·47	13	
·492		29	277·59	...	13	clouds.
·495		30	276·84	38·46	12·7	
·511	July	6	277·06	38·34	12	
·514		7	277·09	38·51		
·533		14	276·52	38·37		
·549		20	276·34	38·50		
·626	Aug.	17	276·92	38·45		
·645		24	276·49	38·11		
·664		31	276·70	38·21		
·725	Sept.	22	276·74	38·16		
·741		28	276·87	37·80	15	
·782	Oct.	13	277·06	38·15		
·799		19	277·14	38·07	14·5	
·818		26	277·31	38·80		
·821		27	277·36	37·79	15·0	
·897	Nov.	24	277·35	38·29	14·5	
1903·652			276·98	38·28	13·7	

A F—*continued.*

				mag.	mag.
1904·335	May 2	278°·20	37″·76		12·7
·393	23	277·89	38·05		12·5
·560	July 23	277·71	37·39		14
·570	27	277·64	37·62		
·584	Aug. 1	277·36	37·35		15
·598	6	277·01	37·90		
·655	27	277·74	37·15		14
·675	Sept. 3	277·67	37·40		15
·751	Oct. 1	277·84	37·15		14·5
·790	15	278·17	37·38		15
·795	17	277·64	36·83		14·5
·828	29	278·19	37·18		14·7
·833	31	277·87	37·34		14·5
·905	Nov. 26	278·16	36·99		14·5
1904·734		277·79	37·39		14·2

1907·667	Sept. 1	280·56	35·23		14·5
·706	15	280·76	34·47		
·749	Oct. 1	280·75	34·73		14·5
·782	13	281·05	34·67		14
1907·726		281·78	34·77		14·3

1908·415	May 31	281·80	34·94		
·417	June 1	282·06	35·16		
·428	5	282·13	35·15		
·467	19	281·58	34·63		
·472	21	281·40	34·78		
1908·440		281·94	34·93		

C E.

1903·552	July 21	129°·71	46″·19		
1907·634	Aug. 20	129·41	45·84		
·782	Oct. 13	129·88	45·98		13·7
1907·708		129·65	45·91		13·7

It will be seen from the measures that the distance A B is beginning to lessen, having attained a maximum about 1903 or 1904. The position-angle A C seems to have reached its greatest value about 1907 and is now slowly diminishing. Both changes are due to the orbital motion of A in the system A B.

The following estimates of the magnitude of C were made at the time of the observations which have been included in Table I.

	m
1900 Dec. 8	9·5
1901 Sept. 24	9·5
Oct. 21	9·7
1902 Sept. 29	9·9

At the observation of 1900 Dec. 9 the following note occurs:—"Both A and C are slightly yellowish."

Two small Double Stars near Krueger 60.

During these measures two small double stars were found and measured. Though they are not in any way connected with Krueger 60, their proximity to this star, and the general resemblance of one of them to A B, would seem to warrant their insertion here.

No. 1.

(1855·0 α 22^h 19^m 42^s·8, δ +57° 6′·3.)

		°	″	mag.	mag.
1901·731	Sept. 24	247·04	2·99	9·5	12
·805	Oct. 21	247·19	3·11	9	11
1901·768		247·11	3·05	9·3	11·5
1902·766	Oct. 7	247·23	2·67	9·5	11
1903·454	June 15	246·70	3·20	9·8	11
·473	22	248·79	3·09		
·511	July 6	247·62	3·04		
·514	7	246·68	3·25		
1903·488		247·45	3·14	9·8	11
1908·428	June 5	248·05	3·04		
·467	19	245·08	3·22	9·3	11·2
1908·447		246·57	3·13	9·3	11·2

No. 2.

(1903·0 α 22^h 22^m 52^s·1, δ +57° 13′ 33″.)

					mag.	mag.
1902·744	Sept.	29	260°·15	1″·37	10	11
1903·454	June	15	262·50	1·41		
1907·749	Oct.	1	259·05	1·37		
1908·382	May	19	261·32	1·35	10	11
·401		26	260·95	1·45	9·6	10·5
·467	June	19	259·95	1·46		
1908·417			260·74	1·42	9·8	10·7

The Following Component of No. 2 and a Faint Star North.

					mag.	mag.
1908·382	May	19	340°·59	87″·67		12
·401		26	340·56	87·70		13·2
1908·391			340·57	87·69		12·6

Additional measures of some of the stars have been inserted since this paper was submitted to the R.A.S.

YERKES OBSERVATORY:
1908 *May* 18.

Postscript.

I am very glad to be able to add to this paper the value of the parallax of Krueger 60 determined by Dr. Henry Norris Russell from photographs with the Cambridge (England) refractor. Dr. Russell has kindly supplied me with his value since my paper was put in type.

His result is:—

$$\pi = +0''\cdot258 \pm 0''\cdot013$$

from measures of $\Delta\alpha$. Nine comparison stars were used, and the probable error for a single plate was $\pm 0''\cdot026$.

The following, therefore, are all of the values of the parallax of this star known to me:—

Schlesinger = $+0''\cdot248 \pm 0''\cdot009$
Barnard = $+0\cdot249 \pm 0\cdot010$
Russell = $+0\cdot258 \pm 0\cdot013$

Photograph containing three exposures of the double star Krueger 60 A B and the comparison star C. Taken with the 40-inch refractor of the Yerkes Observatory, 1908 July 10 (enlarged). Exposures 10 minutes each. —E. E Barnard.

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Additional Measures of Krueger 60.

A B.

1908	Aug.	30	100°·71	3″·14
	Sept.	4	103·10	3·17
		6	105·13	3·15
		15	104·69	3·24
		20	103·92	3·14
	Oct.	4	102·67	3·21
			103·37	3·17

A C.

1908	Aug.	30	59°·62	43″·93
	Sept.	4	59·69	43·94
		6	59·55	43·91
		15	59·38	44·19
		20	59·74	43·87
	Oct.	4	59·55	44·08
			59·59	43·99

I have also added a photograph of Krueger 60 (A B) and the comparison star C, which I made on 1908 July 10 with the 40-inch refractor and a yellow colour screen, with Cramer Instantaneous Iso plate. There are three exposures of 10^m each.

1908 *Oct.* 5.

On possible Periodic Inequalities in the Epoch of the Sun-spot Variation. Papers of the I.U.S.R. Computing Bureau, No. III.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. When any point of importance arises with regard to the stars, it is natural to inquire whether it is represented in the behaviour of the one star which we can observe to special advantage, viz. our own Sun. Reasons have been given elsewhere * for regarding the Sun, provisionally at any rate, as a long-period variable with a period of 11⅛ years, but with a very small range of variation. Reasons have also been given † for regarding the oscillations of maximum of long-period variables as systematic; so that when Chandler writes for the epochs of maximum an expression such as

$$\epsilon + P.E + C \sin (A^\circ.E + \alpha),$$

where P is the period in days, and E the number of periods elapsed since the epoch ϵ, and C, A, α three constants, we have approximately

$$C = 7^d.3 + .064\ P$$
$$A = 2^\circ.6 + .018\ P.$$

Hence the questions arise,—Is there an oscillation of this kind in the Sun-spot maxima? and if so, how far does it fit in with these formulæ? The period of the Sun (say 4060 days) is so very much longer than that of any of the stars used in deriving the above formulæ that we may be prepared for some error, even a considerable error, in the extrapolation; but we may at least expect both A and C to be larger than for the stars, if the Sun falls into line at all.

2. Dr. Wolfer has given (on page 96 of the *Astron. Mitteilungen*, No. xciii.) a list of deviations of 26 minima and 26 maxima from a uniform arithmetical progression. These are shown in Table I.

TABLE I.

Wolfer's Minima and Maxima of Sun-spots.

Min.	Max.	Min.	Max.	Min.	Max.
y	*y*	*y*	*y*	*y*	*y*
+0·3	−0·8	+1·2	+2·1	−0·5	+0·5
−2·7	−1·4	+1·6	+0·3	+1·1	+2·9
+1·2	+1·0	+0·9	+0·4	+0·6	−0·9
+1·0	−0·6	+0·8	+0·9	−1·0	−1·1
−0·1	−0·6	−0·2	+1·0	+0·4	−0·2
−0·2	+3·3	0·0	−1·9	+0·4	−0·8
+2·1	+2·2	−2·1	−4·2	+1·0	+1·4
+1·0	−0·9	−4·1	−5·6	+0·5	+0·5
−1·7	+0·5	−1·6	+0·4		

(The unit is one year.)

* See *Mon. Not.*, lxiv. p. 549 and lxvii. p. 334.
† *Mon. Not.*, lxviii. p. 544 (June 1908).

3. The problem is to find whether these residuals are affected with a periodicity C sin (A°.E + a), of which both the coefficient C and the argument A are unknown. In such a case, Professor Schuster has insisted that we have no resource but to try *all* values of A; or rather a series of values separated by small intervals, the size of the interval depending on the "resolving power" of the available material. In the present case, successive values of A differing by 2° were tried, and the resulting "periodogram" shows that this interval is sufficiently small. For each value of A we form

$$a^2 + b^2 \equiv \left(\sum r_E \sin A^\circ.E\right)^2 + \left(\sum r_E \cos A^\circ.E\right)^2$$

E being used by Chandler to denote the number of periods after a given epoch, and r_E being the corresponding residual in Table I. If we had *exactly*

$$r_E = C \sin (A^\circ.E + a)$$

then we should have

$$a = C\left[\cos a \sum \sin^2 AE + \sin a \sum \sin AE \cos AE\right]$$

$$= \tfrac{1}{2}C\left[\cos a \sum (1 - \cos 2AE) + \sin a \sum \sin 2AE\right]$$

$$= \tfrac{1}{2}C\left[n \cos a - \sum \cos (2AE + a)\right]$$

where n is the number of terms in the series—viz. 26 in our case. Now the sum $\Sigma \cos (2AE + a)$ is generally small compared with n. It will be seen later that the smallest value of A which seriously concerns us is about 20°; so that as E increases from 0 to 26, 2AE increases from 0° to $40^\circ \times 26 = 3 \times 360^\circ$ nearly. Hence cos (2AE + a) runs through its cycle nearly three times; and since in each cycle the positive and negative terms cut one another out, what is left is small. Neglecting it, we should have, in the ideal case,

$$a = \tfrac{1}{2}C\, n \cos a \quad \text{and} \quad b = \tfrac{1}{2}C\, n \sin a$$

$$\text{or} \quad 13C = \sqrt{a^2 + b^2}.$$

But accidental errors will give a sensible value for $a^2 + b^2$ for any value of A. We must look for values of A which give *large* values of $a^2 + b^2$. These values are tabulated in Table II., which was calculated in the Computing Bureau of the I.U.S.R.

TABLE II.

Periodogram of Wolfer's Residuals.

A	Min.	Max.	A	Min.	Max.	A	Min.	Max.
°								
2	·00	·04	48	1·06	1·97	94	1·04	1·08
4	·01	·09	50	1·01	2·81	96	·92	·92
6	·07	·16	52	1·05	3·52	98	·75	·80
8	·26	·39	54	·95	3·34	100	·82	·53
10	·33	·70	56	·66	2·98	102	·72	·28
12	·45	·99	58	·43	2·57	104	·66	·46
14	·69	1·51	60	·41	2·39	106	·60	·52
16	1·27	1·80	62	·26	1·70	108	·57	·59
18	1·83	2·16	64	·27	1·30	110	·44	·77
20	1·82	2·56	66	·36	1·13	112	·40	·83
22	1·76	2·50	68	·46	·95	114	·35	·72
24	1·60	2·26	70	·45	·95	116	·26	·62
26	1·52	2·13	72	·47	·78	118	·27	·40
28	1·07	1·59	74	·28	·80	120	·22	·28
30	0·78	1·04	76	·13	·48	122	·17	·21
32	1·52	·92	78	·19	·75	124	·19	·29
34	·50	·56	80	·29	·97	126	·31	·34
36	·46	·43	82	·48	1·23	128	·20	·48
38	·50	·41	84	·71	·93	130	·16	·48
40	·56	·62	86	·85	1·09	132	·14	·44
42	·77	·91	88	·98	1·06	134	·10	·28
44	·94	1·25	90	·87	1·15	136	·10	·16
46	·96	2·12	92	1·08	1·12	138	·23	·09
Means	0·86	1·18		0·60	1·56		0·41	0·50

4. We have to look for cases where the value of a^2+b^2 strikingly exceeds its mean value. The mean value should, however, be taken from the undisturbed portions of the periodogram, and there is always a little doubt in the first instance which these are. The means of the columns have been taken, and the mean of the six is 0·82. The highest value reached, 3·52, is only 4·4 times this mean; but if we exclude the column in which it occurs the mean falls to ·71, and the highest value, 3·52, is very nearly five times this. There is just a possibility that this peak indicates a real periodicity, though the probability is not high. It will be seen that there is a slight rise in the column for minima near the same value of A (viz. 52°), though the rise is far too slight to be worth notice independently. Still, there are obvious reasons why the minimum may be not so well determined as the maximum.

5. There is also a rise at A = 20°, though not quite so marked.

It is worth noticing, however, that the accompanying rise in the column for minima is greater, and that there are indications in some of the variable stars of another term accompanying the main inequality (see the case of S Serpentis in *M.N.*, lxviii. p. 563).

6. To state the case as favourably as possible for the existence of a periodic inequality in the sun-spot period, we may exclude from both columns the values

$$\text{from } A = 14^\circ \text{ to } A = 30^\circ, \text{ and from } A = 44^\circ \text{ to } A = 66^\circ,$$

and then the mean value for the periodogram for minima becomes 0·46 and for the maximum 0·62. The ratio of 3·52 to its own mean is thus nearly 6, and to the mean for both maximum and minimum is nearly 7.

7. Recurring now to the quantities which give us the inequality, if it exists, we find for the formulæ representing the maxima, when $A = 53^\circ$ (which seems the best value),

$$-1{\cdot}4 \sin 53^\circ E - 0{\cdot}2 \cos 53^\circ E,$$

and for the minima

$$-0{\cdot}7 \sin 53^\circ E - 0{\cdot}1 \cos 53^\circ E.$$

But the epochs are not quite the same. The minima begin with 1610·5, and the maxima with 1616·3, or 5·8 years later, which is $5{\cdot}8/11{\cdot}13 = {\cdot}52$ of a period. The phase of the inequality is thus advanced by $53^\circ \times {\cdot}52 = 27^\circ$ for the maxima, and to compare the expressions we must write

$$-1{\cdot}4 \sin (53^\circ E - 27^\circ) - 0{\cdot}2 \cos (53^\circ E - 27^\circ)$$

$$= -1{\cdot}3 \sin 53^\circ E + 0{\cdot}4 \cos 53^\circ E \quad \text{for maxima,}$$

and, as before,

$$-0{\cdot}7 \sin 53^\circ E - 0{\cdot}1 \cos 53^\circ E \quad \text{for the minima.}$$

8. In Chandler's notation, the sun-spot maxima would therefore be given by some such formula as

$$\text{const} + 4060\ E + 490 \sin (53^\circ E + \text{const}),$$

and the question now arises how far this formula for the Sun accords with those found for the stars and quoted in § 1. Putting $P = 4060^d$ in these formulæ, we have

$$C = 7^d{\cdot}3 + 0^d{\cdot}064 \times 4060 = 267^d \text{ as against } 490$$

$$A = 2^\circ{\cdot}6 + 0^\circ{\cdot}018 \times 4060 = 76^\circ \text{ as against } 53^\circ.$$

There is, of course, no reason why these formulæ should be strictly linear, so far as we know at present; and if we remark that the value of the coefficient C obtained from the minima is only about half that obtained from the maxima (*i.e.* in sensible agreement with the star-formula), the accordance is sufficiently good

to suggest further inquiry. One fact emerges from the discussion, viz. that from the available material it is difficult to make sure of the existence of an inequality similar to those shown by the stars. We may take it that the coefficient C is of the right order of magnitude to fit in with the star-formula: and our periodogram shows that in this case it is too small to stand out clearly from the accidental inequalities. It will not be possible to affirm or deny the existence of such an inequality with confidence until the material is improved by extending the series of observations, or possibly by reducing the accidental errors of the older observations by improved discussions of them. One hope of reducing the accidental errors proved vain. It was thought that, since there two independent series of maxima and minima, they might be ... in combination in some way, so that the effect of accidental errors of one series might be reduced by the other. But apparently the two series run together so closely that not much can be gained in this way.

9. The value $A = 20^\circ$ gives a value of C not much smaller than that for $A = 53^\circ$; and from the material it is not easy to say which of these two possible terms corresponds to the terms found for variable stars. Are there possibly two terms in general? The case of S Serpentis has already been quoted, where the existence of a long-period term had masked the short-period term. In other cases there may be long-period terms affecting the short-period terms to a smaller extent, and this may account for some of the large deviations from the formula. And these two values $A = 53^\circ$ and $A = 20^\circ$ for the Sun may help us, by suggestion, in getting at the facts for the variables. The series of observed maxima for the Sun is much longer and more continuous than those for most of the variables; and it would not be surprising if we got suggestions from it which would help in elucidating the shorter series.

10. Assuming that the value $A = 53^\circ$ corresponds to the terms that have attracted attention for the variables, then the formula

$$A = 2^\circ{\cdot}6 + 0^\circ{\cdot}018 \times P$$

does not hold for so large a value of P as $P = 4060$; for which we should get $A = 75^\circ$, as remarked in § 7. Can this distant point on the curve be used to improve the formula?

11. Firstly, let us examine the consequence of assuming the formula still linear, and let us determine a and b in the expression

$$A = a + bP$$

so as to satisfy the Sun and the mean of the stars; that is, put

$$53^\circ \quad = a + b \times 4060 \qquad \text{(the Sun)}$$

$$8^\circ{\cdot}16 = a + b \times \ 311 \qquad \text{(mean of stars)}.$$

From these we get $b = 0^\circ{\cdot}012$, $a = 4^\circ{\cdot}43$.

This new formula $A_2 = 4^\circ\cdot4 + 0^\circ\cdot012\ P$
differs from the former $A_1 = 2\cdot6 + 0\cdot018\ P$

as follows, for different values of P :—

	P =	100^d	200^d	300^d	400^d	500^d	600^d
New formula	A_2 =	5°·6	6°·8	8°·0	9°·2	10°·4	11°·6
Old formula	A_1 =	4·4	6·2	8·0	9·8	11·6	13·4
	$A_2 - A_1$ =	+1·2	+0·6	0·0	−0·6	−1·2	−1·8

In doubtful whether our present material is sufficient to enable us to discriminate between these two formulæ, for most of the stars have periods between 200 and 400 days. But we may notice one other supposition, viz.—

12. Secondly, let us adopt the suggestion of a curve of some kind rather than a straight line. The appropriate indices for A and P will be suggested by finding m in the formula

$$A^m = P$$

for the large values of A and P, *i.e.* for the case of the Sun. We have

$$m = \log P/\log A = \log 4060/\log 53$$
$$= 3\cdot61/1\cdot72 = 2\cdot1.$$

This suggests some formula such as either

$$A^2 = a(P + p)$$

or $$(A + \alpha)^2 = bP.$$

13. Determining the constants from the two cases of the Sun and mean of the stars, we find for the two suppositions

$$A^2 = 0\cdot73\ (P - 220)$$
$$(A + 9^\circ)^2 = 0\cdot94\ P.$$

Of these, the former gives impossible values of A for periods below 220 days, and is thus unsuitable. The latter gives values of A for different values of P, as below.

P =	100^d	200^d	300^d	400^d	500^d	600^d	
A_3 =	0°·7	4°·7	7°·8	10°·4	12°·7	14°·8	(new formula).
A_1 =	5·6	6·8	8·0	9·2	10·4	11·6	(linear formula).
$A_3 - A_1$ =	−4·9	−2·1	−0·2	+1·2	+2·3	+3·2.	

These differences are larger than the former, and it seems probable that we can discriminate even now in favour of the original formula.

SUMMARY.

The sun-spot maxima (and minima) occur on the average at intervals of 11·125 years. But the individual maxima (and minima) show discordances which have been tabulated for 26 periods by Wolfer. Analysing these by the periodogram method of Professor Schuster, there are indications of two periodicities, one of which the phase advances 53° per period of $11\frac{1}{8}$ years, the other of which the phase advances only 20°. The cycles are completed in about 75 and 200 years respectively. The amplitude of each inequality is about a year, but the accidental errors are so large that either or both of these inequalities may be spurious.

The quicker moving inequality (53°) can be brought into line with similar inequalities for the long-period variables; the best formula connecting A (advance of phase in degrees per period) with P (the period in days) being the simple linear formula

$$A = 4^{\circ}\cdot 4 + 0^{\circ}\cdot 012\ P.$$

The slower moving inequality (20°) may quite possibly have analogies in the stars, but as yet the material is not sufficient to declare.

Researches on the Solar Constant and the Temperature of the Sun. By Dr. J. Scheiner, Assoc. R.A.S.

In No. 55 of the Publications of the Astrophysical Observatory, Potsdam, I have published an extended paper on this subject, and I should like to give a short report of the results to the readers of the *Monthly Notices*.

The measures of the Sun's radiation were made with the Ångström Electric Compensation Pyrheliometer, to which I had given a modified exterior form and a parallactic motion with clockwork. On eleven days in June and July 1903 I made a long series of observations on the top of the Gorner Grat in Canton Wallis (Switzerland), from which I could derive the radiation of the Sun outside our atmosphere. This part of the problem is the most difficult one, and, according to my view, it cannot be solved from measurements of the solar radiation alone. From such observations a portion only of the real solar constant can be obtained, because only that portion of the loss by absorption in our atmosphere can be calculated which is based upon the *continuous* increase of absorption with growing thickness of the atmospheric layer traversed by the radiation. With carbon dioxide and water vapour there exists a nearly sudden absorption in the highest thin layers of the atmosphere, which must be treated as a constant to be added to the

result from the radiation-curves. Therefore this latter result is not the solar constant as generally supposed, and I have chosen for it the term "Strahlungsconstante" or "Constant of Radiation." From my observations on the Gorner Grat it amounts to 1·95–2·02 gr. cal. The remaining constant, which must be added to it for obtaining the Solar Constant, can be found only from experimental researches in the laboratory. To this part of the problem I have devoted much labour in measuring the absorption of carbon dioxide and superheated water vapour with varying depth of layer.

This very complicated research cannot be described in a short abstract, and I must therefore refer to the original paper. The result is that for reducing the Radiation Constant to the Solar Constant there must be added for carbon dioxide 1%, for water vapour 7%, and for the ultra violet absorption 1½%, whence the Solar Constant for the unit of distance is found to be 2·22–2·29 gr. cal., with a probable error of 2%.

The constant of the Stefan law, which is necessary for calculating the effective temperature of the Sun from the Solar Constant, I have ascertained by different methods, and with the same pyrheliometer, thus eliminating the constant error of the apparatus which cannot exceed 1 per cent. The "black radiation" of known temperature was measured from black platinum, rendered incandescent by an electric current, from light flames of different thicknesses, and from the artificial "black body." The latter results were the most exact and the effective temperature of the Sun based upon them was found to be 6196°–6252°.

Further on, I have endeavoured to calculate the real temperature of the solar photosphere from the effective temperature by employing the known data on the absorption of the solar atmosphere. Of course this research cannot be of the same exactness as the foregoing one, especially because the photosphere has no definite temperature, consisting as it does of layers of very different temperature.

Neglecting the errors arising from our ignorance of the structure of the photosphere, its average temperature comes out as 7065°.

The Total Solar Eclipse of 1910 *May* 8.

By A. M. W. Downing, D.Sc., F.R.S.

This eclipse is observable in Tasmania, though not under very favourable conditions, owing to the Sun's low altitude at the time of totality.

The particulars for Hobart are given on page 444 of the *Nautical Almanac* for 1910. It will be noticed that the Sun sets before the ending of the partial phase of the eclipse.

Port Davey, in the south-west of Tasmania, is, however, a more favourable station from which to observe the total phase of this eclipse, the duration of totality there being half a minute longer than the duration at Hobart. Some particulars of the eclipse as seen from Port Davey are given, as an example of eclipse calculations, on page 590 of the *Nautical Almanac* for 1910. For the convenience of observers these are reproduced here, and some additional particulars added.

Port Davey. *Long.* 146° 0′ *E.* *Lat.* 43° 22′ *S.*

Standard Mean Time (10^h *East*).

		d	h	m	s	Angle from North Point.	Vertex.
First contact	May	9	3	4	2	247°	99°
Total Eclipse {		9	4	11	55	52	272
		9	4	15	25	267	127
Sun sets		9	5	8			

With the existing errors of the lunar tables, the predicted times for the phases of the eclipse given above will be several seconds too late. It may be useful, therefore, to add the intervals in time from the instant when the cusps subtend a given angle at the Sun's centre to the commencement of totality.

Angle of Cusps.	Time before commencement of totality.
°	s
90	42
60	16
45	9
30	4
15	1

The Sun's altitude at the time of totality is 8°.

The Perturbations of Halley's Comet in the Past. Fifth Paper. The period B.C. 240 *to* A.D. 760. By P. H. Cowell and A. C. D. Crommelin.

We have once more to commence by acknowledgment of the continued assistance that we have received from Dr. Smart and Mr. Cripps in the calculation of the mechanical quadratures.

The fourth paper of this series (*M.N.*, May 1908) traced the comet back to A.D. 760 June 10 (Julian day 1998810), the corrected value of n at that epoch being $46''\cdot113$. Hind's date for the previous passage is 684 Oct. 18, though the observed positions are too vague to fix the time of perihelion within several days. Mr. Knobel has shown that this comet was also observed in Japan (*M.N.*, lxvi., 2, p. 72) the date of first apparition, 684 Sept. 7, being in good accord with the date Sept. 6 of the Chinese records; he has, however, by a slip, given Hind's date of perihelion passage as Sept. 18 instead of Oct. 18.

Our computations indicate that Hind's identification is correct, the calculated date being 39 days later than his, which is a reasonable discordance for that remote epoch.

Revolution 684–760.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0°–30°	$-''\cdot002$	...	$-''\ 56$
,,	330–360	$-\cdot006$	...	...
Earth	0–30	$+\cdot011$	...	+ 308
,,	330–360	$-\cdot002$	...	...
Jupiter	0–90	$+1\cdot137$	+560	+31720
,,	90–270	$+\cdot148$	−523	+ 636
,,	270–360	$-\cdot983$	+ 48	+ 170
Saturn	0–90	$+\cdot0219$	+ 34	+ 612
,,	90–270	$-\cdot0312$	−129	− 466
,,	270–360	$+\cdot0381$	+ 54	− 7
Uranus	0–360	$+\cdot012$	...	+ 280
	Sums	$+\cdot344$	+ 44	+33197

Taking n in 760 as $46''\cdot113$, n in $684 = 45''\cdot769$, and period in days $= \frac{1296000 - 33197}{45\cdot769} = 27591$, which brings us to J.D. 1971219 $= 684$ Nov. 26.

The observations will scarcely permit so late a day as this for the perihelion passage. If we take the mean of Hind's value (Oct. 18) and our value Nov. 26 as the actual date, we must take n in 684 as $45''\cdot737$.

Proceeding to the revolution before this, an approximate computation quickly showed that Hind's date 608 Oct. 19 was about $1\frac{1}{2}$ year too late. Dr. Ångström had already deduced from his empirical curve that "l'apparition pour l'année 608, présentant l'écart le plus grand, est probablement douteuse." Every alteration to Hind's dates that we have found brings the results into closer accord with Ångström's curve, a fact which renders its failure for the next return more surprising. We propose examining this question further in a subsequent paper.

The observations of 607 are in a decided tangle; it seems clear that they refer to, at least, two different comets (some say that no less than four appeared in this year). Halley's comet seems to have appeared in the spring, and the date March 20 was selected for the purpose of computing the perturbations.

Revolution 607–684.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0°–30°	+″·001	...	+″·28
,,	330–360	−·005	...	...
Earth	0–30	−·007	...	−196
,,	330–360	+·003	...	...
Jupiter	0–90	−·189	−452	−5954
,,	90–270	−·126	...	+4877
,,	270–360	+·491	−157	−122
Saturn	0–90	−·051	−37	−1438
,,	90–270	+·062	...	+2044
,,	270–360	−·185	...	+30
Uranus	0–360	+·006	...	+80
	Sums	−·000	...	−651

Taking n in 684 as 45″·737, n in 607 = 45″·737, and period in days $= \frac{1296000+651}{45{\cdot}737} = 28350$ days. Subtracting this from 1971199 (the adopted Julian day in 684) we obtain J.D. 1942849 = 607 March 26.

At this point we have somewhat varied the former procedure; owing to the difficulty of deciding from observation the exact day of perihelion in 607, we took the well-established return of 451 July 3·5 = J.D. 1885969·5 as a starting-point and assumed in a preliminary computation that the subsequent passage occurred in 530 November, in accordance with Hind. The verification of the date 530 November makes the revolution 451–530 the longest on record; it is about three months longer than 1066–1145 or 1222–1301,

which are very nearly equal in length. The adopted date is 530 November 15 = J.D. 1914959.

On this basis the perturbations were carried forward to 607, and it was found that nearly the same date was arrived at as in the previous backward reckoning. The value obtained for n in 607 differs somewhat, but this is easily accounted for when we note that in the three consecutive returns of 530, 607, 684 the observations are so vague that there is an uncertainty of a fortnight or so in the date of perihelion in each case. It is sufficient to know that our comet is correctly identified in each year, and we must be content with this small uncertainty in the dates.

Perturbations for the two revolutions 451–530, 530–607.

		Revolution 451–530.			Revolution 530–607.		
Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0°– 30°	+″·018	... ″	+ 504″	+″·020	...	+ 560″
,,	330–360	−·013	...	...	−·007	...	...
Earth	0– 30	+·003	...	+ 84	+·014	...	+ 392
,,	330–360	+·003	...	...	−·011	...	...
Jupiter	0– 90	−·2455	−213	−7179	+1·106	+633	+30974
,,	90–270	−·0134	−189	+7852	+·222	−348	+ 1969
,,	270–360	+·6501	−240	− 164	−·734	−295	+ 193
Saturn	0– 90	−·0077	+ 56	− 223	+·196	− 18	+ 5498
,,	90–270	+·0248	−101	+ 49	−·073	− 10	− 237
,,	270–360	−·0047	+ 4	+ 10	+·056	− 20	− 10
Uranus	0–360	−·001	...	+ 17	+·002	...	− 15
	Sums	+·425	−683	+ 950	+·791	− 58	+39324

For the revolution 451–530 we have the equation $\frac{1296000 - 950}{n \text{ at } 451} = 28990$, the observed period in days. Hence n at 451 = 44″·672, n at 530 = 45″·097.

For the revolution 530–607 we have the equation $\frac{1296000 - 39324}{45''·097} =$ period in days $= 27866$. Adding this to J.D. 1914959 we obtain J.D. 1942825 = 607 March 2, and n in 607 = 45″·097 + ″·791 = 45″·888. As the actual date of perihelion in 607 was probably at the end of March, we should have to diminish n in 530, 607 by about 0″·05, which would make a difference of only 0″·09 between the value of n in 607 found by forward reckoning, and that already reached by backward reckoning.

In carrying the research still further back, we have changed our method at this point, and used only the approximate tables given on p. 458 of the present volume, combined with the definite integral table on p. 178. This method is less accurate than that hitherto followed, and we must be prepared for errors of two or

three months in the computed periodic time, corresponding to 0″·15 or 0″·20 in the value of n; but this is near enough to test the accuracy of Hind's dates, which may be taken as presumably right if they fall within two months or so of the time indicated by our method; and, as a matter of fact, the results establish the accuracy of all Hind's dates from A.D. 451 back to B.C. 12, which is a very satisfactory conclusion. In his whole series there were only four errors, viz. A.D. 1223 (eleven months too late), A.D. 912 (four months too early), A.D. 837 (one month too late), A.D. 608 (one and a half years too late).

The research has been carried beyond the limits of Hind's list, and one fairly certain return has been added, that of B.C. 87. The description in Williams is, "In the second year of the epoch How Yuen (*i.e.* B.C. 87), the seventh moon (August), there was a comet in the east." Perihelion would be B.C. 87 August or September. Calculation indicates B.C. 163 June for the preceding passage, but no definite observation can be found in this year; Pingré has several vague references to comets about this epoch, but they are so wanting in precision that no use can be made of them. Going back another round, the date B.C. 239 January was found; we think it not unlikely that the comet observed in the spring of B.C. 240 was Halley's.

The discordance from our date is not greater than we might reasonably expect when the approximate method is used without check for two revolutions, and the characteristics of the comet of B.C. 240 are very like those exhibited by Halley's comet when perihelion falls about April.

Williams says, "In the seventh year of the reign of Che Hwang (B.C. 240) a comet first appeared in the east. It was afterwards seen in the north. In the fifth moon (May) it was seen for 16 days in the west."

For the two preceding passages there are no comets in either Pingré or Williams that could possibly be Halley's. Three revolutions earlier, there is the following in Williams: "In the second year of the Emperor Ching Ting Wang (B.C. 467), a comet was seen." This is at about the time when we should expect Halley's comet, but the identity cannot become more than a vague conjecture.

To sum up, we have carried the comet with fair certainty back to B.C. 87, with some probability back to B.C. 240; at this point we are brought to a standstill by the complete absence of earlier observational material.

The perturbations for the different rounds are given in the following tables. In each revolution $\frac{1296000 - \int d\zeta}{n \text{ at beginning of rev.}} =$ period in days; an equation which gives n when the period is assumed, or which gives the period when the value of n at the end of the period is assumed, and the value at the beginning deduced by applying the calculated perturbation of n.

Dates of beg. and end of Rev.			373 November 7; 451 July 3	295 April 7; 373 November 7	218 April 6; 373 April 7
Values of g' for $u=$ 0°, 90°, 270°, 360°	Jupiter		352°, 218°, 322°, 189° 247, 338, 25, 116	126°, 355°, 123°, 352° 7, 99, 155, 247	309°, 173°, 262°, 126° 146, 236, 277, 7
			$\int dn.$ $\int d\varpi.$ $\int d\zeta.$	$\int dn.$ $\int d\varpi.$ $\int d\zeta.$	$\int dn.$ $\int d\varpi.$ $\int d\zeta.$
Jupiter	0°	90°	+″·770 −290″ +21972″	−″·120 +258″ −3539″	+″·330 −583″ +9369″
,,	90	270	− ·074 −242 − 3697	+·103 −162 +7862	− ·249 −163 −2282
,,	270	360	−1·040 +190 + 168	+·210 −500 − 45	−1·270 +700 − 6
Saturn	0	90	− ·02 − 77 − 567	+·150 − 20 +4330	− ·036 + 55 −1018
,,	90	270	+ ·037 + 50 + 2142	−·059 − 31 − 280	+ ·047 −110 + 608
,,	270	360	− ·110 − 66 + 20	+·060 + 17 − 10	− ·050 + 3 + 10
	Sums		− ·437 −435 +20035	+·344 −438 +8318	−1·228 − 98 +6681

Dates of beg. and end of Rev.			141 March 25, 218 April 6	66 January 26, 141 March 25	B.C. 12 Oct. 8, A.D. 66 Jan. 26	B.C. 87 Aug. 15, B.C. 12 Oct. 8
Values of g' for $u=$ 0°, 90°, 270°, 360°	Jupiter		132°, 355°, 85°, 309° 285, 15, 56, 146	12°, 231°, 274°, 132° 86, 174, 197, 285	217°, 79°, 150°, 12° 234, 324, 357, 86	95°, 314°, 358°, 217° 36, 124, 146, 234
			$\int dn.$ $\int d\varpi.$ $\int d\zeta.$	$\int dn.$ $\int d\varpi.$ $\int d\zeta.$	$\int dn.$ $\int d\varpi.$ $\int d\zeta.$	$\int dn.$ $\int d\varpi.$ $\int d\zeta.$
Jupiter	0°	90°	−″·190 +200″ −5459″	+″·940 −114″ +25968″	−″·217 −373″ −6128″	+″·848 +640″ +23305″
,,	90	270	+·244 − 68 +7788	− ·224 −464 − 3417	− ·018 +246 +6584	+·269 −492 + 3568
,,	270	360	− ·047 −600 + 43	−1·280 +710 + 23	+·328 −447 − 94	− ·853 − 97 + 197
Saturn	0	90	+·055 − 95 +1578	+ ·010 + 16 + 285	− ·036 − 58 −1003	+·058 − 54 + 1606
,,	90	270	− ·005 + 78 +1630	− ·034 −129 − 462	+·063 + 33 +2142	− ·075 − 64 − 596
,,	270	360	− ·038 − 86 + 4	+ ·035 + 50 − 10	− ·192 − 22 + 30	+·060 0 − 10
	Sums		+·019 −571 +5584	− ·573 + 69 +22387	− ·072 −621 +1531	+·307 − 67 +28070

Dates of beg. and end. of Rev.				B.C. 163 May 20; B.C. 87 Aug. 15			B.C. 240 May 15; B.C. 163 May 20		
Values of g' for $u=$ 0°, 90°, 270°, 360°			Jupiter / Saturn	302°, 163°, 233°, 95° / 185°, 274°, 307°, 36°			125°, 349°, 78°, 302° / 323°, 53°, 94°, 184°		
				$\int dn.$	$\int d\varpi.$	$\int d\zeta.$	$\int dn.$	$\int d\varpi.$	$\int d\zeta.$
Jupiter	0°	–	90°	+·260	−607″	+7321″	−·090	+280″	−2596″
,,	90	–	270	−·307	− 63	−1561	+·254	− 89	+ 7326
,,	270	–	360	+·054	+ 15	− 120	−·093	−600	+ 62
Saturn	0	–	·90	−·060	+ 15	−1691	+·163	− 73	+ 4569
,,	90	–	270	+·074	− 66	+1484	−·049	+ 52	+ 704
,,	270	–	360	−·174	+ 16	+ 22	+·018	− 76	− 24
			Sums	−·153	−690	+5455	+.203	−506	+10041

The date B.C. 240 May 15 was deduced from observation, not from calculation. The date calculated is B.C. 239 Jan. If we suppose an error of two months in the same direction in each of the calculations of the last two revolutions (which is quite probable), the discordance would almost disappear. It appears worth while to calculate the three revolutions B.C. 12 to B.C. 240 by more exact methods, and we hope to undertake this at a later date.

The following table gives the values of n calculated from the above results for the different revolutions, and also smoothed values resulting from the combination of values from consecutive revolutions.

Perihelion Passage.		n.	Smoothed n.	Perihelion Passage.		n.	Smoothed n.
		″	″			″	″
			45·742				45·862
A.D.	684	45·737		A.D.	218	45·88	
	607	45·747			141	45·86	45·835
			45·818				
	607	45·888			141	45·817	
	530	45·097			66	46·39	46·397
			45·085				
	530	45·097		A.D.	66	46·398	
	451	44·672		B.C.	12	46·47	46·483
			44·604				
	451	44·54			12	46·497	
	373	44·98			87	46·19	46·185
			45·095				
	373	45·19			87	46·185	
	295	44·845		B.C.	163	46·34	
			44·715				
	295	44·617					
A.D.	218	45·845					

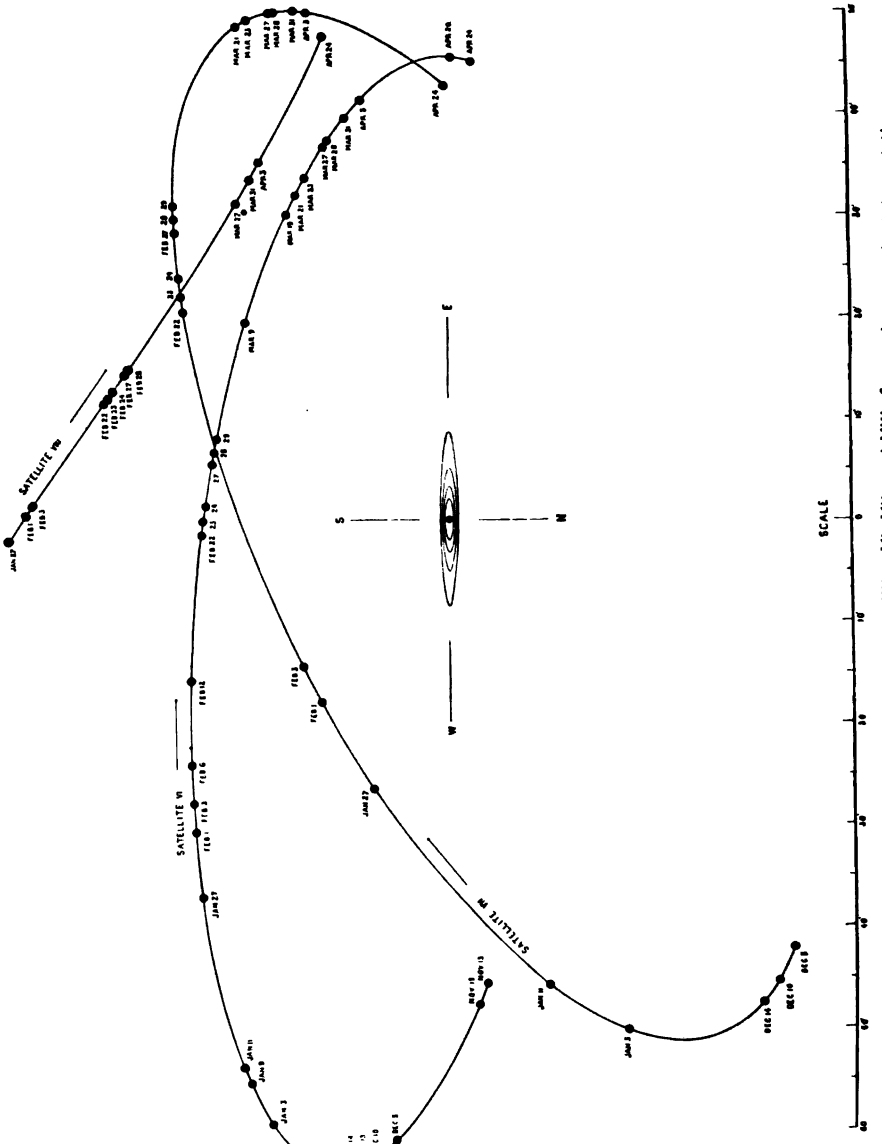

Diagram showing the positions of Jupiter's Satellites VI, VII, and VIII, from photographs taken at the Royal Observatory, Greenwich, during the Opposition of 1907-8.

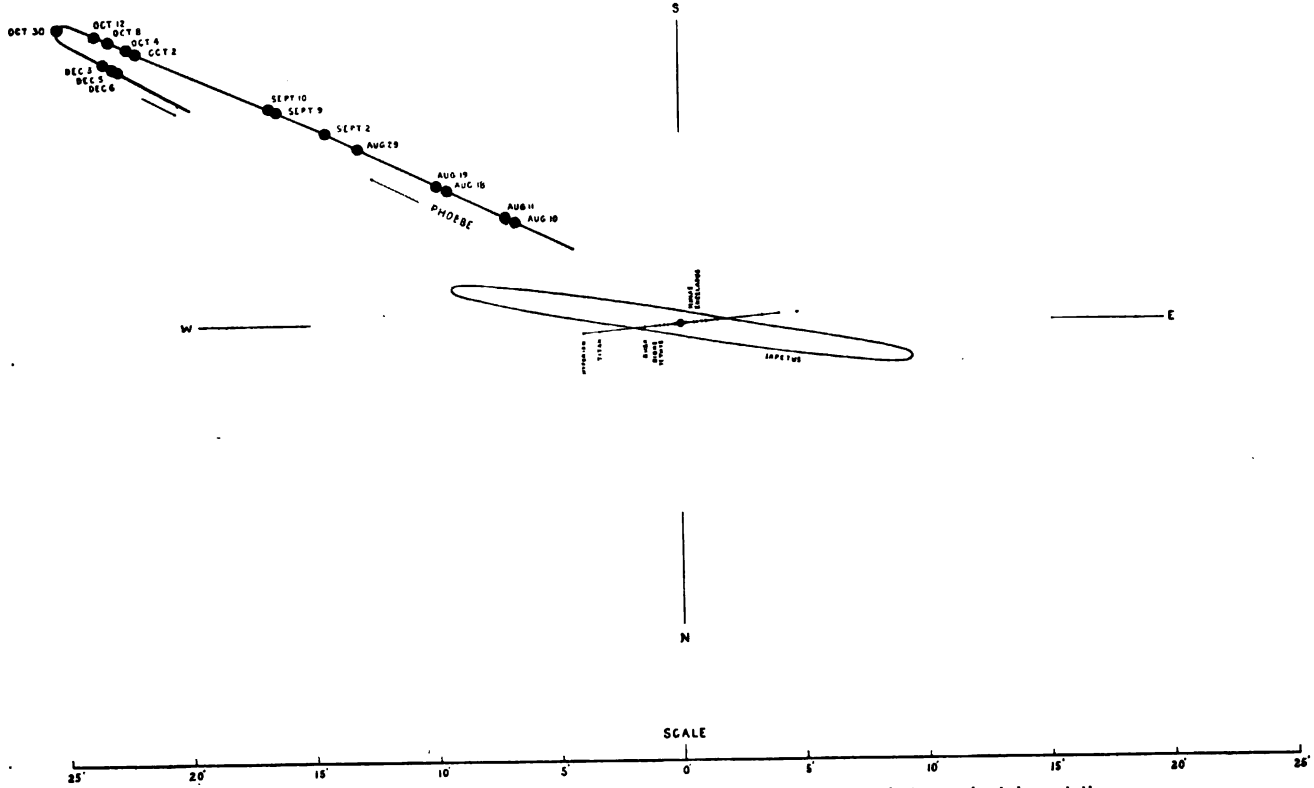

Diagram showing the position of Saturn's Ninth Satellite—Phœbe, from photographs taken at the Royal Observatory, Greenwich, during the Opposition of 1907.

Diagram showing the positions of Jupiter's Satellites VI, VII, and VIII, from photographs taken at the Royal Observatory, Greenwich, during the opposition of 1907–8. (Plate 15.)

(*Communicated by the Astronomer Royal.*)

The diagram is a graphical representation of the observations of Jupiter's Satellites VI, VII, and VIII, from photographs taken at the Royal Observatory, Greenwich, with the 30-inch reflector during the opposition of 1907–8, printed in the *Monthly Notices*, vol. lxviii. p. 582. In all, 38 photographs of Satellite VI, 21 of Satellite VII, and 13 of Satellite VIII have been obtained.

The positions plotted in the diagram are the means when two or more photographs were taken on any night, and the curves are drawn through these points.

For comparison, the orbits of the four inner large satellites are shown, the major axes being plotted to scale, but the minor axes intentionally exaggerated.

The apparent convexity of the path of the eighth Satellite with regard to Jupiter is due to the Earth's motion during the period over which the observations extend (about three months), and the consequent change in the point of view.

Royal Observatory, Greenwich:
1908 *September* 23.

Diagram showing the positions of Saturn's Ninth Satellite—Phœbe, from photographs taken at the Royal Observatory, Greenwich, during the opposition of 1907. (Plate 16.)

(*Communicated by the Astronomer Royal.*

This diagram is a graphical representation of the observations of Saturn's ninth Satellite—Phœbe—from 16 photographs taken at the Royal Observatory, Greenwich, with the 30-inch reflector during the opposition of 1907, printed in the *Monthly Notices*, vol. lxviii. p. 211.

The observed position of Phœbe with respect to Saturn on each day of observation is plotted in the diagram, and the curve drawn through these points.

For comparison, the orbits of the inner satellites are shown plotted to scale.

It may here be noted that during the present opposition (1908) 23 photographs on 15 nights have already been obtained up to the present time, the first being on July 31, some few days before Phœbe reached eastern elongation.

Royal Observatory, Greenwich:
1908 *September* 23.

The Relation between Star Colours and Spectra.

By W. S. Franks.

In a former paper on this subject (*Monthly Notices*, June 1907) I gave the result of an investigation into the relation between the colours and spectra of 1360 bright stars, in both hemispheres; the colours depending on the observations made by members of the British Astronomical Association in England and Australia. The stars observed in this country, from the N. pole to 25° south declination, amounted to 928 of the total number. The recent publication of the *Revised Harvard Photometry*, which contains over 9000 stars down to 6½ magnitude, has enabled me to carry this inquiry a stage further. Making use only of my own colour observations, contained in a MS. synoptic catalogue of 4175 stars (which includes *all* my previous work in this direction), I have carefully compared this with the Harvard revised spectra. The number of stars between the N. pole and −25°, common to both lists, amounts to 3497; which is nearly four times greater than the B.A.A. total, and it includes stars down to 6½ magnitude. As the colour estimates in my earlier catalogues were not expressed by symbols, like the B.A.A. and my later observations, I have used the old notation in this paper—dividing the colours into seven groups, as here shown :—

		Symbol.
1. White	Stars—including bluish-white and greenish-white . .	O
2. Yellowish White	,, —an important and well-marked class . . .	Y^1
3. Pale Yellow	,, —including a few pale orange-yellow . . .	Y^2
4. Yellow	,, —(normal tint) including orange yellow and full yellow	Y^3
5. Pale Orange	,, —a distinctive group	Or^2
6. Orange	,, —(normal tint) including also full orange . .	Or^3
7. Orange Red	,, —including all ruddy tints	OrR^5

The foregoing colours are distributed as follows :—

Colour.	Number.	Percentage.
1. White	1083	31·0
2. Yellowish White	729	20·8
3. Pale Yellow	782	22·4
4. Yellow	413	11·8
5. Pale Orange	271	7·7
6. Orange	195	5·6
7. Orange Red	24	0·7
Total	3497	100·0

Analysis of Colours and Spectra.

TABLE I.

Colour.	*Spectrum Types.*																	
	Od	Oe	Oe5B	B	B1A	B2A	B3A	B4A	B5A	B8A	B9A	A	A2F	A3F	A4F	A5F	A8F	
White	...	...	5	23	9	22	88	...	49	69	21	605	79	18	1	28	2	(continued below.)
Yelsh. Wh.	...	...	4	3	8	10	11	1	17	13	10	254	41	20	...	63	2	
Pale Yell.	1	1	1	...	...	...	1	...	...	3	2	38	12	4	...	13	1	
Yellow	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
Pale Orange	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
Orange	...	...	...	...	...	...	...	...	...	...	...	..	...	...	...	...	...	
Orange Red	...	...	...	...	...	...	...	...	...	...	...	...	...	..	...	...	...	
Total	1	1	10	26	17	32	100	1	66	85	33	897	132	42	1	104	5	

Colour.	*Spectrum Types.*																
	F	F2G	F5G	F7G	F8G	G	G2K	G5K	G8K	K	K2M	K5M	Ma	Mb	Mc	Md	N	Total.
White	44	4	7	...	2	7	...	...	...	...	...	...	...	...	...	...	...	1083
Yelsh. Wh.	134	5	47	1	18	37	3	10	...	17	...	...	...	...	...	...	...	729
Pale Yell.	110	19	51	1	36	102	15	88	...	260	2	10	8	3	...	...	...	782
Yellow	8	1	4	...	6	22	4	35	1	274	7	25	20	6	...	...	...	413
Pale Orange	...	...	2	...	1	13	3	24	...	155	2	27	30	14	...	...	...	271
Orange	...	...	...	...	...	2	2	9	...	84	3	33	45	12	1	1	3	195
Orange Red	...	...	...	...	...	...	...	2	...	4	...	...	6	1	...	3	8	24
Total	296	29	111	2	63	183	27	168	1	794	14	95	109	36	1	4	11	3497

Summary of Analysis.

TABLE II.

Group.	Spectrum Type.	White.	Yelsh. Wh.	Pale Yel.	Yellow.	Pale Or.	Orange.	Orange Red.	Total.
Od and Oe	Bright line spectra	...	...	2	...	...	...	...	2
Oe5B	Intermediate	5	4	1	...	...	...	...	10
B to B5A	Orion type	191	50	1	...	...	...	...	242
B8A ,, A4F	Sirius ,,	793	338	59	...	...	...	...	1190
A5F ,, F2G	α Carinæ ,,	78	204	143	9	...	...	...	434
F5G	Procyon ,,	7	47	51	4	2	...	...	111
F7G to G8K	Capella ,,	9	69	242	68	41	13	2	444
K	Arcturus ,,	...	17	260	274	155	84	4	794
K2M and K5M	Aldebaran ,,	...	...	12	32	29	36	...	109
Ma to Md	Betelgeuse ,,	...	...	11	26	44	59	10	150
N	19 Piscium ,,	...	...	...	...	...	3	8	11
	Total	1083	729	782	413	271	195	24	3497

Percentage of Colours in each Group.

TABLE III.

Group.	White.	Yelsh. Wh.	Pale Yel.	Yellow.	Pale Or.	Orange.	Orange Red.	Total.
Od and Oe	...	...	100·0	...	...	...	...	100·0
Oe5B	50·0	40·0	10·0	...	...	...	..	100·0
B to B5A	78·9	20·7	0·4	...	...	...	...	100·0
B8A ,, A4F	66·6	28·4	5·0	...	...	...	...	100·0
A5F ,, F2G	18·0	47·0	33·0	2·0	...	...	...	100·0
F5G	6·3	42·3	46·0	3·6	1·8	...	...	100·0
F7G to G8K	2·0	15·5	54·5	15·3	9·2	3·0	0·5	100·0
K	...	2·1	32·8	34·5	19·5	10·6	0·5	100·0
K2M and K5M	...	...	11·0	29·4	26·6	33·0	...	100·0
Ma to Md	...	...	7·3	17·3	29·3	39·4	6·7	100·0
N	...	...	...	...	...	27·3	72·7	100·0

Classification of Spectra in Tables II. and III.

Photo. Spectra.				Visual Spectra
Oa to Oe	—Bright line spectra	. . . .	=	V.
Oe5B	—All dark lines (intermediate between O and B)		=	V–I.
B to B5A	—Helium ,, Orion type	B . .	=	I*a*.
B8A to A3F	—Hydrogen ,, Sirius ,,	A . .	=	I*b*.
A5F to F2G	— do. ½ intensity *a* Carinæ ,,	F . .	=	I–II*a*.
F5G	— (Intermediate) Procyon ,,	F5G . .	=	I–II*b*.
F8G to G5K	— Solar lines Capella ,,	G . .	=	II.
K	— "K" line intense Arcturus ,,	K . .	=	II–III*a*.
K2M and K5M	— (Intermediate) Aldebaran ,,	K5M . .	=	II–III*b*.
Ma to Md	— Banded spectra Betelgeuse ,,	Ma . .	=	III.
N	— do. (reversed) 19 Piscium ,,	N (ruddy stars)	=	IV.

The last class (N) would be considerably increased but for the difficulty of photographing objects near the red end of spectrum—hence they fall short of the number that can be observed visually. A number of red and orange stars of 6½ magnitude, contained in my catalogue, are, from this cause, entirely omitted from the Harvard spectra results.

Deductions from preceding Tables.

Spectrum O—	*Maximum*	in white, gradually decreasing to pale yellow.
,, B—	,,	,, ,, (practically 79 per cent.). The *whitest* stars.
,, A—	,,	also ,, but less wh. and more yelsh. wh. than B.
,, F—	,,	,, yelsh. white, but the pale yellow not far behind.
,, F5G—	,,	between yelsh. wh. and pale yel.—the latter slightly in excess.
,, G—	,,	in pale yellow—very pronounced.
,, K—	,,	between pale yel. and yellow—nearly equal.
,, K2M and K5M	,,	pretty evenly distributed between yel., pale or., and orange.
,, M—	,,	in orange, but pale orange not far behind.
,, N—	,,	orange red—very pronounced.

The *sequence* of *colours* corresponding to the *sequence* of the *spectra* is thus unmistakably indicated. But in the A class there is a definite number of pale yellow stars; their occurrence is too frequent and regular to be accidental; they show an increase in frequency in the regions where coloured stars are predominant. Likewise there is a still larger fraction of white stars in the F class—contrary to the normal. This serves to emphasise the fact that in some cases two stars of exactly the same colour visually are of different spectral types—some noteworthy examples of which have come to light during the present investigation. And, conversely, some stars of identical spectra differ very perceptibly in colour—whatever the cause of such discrepancy may be. But perhaps these exceptions, which after all are not very numerous, merely point the rule which governs the relation between colour and spectrum in the great majority of cases. If I have succeeded in establishing this connection, it may be also a plea that star colour observations *do* possess some value; and I shall consider that these thousands of independent estimates, which have so long lain dormant, were not altogether made in vain.

Uxbridge: 1908 *August* 12.

Note on Y Cephei. By Rev. J. G. Hagen, S.J.

(Communicated in a letter to the Astronomer Royal.)

"The advance prints of your Astrographic Plates 3619 and 6983, which you kindly sent me for the examination of Y Cephei and its surroundings, have enabled me to measure the additional stars to the Atlas Chart of this variable which were invisible in the 12-inch refractor. They will be recorded in the Catalogue sheet No. 188 of series vi., with due acknowledgment.

"Since the magnitudes of this series are all reduced to the Harvard scale, it is well to put the magnitudes of Y Cephei, as shown on your plates, on record for future corrections of the elements of variation. The following extract from the Catalogue will designate four comparison stars:—

No. 3.	$\Delta\alpha = -0^m\,20^s$	$\Delta\delta = -11'{\cdot}5$	H.P. $8^m{\cdot}7$	letter c
4.	,, $+3\,20$	,, $-12'{\cdot}1$	,, $8^m{\cdot}8$	,, d
37.	,, $0\,0$	,, $-1'{\cdot}5$	,, $12^m{\cdot}7$	,, ϕ
40.	,, $+0\,30$	,, $-1'{\cdot}0$	,, $13^m{\cdot}1$	,, τ

Your plates give the following estimates, which anyone can verify:—

Pl. 3619,	1897 Sept. 9:	c2Y4d,	Magn. $8^m{\cdot}7$ H.P.
,, 6983,	1908 Jan. 12:	ϕ4Y1τ,	,, $13^m{\cdot}0$,,

"The provisional elements of variation give the following corresponding epochs of maxima:—

Max. 1897 Sept. 24 = Plate 3619 + 15 days.

,, 1907 Nov. 26 = ,, 6983 − 47 days.

"The first plate agrees well with the computed maximum and with the remark of Ceraski (*A.N.* 3644) that the brightness of Y was diminishing in October 1897. The second plate, however, will probably demand a correction to the assumed elements (*A.N.* 3744 and *V.J.S.* 41, page 317), as it is unlikely that the star should have diminished in brightness by $4\frac{1}{2}$ magnitudes in 47 days."

Rome: July 22, 1908.

Corrections to Professor Turner's paper on Correlation,—
(*M.N.*, *p.* 551).

An unfortunate slip on p. 551 has been kindly pointed out by Mr. J. B. Dale.

Both σ_A and σ_B should have been divided by 3, the square root of the number of observations; so that

$$\sigma_A = 1{\cdot}15 \qquad \sigma_B = 0{\cdot}82$$

and

$$r = {\cdot}72 \text{ } \textit{instead of} \text{ } {\cdot}08.$$

The correlation between A and B is therefore by no means negligible as stated on p. 552, and again on p. 545.

The two "regression lines" are

$$(A-7) = r.\frac{\sigma_A}{\sigma_B}(B-7) \quad \text{and} \quad (B-7) = r.\frac{\sigma_B}{\sigma_A}(A-7)$$

or

$$(A-7) = 2(B-7) \quad \text{and} \quad (B-7) = (A-7)$$

which are the two solutions mentioned on p. 545 obtained by grouping according to B and to A respectively.

The example was subsidiary, and the slip does not affect the main argument; but it is not easy to understand how so gross an error passed undetected. H. H. T.

Errata in Mr. J. M. Baldwin's paper on Neptune.

p. 618, line 26, *for* include *read* exclude.

p. 620, line 4, *for* showing in *read* showing an.

LIST OF ADDITIONS

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An asterisk (*) indicates that the work is an excerpt.

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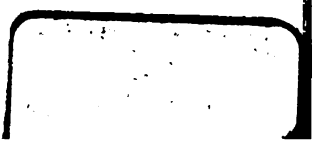
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